In a lattice of three dimensions an arbitrary closed contour of bonds is considered to be the boundary of a large number of surfaces which are formed by squares between nearest neighbors in the lattice. Two of these surfaces define a closed surface and accordingly the direction of an outer normal for each square on either surface. Having assigned on one surface an outer normal to each square, q is defined as contour integral taken counterclockwise over one square as seen from the outer normal and Q is defined as the sum of all q taken over the surface. According to the above argument, the number of O bonds on the contour is then seen to be even.

So far the number ν of A atoms in the lattice of n atoms is regarded as variable so that the configurational partition function is

$$P=\sum_{\nu=0}^{n} p(\nu),$$

where $p(\nu)$ is the partial sum over the configurations with ν A atoms. The partition function of an equimolecular alloy is

$$P^*=\sum_{\nu=0}^{n} p(\nu)\cdot\exp(-\alpha\nu),$$

where the parameter α is determined by

$$n/2=-\partial \log P^*/\partial\alpha.$$

Since $p(\nu)=p(n-\nu)$, it follows from the former equation that $\alpha=0$ for all temperatures. The specific heat curve of an equimolecular alloy is accordingly equal to the specific heat curve of an alloy with variable composition.

The writer[2] recently applied the matrix method to the statistical calculation of the specific heat. The configurations of the lattice are specified in this calculation by the pattern of bonds and only such patterns are admitted in which every square has an even number of O bonds. It appears that in this case the matrices show no "degeneracy." While no information is obtained on the superstructure, the specific heat is evaluated for all temperatures; it has no singularity. A note on the corresponding result for a two-dimensional net has been published previously.[3]

[1] J. Ashkin and W. E. Lamb, Phys. Rev. **64**, 159 (1943).
[2] In print in the Proceedings of the Royal Society.
[3] R. Eisenschitz, Nature **147**, 779 (1941).

Relativity and Supplementary Indeterminacy

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In recent years experimental evidence has been accumulated concerning the production of groups of mesotrons in one collision and processes in which the hypothetical neutrinos take part.[1] It was pointed out[2] that in view of the irreversibility of many of the processes observed with cosmic rays the statistical mechanics of an assemblage of particles having an average energy per particle $E \gtrsim 200$ mc^2 is quite different from the usual one. For instance, the introduction of temperature and entropy is not allowed in the cases in which statistical equilibrium is impossible. New limitations arise because of the following circumstances: In a collision accompanied by the production of several mesotrons, the rest mass of the group changes, and the control of the conservation laws of momenta and spins appears limited by the decay processes and fluctuation of the number of created mesotrons and neutrinos. The wave properties of particles, based on diffraction and interference phenomena, are also altered and probably unobservable in high energy collisions, in accordance with the non-linear character of the laws (differential equations, interaction operators) governing such collisions. The representation of high frequency photon-field (for $h\nu>200$ mc^2) as a system of harmonic oscillators, and the statistical laws of Planck, Fermi, Bose at extremely high temperatures are invalidated for similar reasons. From the foregoing considerations, it follows that the behavior of high energy particles (observed as cosmic rays in a terrestrial frame of reference) is qualitatively different from the usual one, in accordance with the idea of supplementary indeterminacy.[3]

In the theory of relativity we must distinguish the postulate of general covariancy referring to an arbitrary transformation of coordinates (mathematical symbols used to denominate the events in S_4) from the equivalence of different physical reference frames, each individualized by a system of measuring devices (diffraction gratings, microscopes).[4] Obviously our possibilities of building a reference frame corresponding to a given mathematical transformation are limited (we cannot use rigid bodies or apparatus moving with $v>c$). Let us consider a terrestrial frame of reference (neglecting the rotation) as a local "co-moving" frame in the Friedman universe in expansion. The velocities of the receding nebulae and the cosmic rays are distributed around the earth with spherical symmetry. Let us consider another Lorentz observer moving with an ultra-relativistic velocity βc $[(1-\beta^2)^{-\frac{1}{2}}>1000]$. Any particle of the apparatus used by this second observer will constitute for the terrestrial observer a high energy particle capable of producing a cascade shower or a mesotron shower. Now we introduce the assumption that the "ultra-relativistic" frames of reference are not equivalent to the co-moving ones because the apparatus moving with the velocity of high energy cosmic rays could not function or could not be used as measuring devices in the actual world. Thus we can exclude from our relativity postulate such non-allowed reference systems.

In this way we overcome the incompatibility between the relativity and the theory of a world in which a universal length marks the lower limit for measurement of intervals and a supplementary indeterminacy exists in the region of high energies and momenta.

[1] See cloud-chamber photographs of groups of three, five, seven, mesotrons; experiments with counters on mesotron showers produced in air, lead, and also in clay, underground; high altitude researches on the production of protons, neutrons, and mesotrons in lead and paraffin. Some of the secondary protons and neutrons behave like primaries and thus generate mesotron showers.
[2] G. Wataghin, Phys. Rev. **63**, 137 (1943), and **64**, 248 (1943).
[3] Nature **142**, 393 (1938).
[4] Zeits. f. Physik **66**, 650 (1930); Comptes rendus **207**, 421 (1938).