The Ionization Cross Section of the Silver L_{III} State

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The ionization cross section of the silver L_{III} state has been measured, in arbitrary units, by observing the intensity of the $L\alpha$ doublet radiation from a thin target of silver bombarded by cathode rays whose energies ranged up to nine times the excitation energy of the L_{III} state. The observed data are corrected for the effects of diffusion, rediffusion, and retardation of the cathode rays. The measured values of the cross section agree fairly well with calculations based on the Born approximation, but the agreement is not within the estimated experimental error.

THE dependence of the ionization cross section of an atom on the energy of the impinging electron is a matter that has received considerable attention since it affords a means of testing the applicability of wave mechanics to collision processes on the atomic scale. One can measure the ionization cross section of the inner shell of an atom by observing the intensity of an x-ray spectrum line whose initial state involves ionization of the inner shell in question, the ionization being produced by subjecting the atom to cathode-ray bombardment. The chief difficulty arises from the necessity of using monokinetic cathode rays, but if the target be made in the form of a film, then it can be made thin enough so that the cathode rays pass through it with only a small energy loss, for which a correction can be made. Using such targets, Webster[1–3] and his colleagues have measured the ionization cross section of the silver K shell for electrons whose energies ranged from one to seven times the binding energy of the K electrons. On the publication by Burhop[4] of calculated values of the ionization cross sections for the silver L states, it seemed worth while to measure the cross section for the silver L_{III} state in arbitrary units by using the thin target techniques originated by Webster.

I. THE APPARATUS

The x-ray tube designed especially for this investigation is described elsewhere.[5]

The tube voltage was obtained from a motor generator set coupled to a high voltage transformer and full-wave kenotron rectifier, with a pi-section filter composed of two 0.25-microfarad condensers and a large choke. Tests with a cathode-ray oscilloscope failed to reveal any ripple in the high voltage under operating conditions although a ripple of 50 volts would have been detected.

A Siegbahn-Thoraeus[6] vacuum spectrometer, with a selenite crystal and an ionization chamber, was coupled to the x-ray tube by means of a brass pipe whose axis cut the axis of rotation of the crystal and the center of the target, making an angle of $5°\pm1°$ with the plane of the target. The pipe, which had an inside diameter of 0.5 inch, could be blocked by a movable lead stop. The stop was moved into the beam from time to time during the intensity measurements to keep track of the zero reading of the galvanometer connected to the detecting circuit. Two lead jaws fixed on the front of the ionization chamber limited the width of the beam entering the chamber. The other limiting slit in the system was the focal spot itself. Viewed from the crystal, that is, at an angle of 5° from the plane of the target, the focal spot had a width of the order of 0.5 mm and a height not exceeding 6 mm.

The selenite crystal was 10 mm high and 25 mm long, with its axis of rotation 95 cm from the focal spot and 15 cm from the slit at the ionization chamber. This slit, 14 mm high, was 1.65 mm wide, the width being chosen so as to allow a band of radiation 23 x.u. wide to enter the chamber when the spectrometer was set on the Ag $L\alpha$ line

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[1] Webster, Clark, Yeatman, and Hansen, Proc. Nat. Acad. Sci. **14**, 679 (1928).

[2] Webster, Clark, and Hansen, Phys. Rev. **37**, 115 (1931).

[3] Webster, Hansen, and Duveneck, Phys. Rev. **43**, 839 (1933).

[4] E. H. S. Burhop, Proc. Camb. Phil. Soc. **36**, 43 (1940).

[5] J. J. G. McCue, Rev. Sci. Inst. **14**, 339 (1943).

[6] M. Siegbahn and R. Thoraeus, J. Opt. Soc. Am. **13**, 235 (1926).

at 4150 x.u. This ensured the collection, in the chamber, of all of the Ag $L\alpha$ doublet radiation which the crystal reflected in the first order.[7] Moving the ionization chamber through 3 minutes of arc on either side of the peak of the line decreased the intensity reading by only 1 percent. Therefore slight changes in the position of the focal spot could not introduce any error into the measurements, for the focal spot would have had to move 4 mm to shift the peak of the line 3 minutes.

