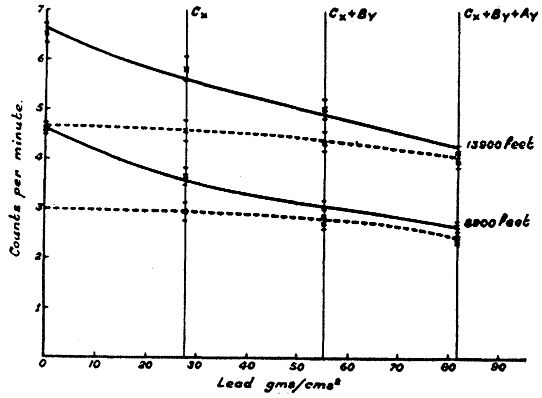

FIG. 2. Curves showing for two altitudes the absorption of the total intensity measured by coincidence rate I, IV (full line) and the meson intensity measured by shower anticoincidence rate I, IV-(II, III) (broken lines).

latitude 25°N show that above 10,000 ft. the meson intensity increases less rapidly than the total intensity and the atmospheric shower intensity. At 13,900 ft. the proportion of mesons was 64 percent of which about 18 percent consisted of slow mesons, but the accuracy of these estimates is not high. A full report of the results obtained at Kashmere and repeated with the same arrangement at Bangalore will appear shortly.[4]

The errors in this experiment might be due to three causes. Firstly, due to the contribution of side showers to the double coincidences that were measured. This would give an under-estimate of the proportion of electrons. Secondly, due to electrons of low energy which do not produce large enough showers to be detected and cut out by the anticoincidence arrangement, and lastly, due to mesons which are associated by other mesons or knock-on electrons and which get cut out by the anticoincidence arrangement. These two errors will tend to give an over-estimate of the proportion of slow mesons. With the help of an accurate experiment that is now being performed with quadruple coincidences measuring the vertical intensity, it is hoped to correct for these errors.

I am indebted to Professor Sir C. V. Raman for supprt and assistance, and to Professor Bhabha for helpful discussions. The Kashmere trip would not have been possible but for the willing help given by the state officials and friends.

[1] P. Auger, Phys. Rev. **61**, 684 (1942).
[2] K. Greisen, Phys. Rev. **63**, 323 (1943).
[3] Bhabha, Proc. Ind. Acad. Sci. (A), in press.
[4] Sarabhai, Proc. Ind. Acad. Sci. (A), in press.

Configurations of a Binary Mixed Crystal

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IN a recent paper Ashkin and Lamb[1] applied the matrix method of statistical mechanics to mixed crystals, especially to the order-disorder equilibrium. In their theory any configuration of the crystal is specified as usually by the kind of atom (A or B) by which every lattice point is occupied. The matrices by which the correlations in the lattice are determined appear to be "degenerated" at low temperatures; the quotation mark indicates that it is the absolute and not the algebraic value of the highest proper value which matters. By this method information is obtained on the superstructure, whereas the evaluation of the specific heat curve seems to be difficult.

In this note an alternative specification of configurations is discussed which is equivalent to the above specification. In a simple cubic lattice of a binary alloy such as β-brass, there are two possible values for the bond energy ϵ, namely, $\epsilon=0$ and $\epsilon=\varphi$ corresponding to bonds between atoms of different kinds and the same kind, respectively. φ is assumed to be positive. The configurations may accordingly be specified by the pattern of bonds, i.e., by the value of ϵ for every bond in the lattice. Not every pattern of bonds corresponds to an atomic configuration; only those patterns are admissible in which there is an even number of "O bonds" on every closed contour of bonds in the lattice. Every pattern which complies with that condition corresponds to two atomic configurations which are transformed into each other by interchanging all A atoms with B atoms and vice versa. The set of all admissible patterns corresponds to the set of all atomic configurations in which not only the positions of A and B atoms but also their ratio is varied.

Instead of making statistics over atomic configurations, it may be made over the admissible patterns of bonds. The central problem is the enumeration of those admissible patterns for which the number of "φ bonds" and accordingly the configurational energy is given. In a square of four nearest neighbors those patterns are admissible in which the number of O bonds is 0 or 2 or 4. There are altogether 8 patterns of this kind. If a pattern of bonds in the lattice is admissible, then every square necesssarily exhibits one of these 8 patterns. This necessary condition is also sufficient. Assuming that the number of O bonds is even in every square of four nearest neighbors, it is proved in the following that this number is even on every closed contour of bonds in the lattice.

In proving this, at first a square net in the plane is considered. Set $q=\int(1-\epsilon/\varphi)ds$ a contour integral taken counterclockwise over the circumference of one square. This integral is equal to a sum of four terms which are 0 for every φ bond and ± 1 for every O bond. Let Q be the sum of all q taken over an area which is enclosed by any arbitrary contour of bonds in the net. Then all bonds in the interior of this area contribute 0 to the sum of contour integrals. Q is equal to the contour integral $\int(1-\epsilon/\varphi)ds$, which is taken over the boundary and equal to a sum of as many terms as there are bonds in the boundary. Since, according to premises, every square contributes in even number to Q, Q is even and so is the sum taken over the boundary. As that sum consists only of terms equal to 1, 0, -1, the number of its non-vanishing terms is necessarily even. There is accordingly an even number of O bonds on the boundary, whatever the contour may be like.

