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Statistical Mechanics at Extremely High Temperatures

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Equilibrium conditions between elementary particles and nuclei at temperatures $T \gtrsim 10^9$ degrees are studied. Three temperature intervals below the upper limit $\sim 10^{12}$ degrees [$kT \sim 10^8$ ev] are considered. It is found that because of the behavior of some high energy particles, known experimentally from observations on cosmic rays and nuclei, some limitations arise to the validity of the laws of quantum statistics, in accordance with the idea of the existence of a supplementary indeterminacy for high energy particles and a lower limit for measurable lengths. Some astrophysical aspects of the phenomena of pair production and of the gravitational effect of light particles are discussed. The results concerning pair production at temperatures $\sim 10^9$ degrees are summarized in Table I. General formulae for thermal equilibrium between nuclei and light particles are given.

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IN some previous papers[1] the statistical phenomena taking place at extremely high temperatures were considered. The purpose of the present research is to give a fuller account of some of the results obtained. Although the idea of the supplementary indeterminacy in the high energy regions served as a guiding principle in all this research, we tried to base our conclusions mainly on the known experiments on cosmic rays and nuclei, and on the obvious irreversibility of some of the processes in which particles are created.

We shall distinguish three temperature intervals below the upper limit $T_u \sim k^{-1}\, 137\, mc^2$.

1. Temperatures well below the critical temperature $T_0 = k^{-1}mc^2$, e.g., $T < k^{-1}mc^2/10$, for which the usual quantum statistical laws hold.

2. Temperatures $\frac{1}{5}k^{-1}mc^2 \lesssim T \lesssim T_0$, characterized by the appearance of positrons and electrons produced by thermal photons, at which an approximate treatment of the assembly, by means of elementary statistical formulae, is still possible. In this temperature interval it is possible to neglect, as a first approximation, the production of few high energy particles with $E > 10^8$ ev. However, the increase of density, thermal capacity, and pressure owing to the created electron pairs must be taken into account.

3. Temperatures $T_0 < T < 137\, T_0$ which can be introduced only for assemblages of heavy particles, e.g., for nuclei.

4. Conditions [$kT \gg 10^8$ ev] at which the average amount of energy per particle is so great that the majority of the processes become irreversible and no equilibrium is possible. The lack of the equilibrium is due to the simultaneous or successive creation of many particles (especially of unstable mesotrons) and to the processes involving emission of neutrinos, in which an appreciable fraction of the energy and momentum escapes all observation.

We shall not discuss here the problem of relativistic invariance examined briefly in a preceding paper.[2]

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Let us consider the equilibrium between photons, electrons, positrons, neutrons, protons, and nuclei at $T \lesssim T_0$. Let N_s, n_{es}, n_{ps}, n_{Ns}, n_{Hs}, $\overset{A}{n}_{Zs}$ be the numbers (per cm³) of photons, electrons, positrons, neutrons, protons, and nuclei of charge Z_e and atomic weight A, which belong to the momentum-interval $p_s\ p_s + dp_s$. We assume the validity of the laws of conservation of charge:

$$\sum n_{Hs} + \sum Z\overset{A}{n}_{Zs} + \sum n_{ps} - \sum n_{es} = n = \text{const.} \quad (1)$$

conservation of energy:

$$\sum_s [N_s h\nu_s + n_{es}E_{es} + n_{ps}E_{ps} + n_{Hs}E_{Hs} + \overset{A}{n}_{Zs}\overset{A}{E}_{Zs}] + \mathcal{E} = E = \text{const.} \quad (2)$$

[where $\mathcal{E}$ indicates the interaction energy, $E_{es} = mc^2(1-\beta^2)^{-\frac{1}{2}}$ is the *total* energy of an

[1] G. Wataghin, Phil. Mag. **17**, 910 (1934); Comptes rendus **203**, 909 (1935); Phys. Rev. **63**, 137 (1943); **64**, 248 (1943); **65**, 205 (1944).