The ionization chamber that took the place of the plate holder with which the spectrometer was originally equipped was a hollow brass cylinder 12 cm long and of 5 cm inside diameter, with a collecting rod mounted parallel to, but displaced from, the axis of the cylinder. The chamber contained air at atmospheric pressure. Checking the variation of ionization current with x-ray tube current, with the x-ray voltage held constant, showed that in the range of intensities encountered, the ionization current was proportional to the x-ray intensity within the accuracy of the observations, which was about 1 percent.

The ionization current was amplified by an FP-54 electrometer tube operating in a Barth[8] circuit connected to a Leeds and Northrup type R galvanometer whose sensitivity was 4×10^{-10} amp./mm/meter. The over-all sensitivity of the amplifying system was about 30,000 mm/volt with the scale 170 cm from the galvanometer. The non-linearity of its response was detectable, but the range of intensities used was so small that the error introduced by assuming a linear response is less than 0.5 percent. During the course of the measurements, the sensitivity varied somewhat because of changes in the temperature of the battery. Frequent measurements of the sensitivity made possible appropriate correction of the data.

The target consisted of a silver film distilled onto a beryllium disk,[9] 24 mm in diameter and 0.8 mm thick, soldered with Alumaweld to a water-cooled beryllium backing. The interference pattern formed when a glass optical flat lay on the disk showed that the target surface was flat within one wave-length of green light over the area covered by the focal spot, while the interference rings showed that the smoothness of the beryllium surface was about the same as that of plate glass.

The silver, 99.98 percent pure, was melted in a horizontal tungsten trough 10 cm beneath the horizontal polished surface of the beryllium disk; an ionization gauge showed that during the distillation the pressure in the chamber was 3×10^{-5} mm of Hg. Beside the beryllium surface lay a clean, clear-glass microscope-slide. A movable baffle plate allowed the beryllium and glass surfaces to be exposed simultaneously, for an appropriate time, to the silver coming from the trough. Since the heating current in the trough was turned on for only about 2 seconds, the silver condensed on a surface whose temperature was near 20°C.

The thickness of the film was obtained by measuring, at three wave-lengths in the visible spectrum, the opacity of the film on the microscope slide. The data published by Strong and Dibble[10] make possible an estimate of the film thickness when the opacity at these wave-lengths is known. The surface density of the target film according to their curves was 23 micrograms per square centimeter, but the data they obtained by direct weighing indicate that the actual surface density is greater than that shown in their curves; giving equal weights to their two methods of weighing leads to an estimated density of 27 micrograms per square centimeter for the target film. It is sufficient to assume that the volume density of the film was the same as that of massive silver, which implies that the thickness of the target was 260A. A thinner film would have been desirable in some respects, but the thickness of the thinner film would be so uncertain that 260A seemed to represent an optimum thickness.

During the measurements, some contamination of the silver by material from the filament occurred. The portions of the film that could "see" the filament became discolored, but the focal spot was not visibly affected. Tests on the excitation potential of the silver $L\alpha$ lines showed that the cathode rays did not experience any

[7] L. G. Parratt, Phys. Rev. **54**, 99 (1938).
[8] G. Barth, Zeits. f. Physik **87**, 399 (1934).
[9] Beryllium disks were generously furnished by the Brush Beryllium Company.
[10] J. Strong and B. Dibble, J. Opt. Soc. Am. **30**, 431 (1940).

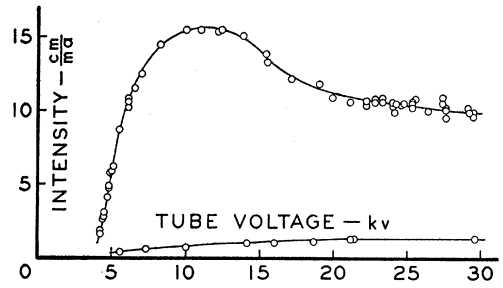

FIG. 1. The observed radiation and the continuous background.

