2.28; from IV it is 3.57. This difference is largely due to a marked decrease in the slope of the N–S curve, which, however, leaves its shape unchanged. If we consider the later curves more reliable, the discrepancy between 3.57 and 2.08 would make the continuous spectrum theory untenable. If the difference is to be attributed to the penumbra, which would raise the northward intensity and lower the asymmetry ratio, then the negative primaries must be included, and the comparison becomes 3.57 to 0.91. It seems very doubtful that the presence of the penumbra could lower the experimental value of the N–S asymmetry by a factor of four. Hence in either case it is difficult to accept the continuous spectrum as determining the conditions at the zenith.

Having thus determined the existence of a bright line in the primary energy spectrum at Mexico City, the next step is the attempt to identify it with one of those found by Millikan, Neher, and Pickering in II. The nearest line found by them is what they call the oxygen-annihilation band, which sets in sharply at Victoria, $4\frac{1}{4}°$ north of Mexico City, but is not observed at all at Valles, only $2\frac{1}{2}°$ north. The intensities seem to depend greatly on method of measurement and depth (considerably greater in II), and afford little basis for comparison. But the theory[4, 9] gives clear indications of the relation between the latitude and zenith angle effects in these latitudes. The vertical direction would correspond to a rigidity difference of 0.040 stoermer between Victoria and Mexico City, and this would correspond to a zenith angle of some 40° at Mexico City. This is not very close to the observed value of 12°.

Another difficulty is with the sharpness of the edge observed. If the solid angle covered by the balloon telescopes is $25°\times45°$ ($\frac{1}{2}$ effective), as in I, then at Valles, well north of Mexico City, and where the present peaks must occur at smaller zenith angle, considerable intensity should be observed. This is not found. It will be noted that these discrepancies are of the sort that can largely be straightened out by further studies of the directional effects at different latitudes and atmospheric depths. Such studies clearly would reveal a great deal about the details of the different atom-annihilation bands.

[9] G. Lemaitre and M. S. Vallarta, Phys. Rev. **49**, 719 (1936).

PHYSICAL REVIEW VOLUME 66, NUMBERS 9 AND 10 NOVEMBER 1 AND 15, 1944

Cascade Showers in Lead

WAYNE E. HAZEN
Department of Physics, University of California, Berkeley, California
(Received September 9, 1944)

Cascade showers of energies up to 500–1000 Mev have been photographed in a cloud chamber that contained eight lead plates. A comparison with theory is made in terms of the size of a shower at its maximum as a function of the total number of particles in the shower. The conclusion reached is that the theoretical cross sections for radiation and pair production are essentially correct for energies up to 500–1000 Mev.

VERY few experiments have been performed that allow an experimental verification of shower theory for monoenergetic electrons. Anderson[1] and others have measured the energy loss of electrons in solid plates as a function of incident energy. Recent developments[2] indicate that an interpretation of atmospheric absorption curves in terms of shower theory is considerably obscured by the additional difficulties of separating shower phenomena from mesotron production, etc. Other experiments[3] have involved the average effects of electrons (and mesotrons) of all energies. On the other hand, detailed theo-

[1] S. H. Neddermeyer and C. D. Anderson, Phys. Rev. **51**, 884 (1937); P. M. S. Blackett, Proc. Roy. Soc. **A164**, 257 (1938); C. D. Anderson, Phys. Rev. **57**, 357 (1940).
[2] M. Schein, W. P. Jesse, and E. O. Wollan, Phys. Rev. **57**, 847 (1940).
[3] M. A. Starr, Phys. Rev. **53**, 960 (1938); see, e.g., D. K. Froman and J. C. Stearns, Rev. Mod. Phys. **10**, 133 (1938).

retical results accurate to perhaps 10 or 20 percent are now available.[4–6]

It is possible to observe the histories, at as many as eight stages, of showers initiated by electrons of energies less than 500 Mev, by placing eight 0.7-cm lead plates in a Wilson cloud chamber.[7] Random photographs with such a cloud chamber have been examined for showers whose entire histories appear to have been recorded. Fifty such showers were found, and the detailed history of one of the best is shown in Fig. 1. Since the total number of showers was rather small, there were no reasonably narrow energy groups containing more than four or five showers. Statistical fluctuations, as predicted,[4] are so large that it is not especially instructive

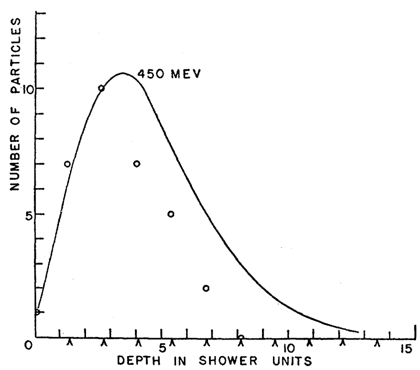

FIG. 1. The complete history of a cascade shower initiated by an electron of energy ~500 Mev. The curve is from the calculations of Bhabha and Chakrabarty (reference 5). The experimental points should be shifted slightly to the right since the electron had passed through several millimeters of wood and glass before reaching the lead plates. The arrows on the abscissa scale indicate depth in terms of lead plates.

to compare individual histories with theoretical curves. This conclusion is particularly true in the present experiment where the original energy can be estimated only from the total number of particles in a shower. When more data are available, it will be possible to compare the average histories of showers in selected energy groups with the theoretical curves.