[2] G. Wataghin, Phys. Rev. **65**, 205 (1944).

electron, etc.], and of conservation of the total number of neutrons and protons:

$$\sum_s[n_{Ns}+n_{Hs}+A\overset{A}{n}_{Zs}]=N=\text{const.} \qquad (3)$$

We assume also that it is possible to specify the states of weakly interacting particles in the usual way, namely, by means of eigenstates having appropriate symmetry. Then, indicating with

$$g_s=8\pi Vh^{-3}p_s^2dp_s=8\pi Vh^{-3}c^{-3}(E_s^2-m^2c^4)^{\frac{1}{2}}E_sdE_s$$

the number of quantum states or eigenvalues of the energy belonging to the sth momentum-interval, we obtain, in the usual way, the following expression for the number of different complexions:

$$W=\Pi\frac{(g_s+N_s-1)!}{(g_s-1)!N_s!}\frac{g_s!}{n_{es}!(g_s-n_{es})!}\cdots. \qquad (4)$$

Neglecting the interaction $\mathcal{E}$, and calculating the maximum of

$$\log W-\alpha n-\beta E-\gamma N,$$

we have, in the usual way:

$$\frac{\partial \log W}{\partial n_{es}}=\log\frac{g_s-n_{es}}{n_{es}}+\alpha-\beta E_{es}=0, \text{ etc.,}$$

and

$$N_s=\frac{g_s}{\exp(\beta h\nu_s)-1}, \quad n_{es}=\frac{g_s}{\exp(-\alpha+\beta E_{es})+1},$$

$$n_{ps}=\frac{g_s}{\exp(\alpha+\beta E_{ps})+1},$$

$$n_{Hs}=\frac{g_s}{\exp(\alpha+\gamma+\beta E_{Hs})+1}, \qquad (5)$$

$$n_{Ns}=\frac{g_s}{\exp(\gamma+\beta E_{Ns})+1},$$

$$\overset{A}{n}_{Zs}=\frac{g_s}{\exp(Z\alpha+A\gamma+\beta E^A_{Zs})\pm 1}.$$

The constant β is $=1/kT$, at least within the limits of applicability of the usual concept of thermodynamical temperature; the other constants α, γ can be determined when we introduce (5) in (1) and (3). In order to take approximately into account the existence of a supplementary indeterminacy in the region of high energy and momenta, the author suggested a more general statistics in which the number g_s of quantum states is: $g_s=8\pi Vh^{-3}G(p_s)p_s^2dp_s$, where $G(p_s)$ is a cut-off factor, which decreases more rapidly than p_s^{-3} for values of $p_s>p_u$ $=137\ mc$ [e.g., $G(p_s)\sim\exp(-p_s^2/p_u^2)$].

For temperatures $T<k^{-1}mc^2$, the following approximation is valid: We can treat the protons, neutrons, and nuclei by the usual statistical methods, neglect the β-ray processes and the concentration of the mesotrons, put $G(p_s)=1$ and $V=1$, and, denoting by n^+, n^- the numbers of positrons and electrons per cm^3, assume:

$$\sum n_{es}-\sum n_{ps}=n^--n^+ =N^+\sim\frac{1}{2}\frac{\rho}{m_H}\sim 3\times 10^{23}\rho_n, \qquad (1')$$

where N^+ is the total number of protons per cm^3 and ρ_n is the density of the nuclear matter (in g cm^{-3}) calculated with exclusion of the masses of the electron pairs.

Indicating with $x_0=mc^2/kT$, $x=E_s/kT$, where $E_s=c(p_s^2+m^2c^2)^{\frac{1}{2}}$ is the total energy of the electron, and putting in (5)

$$g_s=8\pi h^{-3}c^{-3}(kT)^3(x^2-x_0^2)^{\frac{1}{2}}xdx,$$

we obtain easily the following formulae:

$$n^-=\frac{8\pi}{h^3c^3}(kT)^3\int_{x_0}^{\infty}\frac{(x^2-x_0^2)^{\frac{1}{2}}xdx}{e^{-\alpha+x}+1}, \qquad (6)$$

$$n^+=\frac{8\pi}{h^3c^3}(kT)^3\int_{x_0}^{\infty}\frac{(x^2-x_0^2)^{\frac{1}{2}}xdx}{e^{\alpha+x}+1}. \qquad (6')$$

