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Auger Transitions and Widths of X-Ray Energy Levels

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The width of an x-ray energy level depends to a large extent on the probabilities of Auger transitions from that level. Consequently, rapid variations in line width with atomic number are to be expected whenever sudden changes in Auger transition probabilities occur. Atomic number ranges wherein large variations might be expected have been studied. Part of the width variation of $L\beta_3$ for elements $73 \leq Z \leq 92$ is attributed to the Auger transitions $M_{\text{III}} \rightarrow M_{\text{V}} N_{\text{IV, V}}$. The transitions $N_{\text{II}} \rightarrow N_{\text{III}} O_{\text{II, III}}$, $N_{\text{II}} \rightarrow N_{\text{IV, V}} N_{\text{IV, V}}$, and $N_{\text{III}} \rightarrow N_{\text{IV, V}} N_{\text{IV, V}}$ are shown to explain an intensity anomaly of $M\zeta$ for elements $38 \leq Z \leq 58$. Several other irregular line width variations are correlated with changes in the probabilities of Auger transitions. The evidence supports the theory that Auger transitions are primarily responsible for anomalous changes in line width.

I. INTRODUCTION

IN 1935 Coster and Kronig[1] suggested that the relative diffuseness of the lines arising from transitions from L_{I} initial states (hereafter called L_{I} lines) was due to the broadening of the L_{I} level by Auger transitions. Large probabilities of Auger transitions from a given state decrease the mean life of atoms in this state and lead to an increase in the width of the energy level.[2] Theoretical calculations for gold (79) by L. Pincherle[3] and Ramberg and Richtmyer[4] have shown that Auger transitions account for the greater part of the widths of all levels except K. The theory that the level width depended on Auger transition probabilities was confirmed experimentally by the work of the author,[5] which showed that the width of the L_{I} level increased rapidly with atomic number in the range $73 \leq Z \leq 81$ where the probabilities for the Auger transitions $L_{\text{I}} \rightarrow L_{\text{III}} M_{\text{IV}}$ and $L_{\text{I}} \rightarrow L_{\text{III}} M_{\text{V}}$ are known to be swiftly increasing.

Since there are many Auger transition probabilities which vary rapidly with atomic number, it is clear that there should be other levels (in addition to the L_{I}) which show sudden changes in width from one element to the next. It is the purpose of this paper to investigate the regions in which important variations in Auger transition probabilities occur and to correlate with these variations certain apparently erratic changes in level widths.

II. PROBABLE AUGER TRANSITIONS

According to Coster and Kronig,[1] the probability of an Auger transition in which electron A goes from a state with a wave function $\psi_1(A)$ to a state represented by $\psi_2(A)$, while electron B passes from a state represented by $\psi_1(B)$ to one

[1] D. Coster and R. DeL. Kronig, Physica **2**, 14 (1935).
[2] V. Weisskopf and E. Wigner, Zeits. f. Physik **63**, 54 (1930).
[3] L. Pincherle, Nuovo Cimento (N.S.) **81**, 162 (1935).
[4] E. G. Ramberg and F. K. Richtmyer, Phys. Rev. **51**, 913 (1937).
[5] J. N. Cooper, Phys. Rev. **61**, 234 (1942).

TABLE I. Possible Auger transitions and the elements in which they may occur.