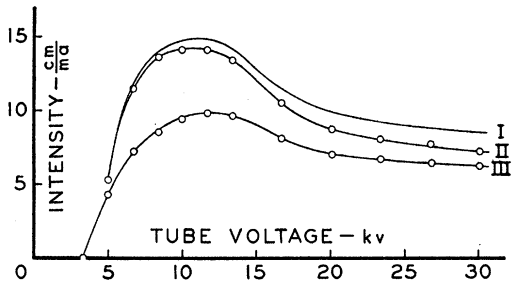

FIG. 2. Specific intensity of the $L\alpha$ doublet. I—uncorrected; II—corrected for fluorescence; III—completely corrected.

serious retardation in a surface film before striking the silver. With a massive silver target that had been used for some time and was very much dirtier than the thin target ever became, $L\alpha$ radiation could be detected when the tube voltage was only 150 volts above the Ag $L\alpha$ excitation potential (3.35 kv). The surface film on the massive target therefore retarded 3.5 kilovolt electrons by not more than 150 volts; the retardation was of course less for cathode rays of higher energy.

The safe limit of power dissipation in the target was about 30 watts. In order to avoid possible injury to the polish of the beryllium surface, it was touched after the polishing operation by nothing but lens paper. There seems to have been enough grease on the beryllium to inhibit good thermal contact between the silver and the backing.

II. THE DATA

After tests had been performed to make certain that the deflection of the galvanometer was proportional to the tube current at constant voltage, and was independent of the size of the focal spot, the intensity of the $L\alpha$ doublet (4.15A) emitted from the film was measured at various voltages from 4 to 30 kv. A set of auxiliary measurements made with the spectrometer set at 4.30A determined the intensity of the continuous radiation in that part of the spectrum. The results appear in the upper and lower curves, respectively, of Fig. 1. The ordinates are centimeters of galvanometer deflection per milliampere of target current, which will be called the "specific intensity." The scattering of the points at the right end of the upper curve is due to unsteadiness in the measuring circuit at the time these points were taken, combined with the fact that the deflections were small because small tube currents had to be used at the high voltages. The difference in the ordinates of the two curves is the specific intensity of the $L\alpha$ doublet radiation from the thin target; it is plotted as curve I in Fig. 2.

III. THE CORRECTIONS

Fluorescence

It is necessary to make allowance for the silver $L\alpha$ radiation that arises from fluorescence of the target foil under the action of the x-ray continuum emanating from the beryllium backing. Let $I(\nu_0, \nu)d\nu$ be the power per unit solid angle emitted by the beryllium disk in the frequency range between ν and $\nu+d\nu$, when ν_0 is the high frequency limit of the continuum. Then the total fluorescent power emitted as $L\alpha$ radiation by the silver foil is

$$u_{L\alpha}\int_{\nu_{L_{\mathrm{III}}}}^{\nu_0} d\nu \int_0^{2\pi} d\phi \int_0^{\pi/2} \frac{\mu_{L_{\mathrm{III}}}\nu_{L_{\mathrm{III}}}}{\mu\nu} I(\nu_0, \nu) \times\{1-\exp(-\mu X_0 \sec\theta)\}\sin\theta d\theta,$$

where $u_{L\alpha}$ = the fluorescence yield of silver for its $L\alpha$ radiation; μ = the absorption coefficient of silver; $\mu_{L_{\mathrm{III}}}$ = the part of the absorption coefficient of silver due to absorption by the L_{III} shell; $\nu_{L_{\mathrm{III}}}$ = the frequency of the silver L_{III} absorption edge; X_0 = the thickness of the silver foil; θ = the angle between an x-ray and the normal to the silver foil; ϕ = the azimuth measured about the normal to the foil.

If the crystal subtends at the target a solid

angle $\Delta\Omega$, then the fluorescent power striking the crystal is

$$\tfrac{1}{2}u_{L\alpha}\Delta\Omega\int_{\nu L_{III}}^{\nu_0}\frac{\mu L_{III}\nu L_{III}}{\mu\nu}I(\nu_0,\nu)\int_0^{\pi/2}\{1-\exp(-\mu X_0\sec\theta)\}\sin\theta d\theta d\nu.$$