Substituting (6), (6′), and (1′) and remembering that:

$$(e^{-\alpha+x}+1)^{-1}-(e^{\alpha+x}+1)^{-1}=\frac{\sinh\alpha}{\cosh x+\cosh\alpha},$$

we can calculate α from:

$$n^--n^+=8\pi\left(\frac{mc}{h}\right)^3 x_0^{-3}\sinh\alpha \int_{x_0}^{\infty}\frac{(x^2-x_0^2)^{\frac{1}{2}}xdx}{\cosh x+\cosh\alpha}, \qquad (7)$$

or from:

$$\sinh\alpha\int_{x_0}^{\infty}\frac{(x^2-x_0^2)^{\frac{1}{2}}xdx}{\cosh x+\cosh\alpha}=1.712\times 10^{-7}\rho_n x_0^3. \qquad (7')$$

For values of $x_0<1$ and $\alpha\ll 1$, Eq. (7′) becomes:

$$3.29\alpha+0.33\alpha^3+\cdots=1.7\times 10^{-7}\rho_n x_0^3 \qquad (8)$$

or

$$\alpha\sim 5.2\times 10^{-8}\rho_n x_0^3.$$

TABLE I. Compilation of value calculated for various temperatures.

T	$x_\infty(mc^2/kT)$	ρ_n(g/cm³)	α	$\rho_p \sim [n^- + n^+]m$	Thermal capacity of electrons per cm³ $[C_T]$ pair	Thermal capacity of radiation per cm³ $\alpha U/\alpha T = 4aT^3$	Radiation density and pressure $U = 3p_r$ (erg/cm³)	Density of energy of electrons pairs $E_e^+ + E_e^-$ (erg/cm³)
1.2×10^6	5000	10^{-6}	4983.0					
1.2×10^7	500	10^{-6}	479.8					
1.2×10^7	500	10^2	498.2					
1.2×10^7	500	10^4	506.8*					
1.2×10^8	50	10^{-6}	23.0					
1.2×10^8	50	10^2	44.6					
1.2×10^8	50	10^7	93.0*					
5.9×10^8	10	10^{-6}	4.8×10^{-8}	5.8×10^{-3}	5.0×10^{10}	6.2×10^{12}	9.3×10^{20}	7.2×10^{18}
5.9×10^8	10	10^2	2.3	5.8×10^{-3}	5.0×10^{10}	6.2×10^{12}	9.3×10^{20}	7.2×10^{18}
1.2×10^9	5	10^{-6}	8.7×10^{-11}	3.1	1.3×10^{13}	5.0×10^{13}	1.5×10^{22}	3.7×10^{21}
1.2×10^9	5	1	8.7×10^{-5}	3.1	1.3×10^{13}	5.0×10^{13}	1.5×10^{22}	3.7×10^{21}
1.2×10^9	5	10^2	8.7×10^{-3}	3.1	1.3×10^{13}	5.0×10^{13}	1.5×10^{22}	3.7×10^{21}
3.0×10^9	2	10^{-6}	6.2×10^{-13}	550.0	8.0×10^{14}	8.0×10^{14}	5.8×10^{23}	5.9×10^{23}
3.0×10^9	2	10^2	6.2×10^{-5}	550.0	8.0×10^{14}	8.0×10^{14}	5.8×10^{23}	5.9×10^{23}
3.0×10^9	2	10^5	6.2×10^{-2}	550.0	8.0×10^{14}	8.0×10^{14}	5.8×10^{23}	5.9×10^{23}
1.2×10^{10}	0.5	10^{-6}	6.5×10^{-15}	4.2×10^4	9.0×10^{16}	5.0×10^{16}	1.5×10^{26}	2.5×10^{26}
1.2×10^{10}	0.5	10^2	6.5×10^{-7}	4.2×10^4	9.0×10^{16}	5.0×10^{16}	1.5×10^{26}	2.5×10^{26}
1.2×10^{10}	0.5	10^5	6.5×10^{-4}	4.2×10^4	9.0×10^{16}	5.0×10^{16}	1.5×10^{26}	2.5×10^{26}
1.2×10^{10}	0.5	3×10^7	1.9×10^{-1}	4.2×10^4	9.0×10^{16}	5.0×10^{16}	1.5×10^{26}	2.5×10^{26}
1.2×10^{11}	0.05	10^5	$[6.5\times10^{-7}]$	$[4.2\times10^7]$	$[9.0\times10^{19}]$	$[5.0\times10^{19}]$	$[1.5\times10^{30}]$	$[2.6\times10^{30}]$
1.2×10^{11}	0.05	3×10^7	$[1.9\times10^{-4}]$	$[4.2\times10^7]$	$[9.0\times10^{19}]$	$[5.0\times10^{19}]$	$[1.5\times10^{30}]$	$[2.6\times10^{30}]$
1.2×10^{12}	0.005	3×10^7	$[1.9\times10^{-7}]$	$[4.2\times10^{10}]$	$[9.0\times10^{22}]$	$[5.0\times10^{22}]$	$[1.5\times10^{34}]$	$[2.6\times10^{34}]$

