offering any reason for suspicion is $B^9(p, \alpha)Be^8$. Although these alphas have a range of only 4.4 cm in air, the cross section for this reaction is larger than that for fluorine by a factor of the order of 1000. Enough long range stragglers might possibly occur to produce an appreciable effect if a large amount of boron were present.

A rough spectroscopic test revealed no boron. The great strength of a boron line on a comparison spectrum made us believe that the proportion of boron in the fluorite was almost certainly less than one part in a thousand, and probably not more than one part in ten thousand, but we have no definite basis for stating an upper limit.

The fact that the targets from two separate pieces of fluorite (one of which was definitely pink, the other white) which would not be assumed to contain equal boron contamination, gave results identical within experimental error, seems to indicate that the distribution observed is indeed due to fluorine alphas only.

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Interpretation of the Angular Distribution of Alphas from $Li^7(p;\alpha)\alpha$

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Proton penetration factors for $Li^7(p;\alpha)\alpha$ are calculated under several assumptions of nuclear potential. The status of the Critchfield-Teller theory is briefly discussed in the light of these calculations and the angular distribution found by Jennings, Swartz, and Rossi. Measurements at higher energies are called for.

A DETAILED knowledge of $Li^7(p;\alpha)\alpha$, one of the most widely studied of nuclear reactions, is quite important for nuclear theory. Since it is one of the simplest reactions, calculations based on various detailed assumptions of nuclear forces can be carried through and checked against observation. Although experiments do not give directly the energy levels of light nuclei, it is convenient to divide the calculations into two stages, *viz.*, the calculation of energy levels based on detailed assumptions of nuclear structure, and the calculation of excitation and angular distribution functions, based on these levels, but not involving very detailed assumptions of nuclear model. Hence a check of the second part of the calculation with experimental results can be used to infer the actual energy levels.

Critchfield and Teller[1] have treated the theory of the angular distribution and excitation functions in the $Li^7(p;\alpha)\alpha$ reaction by means of the usual perturbation theory formulation. More recently Swartz, Rossi, Jennings, and Inglis[2] have extended the range of angular distribution measurement past 400 kev to about 900 kev. They found that the results could be fitted by a factor of the form $1+A(E)\cos^2\theta$ over the entire range of bombarding energies thus far measured. The maximum value of $A(E)$ was found to occur at 675 kev rather than near 400 kev as anticipated on the basis of Young, Ellett, and Plain's data.[3] The later experiments were not sufficiently good in the neighborhood of 400 kev to exclude the possibility of a secondary peak there, though they made this seem unlikely. A secondary peak there would be incompatible with the assumptions of Critchfield and Teller.

Most of the energy dependence of the matrix

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elements involved in the perturbation calculation is due to the Coulomb repulsion of the bombarding proton. Since only P protons are involved in this reaction,[4] it seems legitimate to factor the matrix elements into the so-called penetration factor for P protons impinging on Li^7, which contains most of the energy dependence, and the "internal yield" factor, which is relatively constant. Critchfield and Teller[1] pointed out that the maximum in $A(E)$ would be expected to occur at lower bombarding energy than the maximum in the internal yield at right angles, as deduced in reference 1 from the data of Rumbaugh, Roberts, and Hafstad[5] and others. Actually the opposite was found, and the discrepancy is now accentuated in the light of the more recent data, by the shift of the maximum of $A(E)$ to a higher energy than anticipated.

Since the internal yield is equal to the measured yield divided by the calculated penetration factor, or Y/P, it seems reasonable to examine the calculation of the penetration factor for stability of the result under reasonable changes in the details of the model used. The dispersion formula will be most accurate if the unperturbed wave functions used cause rapid convergence of the perturbation calculation. A proper choice will make the matrix elements $V(r; n, p)$ occurring in the second-order terms as small as possible. Now

$$V(r; n, p) = \int \psi_{Cr} V_{AP} \psi_{An} \psi_{Pp} d\tau,$$

where ψ_{Cr} is the wave function of the compound nucleus as calculated from the Hamiltonian of Be^8, except that it reduces rapidly to zero at about $r = r_0$. ψ_{An} is the wave function of the Li^7 nucleus as calculated from its Hamiltonian, H_A. ψ_{Pp} is the wave function of the incident proton, calculated with the Hamiltonian $T_{AP} + U_P$. The Hamiltonian for the complete system may be written $H = H_A + T_{AP} + U_P + V_{AP}$ and V_{AP} is the perturbation potential.