$I(\nu_0, \nu)\Delta\nu\Delta\Omega$ is the power striking the crystal in the range $\Delta\nu$ in the continuum. (A small and easily evaluated correction must be made for the absorption of the continuum in the silver.) This power has been measured, in arbitrary units, for the frequency corresponding to 4.3A. Let ν_c be this frequency, and I_c be the power per unit solid angle per unit frequency interval at ν_c (corrected for absorption in the silver). The arbitrary units used to measure $I_c\Delta\nu$ and the uncorrected power of the $L\alpha$ radiation are the same. Therefore the fluorescence radiation contributed to the observed specific intensity an amount

$$\phi(\nu_0)=\tfrac{1}{2}u_{L\alpha}\int_{\nu L_{III}}^{\nu_0}\frac{\mu L_{III}\nu L_{III}}{\mu\nu}I(\nu_0,\nu)\{1-G(\mu X_0)\}d\nu,$$

where

$$G(\mu X_0)=\int_0^{\pi/2}\exp(-\mu X_0\sec\theta)\sin\theta d\theta.$$

The function $G(y)$ has been tabulated by Gold.[11]

The data on fluorescence yields cited by Compton and Allison,[12] in conjunction with the relative intensity measurements of Parratt,[7] indicate that $u_{L\alpha}=0.17$. Van Dyke and Lindsay[13] have determined νL_{III} for silver, and the work of Andrews[14] makes available the values of μ and μL_{III} in the pertinent spectral region. The spectral band width admitted to the ionization chamber at ν_c was $\Delta\nu=3.7\times10^{15}$ sec.$^{-1}$.

Kulenkampff's[15] conclusion that $I(\nu_0, \nu)$ is proportional to $\nu_0-\nu$ is based on data corrected for absorption in the target. It implies that the intensity at a given frequency in the continuum varies linearly with tube voltage. Measurements in the present investigation show that the intensity at ν_c, measured at an angle of 85° with the cathode rays, increases less rapidly than the tube voltage up to 20 kv and is constant as the voltage ranges from 20 to 30 kv. Kulenkampff's formula for $I(\nu_0, \nu)$ is therefore not applicable to this case because of absorption of the continuum in the beryllium. Moreover, Kulenkampff worked only with the radiation emitted in a direction perpendicular to the cathode rays, while the fluorescence is excited by radiation passing through the foil in all directions. The numerical calculations show, however, that the foil absorbs strongly only those rays passing through the target rather obliquely. For this reason, and because the cathode rays in the beryllium experience considerable diffusion, it seems safe to ignore the dependence of $I(\nu_0, \nu)$ on θ; the best expression for $I(\nu_0, \nu)$ to use on the basis of available information is $I(\nu_0, \nu)=(\nu_0-\nu)I_c/(\nu_0-\nu_c)$. The function $\phi(\nu_0)$ was evaluated by plotting the integrand of (1) for several values of ν_0 and then integrating with a planimeter. The results appear in column 3 of Table I. The quantity U in the table is the ratio of the tube voltage to the L_{III} ionization potential, while $\phi(\nu_0)$ is expressed in cm of galvanometer deflection per ma of tube current.

Diffusion

The fact that slow cathode rays are likely to experience large deflections in traversing the silver film makes the effective film thickness uncertain at the lowest voltages. For potentials

TABLE I. Observed L doublet intensities, corrections, reduced intensities, and cross sections.

U	I_{obs}	$\phi(\nu_0)$	X_0/X	c_x	c_r	i_0	Q
	cm/ma	cm/ma				cm/ma	
1.5	5.5	0.1	0.58	1.39	0.98	4.3	0.46
2	11.8	0.3	0.62	1.05	0.96	7.2	0.77
2.5	14.0	0.4	0.65	1.01	0.95	8.5	0.91
3	14.7	0.6	0.70	1.00	0.95	9.4	1.00
3.5	14.8	0.7	0.74	1.00	0.94	9.8	1.04
4	14.2	0.8	0.77	1.00	0.93	9.6	1.02
5	11.5	1.0	0.83	1.00	0.93	8.1	0.86
6	9.9	1.2	0.87	1.00	0.93	7.0	0.75
7	9.2	1.2	0.90	1.00	0.93	6.7	0.72
8	8.8	1.2	0.92	1.00	0.93	6.5	0.69
9	8.5	1.3	0.94	1.00	0.92	6.2	0.66