* Degeneracy.

In Table I some values of α, for different temperatures and densities, are given.

The numerical calculations were made on the assumption $N^+ = \frac{1}{2}\rho_n/m_H$, that is, by assuming matter contains an equal number of protons and neutrons. In the case when the stellar matter contains a large fraction of hydrogen atoms, one has $N^+ \sim \rho_n/m_H$ and one obtains for the coefficient in (7′) a value 3.4×10^{-7}. The corresponding values of α in (8) become nearly twice as great.

From (6) we have:

$$\frac{n^- - n^+}{n^- + n^+} = \frac{e^\alpha - e^{-\alpha}}{e^\alpha + e^{-\alpha}} \times \frac{\displaystyle\int_{x_0}^\infty \frac{(x^2 - x_0^2)^{\frac{1}{2}} x dx}{e^x + e^{-x} + e^\alpha + e^{-\alpha}}}{\displaystyle\int_{x_0}^\infty \frac{(x^2 - x_0^2)^{\frac{1}{2}} x dx}{e^x + e^{-x} + e^\alpha + e^{-\alpha}} \left[1 + \frac{e^x}{e^\alpha + e^{-\alpha}}\right]} \sim \frac{e^\alpha - e^{-\alpha}}{e^\alpha + e^{-\alpha}}. \tag{9}$$

Observing that the factor $[1 + (e^{-x}/e^\alpha + e^{-\alpha})]$ assumes values between 1 and 2, one has:

$$\frac{n^- - n^+}{n^- + n^+} \sim \text{tgh}\,\alpha \quad \text{or} \quad \frac{n^+}{n^-} \sim e^{-2\alpha}. \tag{10}$$

If $\alpha \ll 1$ the total number of pairs is approximately given by

$$1/2(n^- + n^+) \sim 1/2\alpha\, N^+.$$

In the case when $\alpha < 1$, $x_0 < 10$ the density of the electron pairs ρ_{pair} can assume values comparable to or greater than ρ_n (see Table I). These values can be calculated from (1′), (8), and (10):

$$\rho_{\text{pair}} = (n^- + n^+)m \sim \alpha^{-1} N^+ m \sim 6\times10^3 x_0^{-3}, \quad n^- \sim n^+ \sim 3\times10^{30} x_0^{-3}. \tag{11}$$

The question of degeneracy can be treated in a manner similar to that of the Fermi statistics. We note that the graphics of the two characteristic functions $(e^{x\mp\alpha}+1)^{-1}$ can be obtained one from the other by means of a translation of modulus 2α parallel to the x axis (or by changing α in $-\alpha$) and that only values of $x \geqslant x_0 > 0$ enter into consideration. Then from (6) follows that no degeneracy can occur for the positron gas because the values of $x \geqslant x_0$ correspond always to the quasi-maxwellian region of the distribution curve.

In the low temperature region (large values of x_0) the negatrons are in a degenerated state when $\alpha > x_0$. Introducing into (6′) the critical value of α: $\alpha = x_0 \gg 1$, we obtain in the non-relativistic approximation:

$$n^- \sim 8\pi \left(\frac{mc}{h}\right)^3 x_0^{-3} (2x_0)^{\frac{1}{2}} x_0 \int_0^\infty \frac{\xi^{\frac{1}{2}} d\xi}{e^\xi + 1} \tag{12}$$

or

$$n^- \left(\frac{h}{4\pi mc}\right)^3 \sim x_0^{-\frac{3}{2}}. \tag{12'}$$

One can easily verify that this relation coincides with the usual Fermi criterion for degeneracy.