The choice of U_P which minimizes $V(r; n, p)$ is not the average potential of a proton in the Li^7 field, which is attractive at small distance, but rather a repulsive potential. This has a reasonable interpretation, *viz.*, that the proton does not exist as such inside the nucleus, and hence its wave function should die out there.[6] Since H is fixed, $U_P + V_{AP}$ is not subject to choice. If U_P is chosen as a large repulsive potential, V_{AP} must be chosen large to compensate. There is thus an optimum U_P which makes $V(r; n, p)$ smallest. For this U_P, ψ_{Pp} is small inside $r = r_0$. Since ψ_{Cr} reduces to zero rapidly at about $r \geqq r_0$, the important contribution comes from the region about r_0, and hence the dependence of $V(r; n, p)$ on bombarding energy is closely the same as that of $(|\psi_{Pp}|)_{r=r_0}$ since the other factors are independent of bombarding energy.

By use of Yost, Wheeler, and Breit's[7] tables of Coulomb wave functions, values of the relative penetration factor $P \sim (\bar{F}_1^2/\rho^3)_{r=r_0}$ were calculated for several different values of r_0 and for several different choices of U_P.

(A) With U_P a constant repulsive potential U_0 for $r < r_0$, and a Coulomb potential for $r > r_0$ (cut off to zero at large distance for convenience of calculation), the result was not significantly dependent on the choice of U_0 for the several trial values 785, 1500, 3000, and 15,000 kev.

(B) Calculations were also made by using for U_P a Coulomb potential down to $r = 0$, again cut off at large distance.

(C) Penetration factors arising from Kapur and Peierls'[8] treatment of the dispersion formula, i.e., $1/\rho G_1^2$ in the notation of reference 7, were also calculated for the case of U_P a Coulomb potential down to $r = 0$. As Breit pointed out,[9] $1/\rho G_1^2$ is almost the same as F_1^2/ρ^3 as long as $\phi_1 \theta_1 \leqq 1$, which is true in this region of bombarding energies.

With $r_0 \leqq 4 \times 10^{-13}$ cm, a maximum was obtained in all cases, below $A(E)$ max., independent of the details of the nuclear model used. For r_0 in this neighborhood the values for the right angle yield[5] Y would have to be badly in error to make the maximum of Y/P occur at higher energy than the maximum of $A(E)$. As r_0 is assumed larger, differences appear in the shape

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TABLE I. Internal yield at right angles, calculated from the observed data for various nuclear radii by several methods of approximation. Values listed are log Y/P plus a constant selected to make yield and penetration factor match at $E=225$ kev.

Bombarding energy (kev)	895	711	565	449	357	225
Energy (K.E. in center of mass system)	783	622	494	393	312	197
Calculation (A)						
I	0.40	0.38	0.38	0.37	0.34	0
II	0.38	0.36	0.36	0.36	0.34	0
III	0.32	0.33	0.34	0.35	0.33	0
IV	0.29	0.32	0.34	0.34	0.33	0
Calculation (B)						
I	0.35	0.35	0.36	0.36	0.33	0
II	0.33	0.34	0.35	0.35	0.33	0
III	0.30	0.32	0.34	0.34	0.32	0
IV	0.28	0.31	0.33	0.34	0.32	0
Calculation (C)						
I	0.50	0.48	0.45	0.43	0.36	0
II	0.47	0.45	0.43	0.40	0.36	0
III	0.38	0.38	0.38	0.37	0.33	0
IV	0.34	0.35	0.36	0.36	0.33	0

I. $r_0=6.17\times10^{-13}$ cm III. $r_0=3.80\times10^{-13}$ cm
II. $r_0=5.50\times10^{-13}$ cm IV. $r_0=3.09\times10^{-13}$ cm

of Y/P depending on the type of calculation used. (See Table I.)