[11] E. Gold, Proc. Roy. Soc. **A82**, 62 (1908).
[12] A. H. Compton and S. K. Allison, *X-rays in Theory and Experiment* (D. Van Nostrand, New York, 1935), pp. 488, 490.
[13] G. D. Van Dyke and G. A. Lindsay, Phys. Rev. **30**, 562 (1927).
[14] C. L. Andrews, Thesis, Cornell, 1938.
[15] H. Kulenkampff, Ann. d. Physik **69**, 548 (1922).

above about 10 kv ($U=3.0$) a satisfactory calculation of the effective thickness of the foil can be made. Throughout the discussion of this and the succeeding calculations the notation of Webster, Hansen, and Duveneck[3] will be followed as closely as possible.

According to Bothe,[16] the most probable deflection experienced by a cathode ray in traversing a thickness x of silver is $\lambda=18.5U^{-1}x^{\frac{1}{2}}$, where λ is in radians, and x is in microns. Let Θ be the deflection of any particular cathode ray at a depth x in the film. For this ray, the path length in a thickness dx at depth x will be dx sec Θ. The effective thickness of the target will therefore be

$$X=\int_0^{X_0}\langle\sec\Theta\rangle_{Av}dx.$$

Here X_0 is the thickness of the film and $\langle\sec\Theta\rangle_{Av}$ is the value of sec Θ, at a particular depth x, averaged over all of the cathode rays. For the distribution-in-angle of the deflected cathode rays, Bothe[16] gives

$$nd\Omega=\frac{n_0}{2\pi\lambda^2}\exp(-\Theta^2/2\lambda^2)d\Omega,$$

where $nd\Omega$ is the number of particles scattered into an element of solid angle $d\Omega$ when n_0 electrons impinge normally on the film. This distribution function leads to a physically absurd result unless Θ remains small. Neher's[17] data indicate that the number of electrons emerging along paths nearly parallel to the foil surface is small, and that

$$nd\Omega\propto\exp(-\Theta^2/2\lambda^2)\cos\Theta d\Omega$$

is more nearly correct. Then

$$\langle\sec\Theta\rangle_{Av}=\frac{\int_0^{\pi/2}\sec\Theta\exp(-\Theta^2/2\lambda^2)\cos\Theta\sin\Theta d\Theta}{\int_0^{\pi/2}\exp(-\Theta^2/2\lambda^2)\cos\Theta\sin\Theta d\Theta}.$$

This reduces to $\langle\sec\Theta\rangle_{Av}=2B(p)/A(p)$, where

$$B(p)=\int_0^{\pi/2p}\exp(-u^2)\sin pudu,$$

$$A(p)=\int_0^{\pi/2p}\exp(-u^2)\sin 2pudu$$

and $p=\lambda\sqrt{2}$.

For small values of p, $B(p)$ and $A(p)$ are approximately equal to $B_\infty(p)$ and $A_\infty(p)$, where

$$B_\infty(p)=\int_0^\infty\exp(-u^2)\sin pudu=\exp(-p^2/4)\int_0^{p/2}\exp(s^2)ds$$

and $A_\infty(p)=B_\infty(2p)$. The function $\int_0^{p/2}\exp(s^2)ds$ has been tabulated by Dawson.[18] Investigation shows that $B_\infty(p)$ and $A_\infty(p)$ approximate $B(p)$ and $A(p)$ within 0.1 percent as long as $p\leqq0.5$, but that the approximations become rapidly worse for larger values of p. In the interval $0.6\leqq p\leqq2.0$, the Gauss 4-point numerical method of integration[19] yields values of $B(p)$ and $A(p)$ that are in error by less than 1 percent.

The effective thickness of the target is

$$X=2\int_0^{X_0}[B(p)/A(p)]dx=\frac{U^2}{170}\int_0^{4.22/U}p[B(p)/A(p)]dp;$$

X was evaluated for various values of U by further application of the Gauss 4-point method, after determining $pB(p)/A(p)$ for the appropriate values of p. The results appear in the fourth column of Table I.