In a lattice of three dimensions an arbitrary closed contour of bonds is considered to be the boundary of a large number of surfaces which are formed by squares between nearest neighbors in the lattice. Two of these surfaces define a closed surface and accordingly the direction of an outer normal for each square on either surface. Having assigned on one surface an outer normal to each square, q is defined as contour integral taken counterclockwise over one square as seen from the outer normal and Q is defined as the sum of all q taken over the surface. According to the above argument, the number of O bonds on the contour is then seen to be even.

So far the number ν of A atoms in the lattice of n atoms is regarded as variable so that the configurational partition function is

$$P=\sum_{\nu=0}^{n} p(\nu),$$

where $p(\nu)$ is the partial sum over the configurations with ν A atoms. The partition function of an equimolecular alloy is

$$P^{*}=\sum_{\nu=0}^{n} p(\nu)\cdot\exp(-\alpha\nu),$$

where the parameter α is determined by

$$n/2=-\partial \log P^{*}/\partial\alpha.$$

Since $p(\nu)=p(n-\nu)$, it follows from the former equation that $\alpha=0$ for all temperatures. The specific heat curve of an equimolecular alloy is accordingly equal to the specific heat curve of an alloy with variable composition.

The writer[2] recently applied the matrix method to the statistical calculation of the specific heat. The configurations of the lattice are specified in this calculation by the pattern of bonds and only such patterns are admitted in which every square has an even number of O bonds. It appears that in this case the matrices show no "degeneracy." While no information is obtained on the superstructure, the specific heat is evaluated for all temperatures; it has no singularity. A note on the corresponding result for a two-dimensional net has been published previously.[3]

[1] J. Ashkin and W. E. Lamb, Phys. Rev. **64**, 159 (1943).
[2] In print in the Proceedings of the Royal Society.
[3] R. Eisenschitz, Nature **147**, 779 (1941).

Relativity and Supplementary Indeterminacy

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IN recent years experimental evidence has been accumulated concerning the production of groups of mesotrons in one collision and processes in which the hypothetical neutrinos take part.[1] It was pointed out[2] that in view of the irreversibility of many of the processes observed with cosmic rays the statistical mechanics of an assemblage of particles having an average energy per particle $E\gtrsim 200$ mc^2 is quite different from the usual one. For instance, the introduction of temperature and entropy is not allowed in the cases in which statistical equilibrium is impossible. New limitations arise because of the following circumstances: In a collision accompanied by the production of several mesotrons, the rest mass of the group changes, and the control of the conservation laws of momenta and spins appears limited by the decay processes and fluctuation of the number of created mesotrons and neutrinos. The wave properties of particles, based on diffraction and interference phenomena, are also altered and probably unobservable in high energy collisions, in accordance with the non-linear character of the laws (differential equations, interaction operators) governing such collisions. The representation of high frequency photon-field (for $h\nu>200$ mc^2) as a system of harmonic oscillators, and the statistical laws of Planck, Fermi, Bose at extremely high temperatures are invalidated for similar reasons. From the foregoing considerations, it follows that the behavior of high energy particles (observed as cosmic rays in a terrestrial frame of reference) is qualitatively different from the usual one, in accordance with the idea of supplementary indeterminacy.[3]

In the theory of relativity we must distinguish the postulate of general covariancy referring to an arbitrary transformation of coordinates (mathematical symbols used to denominate the events in S_4) from the equivalence of different physical reference frames, each individualized by a system of measuring devices (diffraction gratings, microscopes).[4] Obviously our possibilities of building a reference frame corresponding to a given mathematical transformation are limited (we cannot use rigid bodies or apparatus moving with $v>c$). Let us consider a terrestrial frame of reference (neglecting the rotation) as a local "co-moving" frame in the Friedman universe in expansion. The velocities of the receding nebulae and the cosmic rays are distributed around the earth with spherical symmetry. Let us consider another Lorentz observer moving with an ultra-relativistic velocity βc $[(1-\beta^2)^{-\frac{1}{2}}>1000]$. Any particle of the apparatus used by this second observer will constitute for the terrestrial observer a high energy particle capable of producing a cascade shower or a mesotron shower. Now we introduce the assumption that the "ultra-relativistic" frames of reference are not equivalent to the co-moving ones because the apparatus moving with the velocity of high energy cosmic rays could not function or could not be used as measuring devices in the actual world. Thus we can exclude from our relativity postulate such non-allowed reference systems.

In this way we overcome the incompatibility between the relativity and the theory of a world in which a universal length marks the lower limit for measurement of intervals and a supplementary indeterminacy exists in the region of high energies and momenta.

[1] See cloud-chamber photographs of groups of three, five, seven, mesotrons; experiments with counters on mesotron showers produced in air, lead, and also in clay, underground; high altitude researches on the production of protons, neutrons, and mesotrons in lead and paraffin. Some of the secondary protons and neutrons behave like primaries and thus generate mesotron showers.
[2] G. Wataghin, Phys. Rev. **63**, 137 (1943), and **64**, 248 (1943).
[3] Nature **142**, 393 (1938).
[4] Zeits. f. Physik **66**, 650 (1930); Comptes rendus **207**, 421 (1938).