The best procedure that the author has devised for comparing the experimental results with

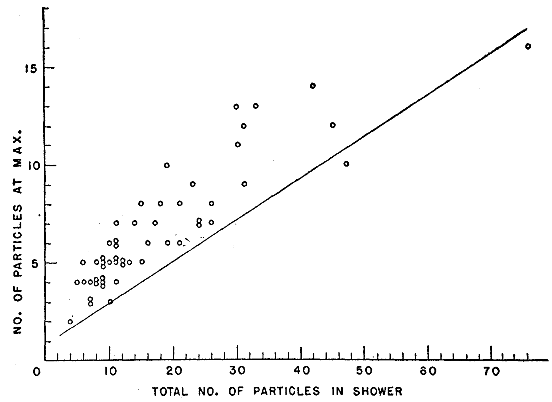

FIG. 2. Experimental points and theoretical curve for the number of particles at the maximum of a cascade shower in lead *versus* total number of particles in the shower.

theory is to plot the number of particles at the shower maximum against total number of particles in the shower. By total number of particles in the shower we here mean the sum total of all the tracks belonging to a particular shower under all of the lead plates. The observed showers are plotted in this manner on the graph of Fig. 2. The theoretical curve of Fig. 2 was obtained from the shower curves of Fig. 3 by plotting the height of a curve at its maximum as a function of the

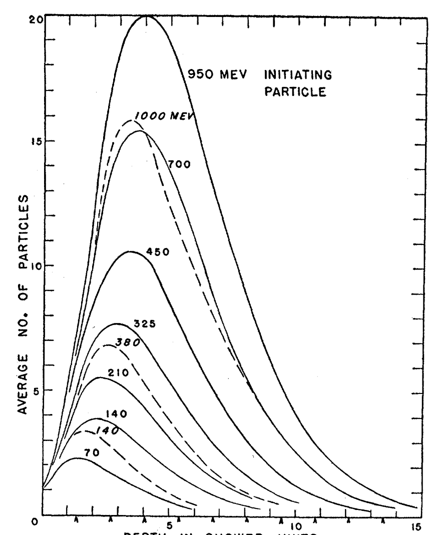

FIG. 3. Theoretical curves for cascade showers in lead. The arrows on the abscissa scale indicate depths in terms of the lead plates used in the experiment. The solid curves were obtained from Rossi and Greisen (reference 4) and the dotted curves from Bhabha and Chakrabarty (reference 5).

[4] B. Rossi and K. Greisen, Rev. Mod. Phys. **13**, 240 (1941).
[5] H. J. Bhabha and S. K. Chakrabarty, Proc. Ind. Acad. Sci. **A15**, 464 (1942).
[6] H. J. Bhabha and S. K. Chakrabarty, Proc. Roy. Soc. **A181**, 267 (1943).
[7] W. E. Hazen, Phys. Rev. **65**, 67 (1944).

sum of the numbers of particles at depth intervals corresponding to the lead plates. The solid curves of Fig. 3 were obtained from Eq. (2.104) of Rossi and Greisen[4] and the dotted curves from the more accurate calculations of Bhabha and Chakrabarty.[5] A comparison of the two sets of curves shows that the more accurate calculations give essentially the same curves as the earlier results, except for the energies of the initiating particles. Since a point on Fig. 2 is determined solely by the shape of a curve in Fig. 3 and not by the initiating energy as such, there is no way in the present analysis to distinguish between the two sets of curves in Fig. 3.

It is seen that almost all of the experimental points lie above the theoretical curve. This result is not entirely unexpected since most of the uncertainties give rise to errors in this direction. In applying the criterion that the entire history of a shower must be observed, it is difficult to eliminate cases in which a few of the divergent particles might have passed out of the illuminated region. Since the lateral spread and hence the likelihood of missing a few particles increases with depth, the total number of observed particles is more likely to be affected than the number of particles observed at the maximum. It is believed that this error is not serious inasmuch as the illuminated volume was large and the change in illumination at the edge of the beam was not abrupt. Furthermore, the lead sheets were covered with steel reflectors of one mm thickness. Iron has a lower atomic number than lead, consequently the transition to iron, which precedes the transition to air, reduces the number of particles. However, the transition thickness is roughly one shower unit[6] whereas one mm of iron is equal to only 1/18 shower unit, and consequently the lead-air transition is not seriously modified.

Let us next consider the approximations in the theoretical calculations. It was assumed that the ionization loss is a constant, independent of energy. This assumption has little effect in the region of high energies since the ionization increases only logarithmically, but it gives a significantly greater range for the low energy particles. The error can be roughly estimated as follows: Assume that multiplication continues until all of the electrons are slowed to the critical energy and then ionization loss predominates. Since the critical energy in lead is seven Mev and the last one Mev of energy is dissipated in a very short distance, the average path of an electron should be shortened by perhaps 14 percent, i.e., the predicted total number of particles in a shower might be too large by 14 percent, whereas the number at the maximum would not be appreciably affected since the average energy is still high at the shower maximum. Scattering also results in a marked reduction of the longitudinal component of the range of the low energy electrons. The calculations of Bhabha and Chakrabarty make use of approximate multiplication cross sections[6] that are too large for all except extremely high energies. This approximation, for the most part, would simply change the scale of a shower curve without marked alteration of the shape of the curve if it were not that the approximation becomes progressively worse for lower energies; this fact leads to a relative increase in height of the tail of a shower curve, where the average particle and photon energies become smaller.

All of the predicted errors, both experimental and theoretical, are in a direction such as to account for the fact that the experimental shower curves are narrower (i.e., fall off more rapidly) than the theoretical curves, and the estimated magnitudes of the errors are sufficient to account for the discrepancy. Therefore it can be concluded that the present experiments, in which detailed observations of the histories of small showers were made, indicate the essential correctness of the radiation and pair production cross sections for energies up to 500–1000 Mev.