Retardation

Let $i(U)dx$ be the $L\alpha$ doublet intensity from an ideally thin target of thickness dx. A real target of thickness X will give an intensity $\langle i(U')\rangle_{Av}$, where $U'e$ is the energy of the electron after it has penetrated to a depth x, and $\langle i\rangle_{Av}$ denotes an average of i taken from $x=0$ to $x=X$. To a first

[16] W. Bothe, *Handbuch der Physik XXII/2* (Springer, Berlin, 1933).
[17] H. V. Neher, Phys. Rev. **37**, 655 (1931).
[18] H. G. Dawson, Proc. Math. Soc. London **29[II]**, 519 (1897/98).
[19] K. Mader, *Handbuch der Physik, III* (Springer, Berlin, 1928).

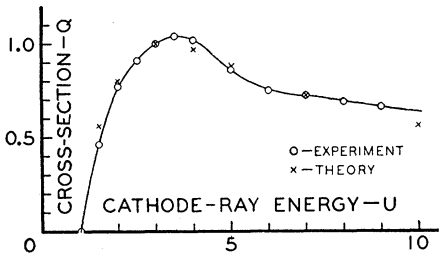

FIG. 3. The ionization cross section of the L_{III} state of silver.

approximation,[3]

$$\langle i(U')\rangle_{Av}=i(U)+\tfrac{1}{2}X\frac{di}{dU}\left(\frac{dU'}{dx}\right)_{U'=U}.$$

The observed intensities, corrected for fluorescence and diffusion of the cathode rays, can be represented to a sufficient degree of approximation by the analytical expression $i(U)=KU^{-1}\log U$. Therefore $\langle i(U')\rangle_{Av}=i(U)+g(U)$, where

$$g(U)=\tfrac{1}{2}KXU^{-2}(1-\log U)\left(\frac{dU'}{dx}\right)_{U'=U}$$

and the observed data, after correction for the effects of fluorescence and diffusion, can be further corrected to compensate for retardation by multiplying them by $c_x=i(U)/[i(U)+g(U)]$. Terrill[20] and Williams[21] have found that, for cathode rays of about the energy used in the present investigation, $(dU'/dx)\propto(1/U')^m$. Terrill finds $m=1$, while Williams gives $m=0.9$, and reports a constant of proportionality much smaller than Terrill's. Since Terrill worked with silver and other metal foils, and Williams' work was done in a cloud chamber, it seems better to use Terrill's results here. Calculation on this basis yields the values of c_x shown in Table I.

Rediffusion

When cathode rays fall normally on a solid body, some of them, experiencing within the body deflections greater than 90°, are "rediffused" so that they appear to have been reflected from the body. The presence of the beryllium backing therefore causes some of the cathode rays to pass through the target twice. The fraction p of electrons experiencing rediffusion when cathode rays are incident normally on a plane surface of a material is called the "rediffusion constant" of the material. Neher's[17] measurements on beryllium show that for 130-kv electrons, $p=0.0248$, and for 70-kv electrons, $p=0.0291$. Using data obtained by Schonland[22] for heavier elements and voltages from 10 to 100 kv, Webster, Clark, and Hansen[2] had previously estimated that $p=0.043$. A reasonable estimate, therefore, is that in the range from 10 to 30 kv $p=0.035$ for beryllium. Following Webster, Clark, and Hansen, let $i(U)=i_0(U)+r(U)$, where $i_0(U)$ is the x-ray intensity per unit length of path of the cathode rays, and $r(U)$ is the intensity per unit thickness of target film. (The diffusion of rediffused electrons is a second-order effect which will be ignored.) It follows[2] that

$$r(U)=2p\int_{1/U}^{1}F(W)\cdot i_0(UW)dW,$$

$pF(W)$ being the fraction of incident electrons which are rediffused with energies between UeW and $Ue(W+dW)$. All of the available evidence[2] on the energy distribution of rediffused electrons indicates that $F(W)=(q+1)(q+2)(1-W)W^q$ is a suitable approximation if q is an appropriate integer. For beryllium $q=3$ seems to be a good choice. An estimate of $r(U)$ satisfactory for the present purpose can be obtained by using for $i_0(UW)$ the empirical approximation $i_0(U)=KU^{-1}\log U$. Setting $c_r=i_0(U)/[i_0(U)+r(U)]$, so that $i_0(U)=c_r\cdot i(U)$, one obtains for c_r the values shown in Table I.