Calculation (B) apparently still shows a maximum in the same region as before, but in (A) and (C) the maximum, if it exists, is moved out of the range of energies thus far used. One may take these results to indicate either that r_0 must be in the neighborhood of 4×10^{-13} cm, as is customarily assumed, or that this perturbation calculation is not particularly trustworthy, as Breit[9] has remarked.

A closer examination also reveals that the first choice above is not necessarily at odds with the Critchfield-Teller theory. It[1] leads to the result:

$$\sigma\sim\left[1+\frac{2(\epsilon-\epsilon_0)\beta\cos\delta\cos\xi+2\beta^2}{1+\epsilon^2}\right]+\cos^2\theta\left[\frac{6\beta\cos\delta\cos\xi(\epsilon_0-\epsilon)}{1+\epsilon^2}\right],$$

where

$$\epsilon_0=-\tan\delta+(u/2)\cos 2\xi$$

and $u=\beta/\cos\delta\cos\xi$, and also

$$A(E)=\frac{6\beta\cos\delta\cos\xi(\epsilon_0-\epsilon)}{1+\epsilon^2+2(\epsilon-\epsilon_0)\beta\cos\delta\cos\xi+2\beta^2}$$

with $A(\epsilon_0)=0$. Here ξ is the angle denoting admixture of triplet and $\beta\exp(i\delta)$ is a complex number depending on the wave function of the compound nucleus, *viz.*,

$$\beta\exp(i\delta)\equiv 10^{\frac{1}{2}}\alpha_2\Gamma_0/2\alpha_0\Gamma_2.$$

From this result it follows that $A(E)$ max. $=A(\epsilon_m)=-3\beta^2/(\beta^2+u\epsilon_m)$ and σ max. $=\sigma(\epsilon_n)\sim1+\beta^2/u\epsilon_n$, where ϵ_m is the position of $A(E)$ max. and ϵ_n is the position of σ max., and

$$\epsilon\equiv(E-E_2)/\tfrac{1}{2}\Gamma_2.$$

For $u>0$, $\epsilon_m=\epsilon_0-(1+2\beta^2+\epsilon_0^2)^{\frac{1}{2}}<0$ and $\epsilon_n=(\epsilon_0-u)+[(\epsilon_0-u)^2+1]^{\frac{1}{2}}>0$,

leading to $\epsilon_n>\epsilon_m$, contrary to observation.

On the other hand, the sign of u is merely a question of the phase of the triplet admixture in the wave function of the compound nucleus, and for $u<0$, $\epsilon_m=\epsilon_0+[1+2\beta^2+\epsilon_0^2]^{\frac{1}{2}}>0$ and $\epsilon_n=(\epsilon_0-u)-[(\epsilon_0-u)^2+1]^{\frac{1}{2}}<0$. In this case $\epsilon_m>\epsilon_n$ as observed. Also

$$\epsilon_m-\epsilon_0=(1+2\beta^2+\epsilon_0^2)^{\frac{1}{2}}>0$$

and hence ϵ_0 is below ϵ_m. Therefore the observed zero in $A(E)$ in the neighborhood of zero bombarding energy occurs at ϵ_0. There is little difficulty in finding sets of consistent values for ξ, β, δ, and E_2 fitting the observed separations $\epsilon_m-\epsilon_0$ and $\epsilon_m-\epsilon_n$. Their direct calculation, however, hardly seems feasible, since detailed considerations of nuclear model are involved. It does seem reasonable to assume, however, that $X_2=X_2^{(5)}\sin\xi+X_2^{(3)}\cos\xi$, the combination of incident triplet and singlet states with $J=2$ which corresponds most strongly to the compound state involved in the alpha-decay, will not be more triplet than singlet, so that $|\xi|<45°$ and $\cos 2\xi>0$.

Measurements at higher energies are needed. The present expectation is that $A(E)$ goes to zero asymptotically at higher energy. Negative $A(E)$, at least below the region where higher levels of Be^{8*} become involved, would force a revision of the theory.

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