At extremely high temperatures, $x_0 \ll 1$, the values of $\alpha \gtrsim x_0$ correspond to values of

$$\rho_n = 2N^+ m_H \sim 2\alpha(n^- + n^+) m_H \gtrsim x_0^{-2} m_H \Lambda_0^{-3} > \rho_{cr},$$

where $\Lambda_0 = h/2\pi mc$ and $\rho_{cr} = m_H/\Lambda_0^3 \sim 2.9 \times 10^7$ g cm^{-3} is the critical value of the nuclear density. Indeed, the approximate value, for $x_0 \to 0$, of $n^- \sim n^+$, is:

$$n^- \sim n^+ \sim 8\pi \left(\frac{mc}{h}\right)^3 x_0^{-3} \int_0^\infty \frac{x^2 dx}{e^x + 1}$$
$$\sim (\Lambda_0 x_0)^{-3} 1.803/\pi^2 \quad [\Lambda_0^{-3} = 1.736 \times 10^{31}].$$

For such values of ρ_n the average distances between the nuclei (and also between the electrons) become $< \Lambda_0$, and the assumptions of the classical or the quantum electrodynamics, on which the preceding calculations are based, are no longer valid. Thus we exclude such cases from our consideration. At $x_0 < 1$ degeneracy of negatrons does not occur at densities $\rho_n < \rho_{cr}$.

The last columns and lines in Table I are reproduced in order to illustrate some interesting features of pair production and radiation processes in regions where we cannot be sure of the validity of the preceding simple statistical calculations.

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Obviously we must expect the failure of the preceding simple calculations at temperatures and densities above certain limits for several reasons. Let us examine some of them.

(1) We neglected the nuclear reactions, which can give rise to β-ray processes connected with the emission of neutrinos. In these processes the laws of conservation of energy and momentum are out of our direct experimental control as long as we have no means to measure the energy of the neutrinos or at least to observe them. In some cases, when we can neglect β-ray processes, an approximate treatment of nuclear reactions is possible on the basis of the formulae (5). One can calculate the equilibrium concentration of the protons, neutrons, and nuclei determining the constants α, β, γ from (1), (2), (3). The calculation of γ leads to especially interesting results, which we shall mention briefly later.

(2) At nuclear densities $\rho_n \gtrsim \rho_{cr} \sim 3 \times 10^7$ g cm^{-3} the average distance between the nuclei becomes of the order of the Compton wave-length. At $T \gtrsim 10^{11}$ degrees $[x_0 \lesssim 0.05]$ the density of the electron pairs $n^- + n^+$ becomes $\gtrsim 2 \times 10^{31} \sim \Lambda_0^{-3}$. In both cases the linear equations of Maxwell are no longer valid. The field strengths become of the order of or greater than $|\mathbf{E}|_{cr} = e/\rho^2$, where $l \sim e^2/mc^2 \sim h/2\pi mc$ is the universal length. The first non-linear correction terms in the Lagrangian are known to be of the form:

$$\alpha_1(\mathbf{E}^2 - \mathbf{B}^2)^2 + \beta_1(\mathbf{E} \cdot \mathbf{B})^2,$$

where α_1, β_1 are constants calculated by several authors. The representation of a field by means of stationary waves or quantum states of the linear field theories also becomes impossible at least in the region of high frequencies. The writer has suggested[3] that in such conditions our possibilities to distinguish eigenvalues, e.g., measure "observables," is limited in a new way, which depends essentially on a new kind of phenomena appearing at high energy collisions, such as pair production, β-ray processes, and mesotron showers. These phenomena are unavoidable in high energy collisions and thus appear necessarily in the interaction between the measuring devices (which are at rest in the respective reference frames) and the objects of observation. The easiest way to obtain converging results in the usual calculations of collision processes, problems of self-energy, and interaction energy, is to abandon the point source models and to use the so-called cut-off prescriptions. In the foregoing formulae (5) these cut-off factors are introduced by means of the factors $G(p_s)$ appearing in g_s.