IV. RESULTS

Curve I in Fig. 2 shows the observed specific intensity of the $L\alpha$ doublet (corrected for the continuous background). To the ordinates must be applied first the correction for fluorescence, $\phi(\nu_0)$, and then the corrections for diffusion, rediffusion, and retardation. Curve II in Fig. 2 is a plot of the observed $L\alpha$ intensity minus the intensity due to fluorescence. The ordinates of curve III, the intensities per unit effective target thickness after all corrections have been made, are obtained from those of curve II by multiplying them by $c_r\cdot c_x\cdot X_0/X$. The finally corrected re-

[20] H. M. Terrill, Phys. Rev. **22**, 101 (1923).
[21] E. J. Williams, Proc. Roy. Soc. **A130**, 310 (1930-31).
[22] B. F. J. Schonland, Proc. Roy. Soc. **A108**, 187 (1925).

sults, which are proportional to the ionization cross section Q, are listed under i_0 in Table I and are replotted in Fig. 3, the ordinates here being arbitrary units chosen to make $Q=1$ when $U=3$. The theoretical values of ionization cross section calculated by Burhop[4] are shown as crosses; they are fitted to the experimental curve at $U=3$. Burhop states that his results have been evaluated numerically to "a few percent." The measured values of Q are tabulated in Table I.

There are four sources of error in the experimental results, namely:

(1) the measurement of target potential,
(2) the measurement of tube current,
(3) the measurement of x-ray intensity, and
(4) the corrections.

The target potential was measured with a low voltage laboratory standard voltmeter and wire-wound multiplier. Checks with a potentiometer and Wheatstone bridge showed the calibration of the combination to be correct within 0.25 percent, when account is taken of the corona and surface leakage in the multiplier. Errors in the measurement of target potential therefore make no significant contribution to the experimental error.

The target currents could be read within about 1 percent.

The electrometer tube circuit introduced uncertainty into the intensity measurements through fluctuations and through changes in sensitivity. Frequent checks on the sensitivity ensured that the errors due to changes in sensitivity are certainly less than 2 percent. Fluctuations are partially neutralized by drawing a smooth curve among the points; they render the location of the curve uncertain by not more than 2 percent. This estimate includes an allowance for zero drift.

The geometrical thickness X_0 of the target is uncertain by about 15 percent, but it transpires that a 15 percent error in X_0 causes an error of little more than 1 percent in the measured value of ionization cross section at $U=9$ relative to that at $U=3$. The errors introduced at other values of U between 1.5 and 9 are even smaller. Departures from the assumed law of distribution-in-angle of the diffused cathode rays may introduce further errors, but no estimate of their size can be made except that they are probably not as large as 1 percent. They will therefore be neglected.

The corrections for rediffusion are so small and are based on such accurate information that they do not contribute significantly to the experimental error. The same is true of the corrections for retardation, except for that at $U=1.5$, which renders Q at that voltage uncertain by about 5 percent.

The square root of the sum of the squares of the errors that have been mentioned is 6 percent at $U=1.5$ and 3 percent for the larger values of U. These are the experimental uncertainties in the ordinates of Fig. 3 at small values of U; at large values of U there is in addition the possibility of errors introduced by the assumptions underlying the correction for fluorescence. Concerning these errors, all that can be said is that they are negligible for small values of U, say $U<7$, and that the computed correction for fluorescence is, if anything, too small. (It is perhaps worth mentioning here that Burhop's results were not consulted until after the calculation of the corrections had been completed.) For $U>7$, the curve does not fall off as rapidly as the theory predicts; a 50 percent increase in the correction for fluorescence would be necessary in order to bring the experimental point at $U=9$ into agreement with the theoretical prediction. Without making actual measurements on the intensity of the x-ray continuum from beryllium in directions making various angles with the target, over the range of tube voltages used in the present experiment, one cannot decide with assurance that, for $U=9$, the present correction for fluorescence is in error by less than 50 percent. It should be pointed out, however, that the experimental results of Webster, Hansen, and Duveneck,[3] of Smith,[23] of Tate and Smith,[24] of Webster, Pockman, and Kirkpatrick,[25] and of Webster, Pockman, Harworth, and Kirkpatrick[26] all show that the theoretical calculations (based on the Born approximation) predict cross sections that decrease too rapidly with increasing U when U is large.

In conclusion, it seems of interest to compare

[23] P. T. Smith, Phys. Rev. **36**, 1293 (1930).
[24] J. T. Tate and P. T. Smith, Phys. Rev. **39**, 270 (1932).
[25] Webster, Pockman, and Kirkpatrick, Phys. Rev. **44**, 130 (1933).
[26] Cited by Burhop, reference 4.

the present measurements on the L_{III} state of silver with those of Webster, Hansen, and Duveneck[3] on the K state of the same element. Two facts are clearly shown:

(1) The cross sections for excitation to the K state and the L_{III} state are both greatest for cathode rays whose kinetic energies are about 3.5 times the ionization energy of the state in question.

(2) When $U>3.5$, the decrease in ionization cross section with increasing cathode-ray energy is more rapid for the L_{III} state than for the K state.

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Finite Self-Energies in Radiation Theory. Part III

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The invariant field theory of Part II is interpreted, in agreement with F. Bopp, as Maxwell's theory with a linear differential relation between the fields E, B and D, H involving a new constant k which measures the reciprocal radius of the electron. The former "mesonic field" of minimum frequency $\nu_0=kc/2\pi$ represents polarization of the vacuum. The electron is a singularity in the D, H field whereas E, B remain finite. Instead of obeying dynamical equations of motion, the electron moves under the condition that the Lorentz force vanishes identically on the singularity, so that no work is done on the particle. All energy is located in the field. In this respect the theory is unitary. Electromagnetic and inert mass are identical. In contrast to Dirac's classical electron which is subject to advanced and retarded potentials and displays self-acceleration, the field theory works with retarded potentials only, and self-acceleration is avoided. Stable equilibrium between electrons and radiation is granted by spontaneous and induced transitions, similar to Einstein's derivation of Planck's radiation formula. In spite of displaying a magnetic moment the electron does not have magnetic self-energy, so that its radius is the ordinary electrostatic radius $1/k=e^2/2mc^2$. In contrast to Born-Infeld's non-linear theory, our field equations allow a Fourier representation as a basis for the quantum theory of Part IV.

1. CLASSICAL FIELD THEORY

THE modification of electrostatics proposed in Part I and its electromagnetic continuation discussed in Part II [1] rest on the assumption that vacuum is polarizable, as described by linear differential relations between the vectors E, B and D, H (for details see Section 6):

$$D=E-k^{-2}\Box E, \quad H=B-k^{-2}\Box B, \qquad (1)$$

where $\Box$ is the Laplace operator in x, y, z, *ict*. The constant k of dimension $[l^{-1}]$ determines the minimum frequency $\nu_0=kc/2\pi$ of waves of polarization or "meson waves." k also plays the role of the reciprocal electronic radius, although the charges ϵ are condensed in mathematical points only. The simplicity and naturalness of our approach are demonstrated by the fact that the same modification of Maxwell's theory has been proposed independently and simultaneously by F. Bopp.[2] Whereas we began in Part I with a Fourier representation of the field of a point particle with finite self-energy, Bopp started from a formal generalization of the Lagrangian function of the field E, $B=f_{\alpha\beta}$, namely,

$$L=-(1/16)\{(f_{\alpha\beta})^2+k^{-2}(\partial f_{\alpha\beta}/\partial x_\gamma)^2\}+J_\alpha\varphi_\alpha \qquad (2)$$

where J is the 4 current and φ is the 4 potential.

The relation to other field theories (Maxwell, Born-Infeld) become obvious if the vectors E, H of Part II are called E, B, and the vectors E'', H'' are called D, H. Our "meson field" $E'=D-H$

[1] A. Landé, Phys. Rev. **60**, 121 (1941). A. Landé and L. H. Thomas **60**, 541 (1941).

[2] F. Bopp, Ann. d. Physik **38**, 345 (1940).