(3) Recently the writer pointed out the irreversible character of the collision processes which give rise to the production of several particles, and which are accompanied by β-ray processes. This irreversibility (due to the fact that the inverse processes involve ternary or multiple collisions and emission of neutrinos) is in contrast with the validity of laws of the type (5) at $T \gtrsim 10^{11}$ degrees, but agrees well with the modified formulae obtained from (5) introducing the cut-off factors $G(p_s)$.

For instance, Planck's formula could not be

[3] G. Wataghin, Nature **142**, 393 (1938); Comptes rendus **207**, 358, 421 (1938).

valid at temperatures of the order of 10^{13} degrees [$kT\sim 10^9$ ev]. At these temperatures the number of photons of energy $>10^9$ ev should be 5×10^6 times the number of those having $h\nu<10^6$ ev. From cosmic-ray observations we know that each of the photons or electrons having $E\gtrsim 10^9$ ev gives rise to a multiplication shower in which the initial energy is subdivided among a great number of created particles. The rate of the inverse processes from many low energy particles to a high energy one is negligible. Indeed, the annihilation of an electron pair can occur with the creation of two photons or of one photon. In the first case the number of light particles is not changed (besides, the cross section for annihilation with two-photon production is maximum at $E\sim 10^6$ ev and decreases rapidly with the energy). The probability of a three-particle-collision (two electrons with a nucleus), necessary for a one-photon annihilation, is much smaller than that of a collision of one electron with one nucleus. Only these three-particle collisions give rise to a reduction of the number of particles, and the rate of these processes is necessarily very low. We conclude that the equilibrium is impossible because the processes of multiplication of particles cannot be balanced by processes which reduce their number.

(4) Let us consider a region near the center of a star during the contractive evolution which can give rise to extremely high temperatures. From Table I (last two columns) one can see that at temperatures $T\sim 10^{12}$ degrees, the density of the radiation energy and the density of the electron pairs are much greater than ρ_n, so that one can neglect the nuclear density. If such conditions are verified for a sufficiently extensive region [$R\gtrsim 10^{10}$ cm], a very peculiar situation arises. The mass of the radiation and of the electron pairs increases $\sim T^4$. The gravitational energy of this mass varies as $-\frac{3}{5}(4\pi/3)^{1/3}GM^{5/3}\rho^{1/3}$. At low densities the gravitational energy is negligible in comparison with the total relativistic energy of the nuclei, radiation, and electrons. But if $\rho_{\mathrm{rad}}+\rho_{\mathrm{pair}}$ is $>10^7$ in a region of linear dimensions $\sim 10^{10}$ cm, the gravitational energy becomes $\sim Mc^2$. The gravitational pressure varies as $\frac{1}{5}(4\pi/3)^{1/3}GM^{2/3}\rho^{4/3}$, whereas the pressure of the relativistic electron gas and the radiation pressure vary $\sim\rho$. Thus at sufficiently high values of T and $(\rho_{\mathrm{rad}}+\rho_{\mathrm{pair}})$ the gravitational pressure becomes and remains greater than the radiation pressure (in the outer layers of the high density region) and determines a contraction of the gas. These conditions are verified when the negative gravitational energy

$$-\tfrac{3}{5}GM^2/R=-\tfrac{3}{5}(4\pi/3)^{1/3}GM^{5/3}\rho^{1/3}$$

becomes comparable to the total energy of the star Mc^2. The increase of the gravitational mass of the star at expense of the potential gravitational energy implies a change of the gravitational field due to the star, and thus the variation of the mass must be accompanied by a gravitational wave which produces this change of the field. A similar variation but of opposite sign is produced when energy is carried away by neutrinos in β-disintegration processes. Obviously the contraction considered above cannot continue indefinitely and must be counterbalanced by losses of energy (and mass) by radiation, electrons, and neutrinos.

There exists also the effect of the gravitational red-shift, which produces an apparent decrease of the energy of the electrons and photons and which becomes appreciable in the considered case:

$$\Delta V/r\sim(GM/R)/c^2\sim(GM^2/R)/Mc^2\sim 1.$$

The paradoxical aspect of the phenomenon in which gravitational mass of electron pairs is created by a gravitational field induces us to admit that on the condition of such strong interaction between the gravitational field and other fields some new physical process counteracts the transformation and makes it impossible. The most plausible assumption seems to be the one that the neutrino emission prevents the increase of the mass and of the gravitational contraction by means of induced β-ray processes.

In this connection it seems noteworthy to us that accepting Friedman's idea of the expanding universe, one can calculate the increase of the gravitational energy of the receding nebulae by means of the Newtonian potential. Assuming that some 10^9 years ago the whole mass of the observed galaxies was concentrated in a reduced volume of linear dimensions ~ 1 parsec, one can make a very rough calculation of how much gravitational energy the observed galaxies have acquired, and one finds that the average energy per proton is $\sim 10^{17}$ ev. Thus, if we accept the idea of Weizsäcker, Chandrasekhar, and Henrich[4] on the prestellar stage of the universe and assume that the gravitational energy of expansion was produced at expense of the particle-energy, we must say that no equilibrium could possibly exist at an earlier epoch with such an amount of energy per particle.[5]

But the analysis of the abundance and distribution of the isotopes of the chemical elements in the universe strongly suggests the idea that nuclei were formed in the whole universe in similar conditions. We assume that such uniformity of distribution could be attained only in conditions which approximate a statistical equilibrium. In order to study this possibility, we can apply the formulae (5) to the equilibrium among nuclei, electrons, and photons. The calculations are similar to those used in the construction of Table I. Here we want to mention only some general results.

At temperatures $T > 10^{10}$, the constant γ is determined essentially by the concentration of neutrons and protons. The concentration of heavier nuclei is entirely negligible and only the concentration of $^4He_2$ is appreciable. The concentration of the heavier nuclei increases gradually in the temperature region between 3×10^9 and 10^9 degrees ($x_0 \sim 2$ or 5). The analysis of the nuclear structure (mass defect as function of Z and $A-Z$) shows that the stability of the nuclei is closely related to the probability of the β-disintegration processes, and the formation of stable nuclei follows closely the evolution of the concentration of electron pairs, photons, and neutrons. The formation of heavier nuclei by the decrease of the temperature from 6×10^9 to 10^9 degrees follows in the order of increasing atomic weights from lightest to heaviest nuclei. An easy calculation shows that at temperatures of a few billion degrees the nuclear photo-effect becomes of great importance and gives rise to the emission of many neutrons, so that equilibrium is possible with a high concentration of neutrons. At temperatures below 10^9 degrees the neutron concentration vanishes and the usual cyclical thermonuclear reactions take place. All these conclusions derive from the formulae (5) corrected by the introduction of the cut-off factors $G(p_s)$.

The limits of applicability of these formulae depend on the induced β-ray processes. In accordance with a fundamental idea of G. Gamow and M. Schoenberg related to the so-called "urca-processes," at temperatures $\gtrsim 10^{10}$ degrees, we can expect the β-ray processes to acquire such an intensity that the energy losses due to the neutrino emission must invalidate the possibility of equilibrium, and thus the law of conservation of energy (2) becomes out of our control. We think that this fast loss of energy by neutrinos was at a certain epoch in competition with the expansion of stellar matter in the non-equilibrium process which increased the gravitational energy of the universe.

Leaving the highly speculative consideration on the prestellar stage of the universe, we want to call attention to the fact that β-ray processes introduce a new limitation to the concept of temperature in an assemblage of nuclei and light particles because the non-applicability of the conservation law (2) implies the impossibility of equilibrium and of identification of the constant β with $(kT)^{-1}$. Of course, one can introduce the notion of temperature also for a nucleus as is done usually in order to measure the average energy per nucleon. But even in nuclear matter the equilibrium becomes impossible when the average energy per particle is greater than the rest energy of the meson $\mu c^2 \sim 10^8$ ev because at higher temperatures irreversible collision processes take place with production of unstable mesons. Thus the temperature $T_u \sim 10^{12}$ degrees must be an upper limit also for heavy particles.

In this paper we shall not discuss the problems of compatibility of the supplementary indeterminacy with the relativistic invariance. Also the production of mesotron showers by collision of high energy nuclei and correlated phenomena were not considered. An account of these problems will be published elsewhere in a short time.

[4] Weizsäcker, Chandrasekhar, and Henrich, Astrophys. J. **95**, 288 (1942); Physik. Zeits. **39**, 633 (1938).
[5] G. Wataghin, Phys. Rev. **63**, 137 (1943).