$L_I \rightarrow L_{II}N_I$	below	$Z=70$	$M_{II} \rightarrow M_{III}N_{IV}$	above	$Z=65$	$N_I \rightarrow N_{III}N_{VI}$	below	$Z=92$
N_{II}	below	75		below	48	N_V	below	53
N_{III}	below	81	N_V	above	65	N_{IV}	below	53
N_{IV}	below	92		below	48			
						$N_{IV}N_V$	below	84
$L_{III}M_I$	below	31	$M_{IV}N_I$	below	88	N_{IV}	below	80
M_{II}	below	36	M_V	below	35			
M_{III}	below	37	M_{IV}	below	35	N_VN_V	below	87
M_{IV}	above	73						
	below	50						
M_V	above	77	$M_{III} \rightarrow M_{IV}N_V$	below	85	$N_{II} \rightarrow N_{III}O_{III}$	above	66
	below	50	N_{IV}	below	84		below	55
			N_{III}	below	71	$N_{III}O_{II}$	below	86
			N_{II}	below	65		above	66
$L_{II} \rightarrow L_{III}M_{IV}$	below	30	N_I	below	55		below	55
M_V	below	30	M_V	below	36	N_{VII}	below	80
M_V	above	90	M_{IV}	below	36	N_{VI}	below	80
			M_VN_V	below	91			
			N_{IV}	below	89	$N_{IV}N_V$	below	60
$M_I \rightarrow M_{II}N_{VII}$	below	91	N_{III}	below	77	N_{IV}	below	60
N_{VI}	below	91	N_{II}	below	70			
N_V	below	72	N_I	below	57	N_VN_V	below	62
N_{IV}	below	71	M_V	below	37			
N_{III}	below	53						
N_{II}	below	53	$M_{IV} \rightarrow M_VO_{III}$	below	85	$N_{III} \rightarrow N_{IV}O_I$	below	87
N_I	below	47	O_{II}	below	82	N_{VII}	below	86
M_V	below	34				N_{VI}	below	85
M_{IV}	below	34				N_V	below	57
			$N_I \rightarrow N_{II}O_{III}$	below	88	N_{IV}	below	56
$M_{III}N_V$	below	87	O_{II}	below	85			
N_I	below	50	O_I	below	79	N_VO_I	below	89
M_V	below	34	N_{VII}	below	80	N_{VII}	below	87
M_{IV}	below	34	N_{VI}	below	80	N_{VI}	below	86
			N_V	below	53	N_V	below	57
$M_{IV}M_V$	below	44	N_{IV}	below	53	$N_{IV} \rightarrow N_VO_{IV,V}$	below	81
M_{IV}	below	43						
M_VM_V	below	45				$N_V \rightarrow N_{VI}N_{VII}$	below	91
						N_{VI}	below	91
						$N_{VII}N_{VII}$	below	91

described by $\psi_2(B)$, is proportional to the square of the matrix element

$$v = \int \frac{\psi_1^*(A)\psi_2(A)d\tau_1\psi_1^*(B)\psi_2(B)d\tau_2}{r_{12}},$$

where $d\tau_1$ and $d\tau_2$ are volume elements separated by r_{12} and the integration extends over all space. In order to obtain a large value of v, $\psi_1^*(A)$ and $\psi_2(A)$ must overlap considerably as must $\psi_1^*(B)$ and $\psi_2(B)$, while r_{12} should be small in the region of appreciable overlapping. $\psi_1^*(A)$ and $\psi_2(A)$ generally overlap most when both initial and final states of electron A lie in the same shell.

The energy W of the ejected electron is very important in determining the overlapping of $\psi_1^*(B)$ and $\psi_2(B)$. If W is either very small or very large, the overlapping is slight and v will be small. The transition probability as a function of W increases to a maximum and then falls off, so that the probability of an Auger transition in which the ejected electron has a high energy is small.

Many Auger transitions are allowed for some elements and forbidden for others. An Auger transition is possible only when the energy of the initial state is greater than that of the final state (of double ionization) and is most probable when the energy difference is not too great. Consequently, rapid fluctuations in the probability of such a transition are most likely in the atomic number range wherein the transition first becomes possible. In view of this fact it is of considerable value to know the atomic number at which various Auger transitions first become permitted. A systematic investigation was made of all Auger transitions which are allowed for some elements and forbidden for others. The result of

TABLE II. Widths of L series lines in electron volts.

Line	Transition	Ag(47)[a]	Ta(73)	W(74)	Rh(75)	Os(76)	Ir(77)	Pt(78)	Au(79)	Tl(81)[b]	Pb(82)[c]	U(92)[d]
$L\beta_3$	$L_{\mathrm{I}}\rightarrow M_{\mathrm{III}}$	6.6	11.7	12.3	13.6	14.7	14.9	17.2	18.4	19.7	18.3	18.8
$L\beta_4$	M_{II}	5.9	14.1	15.1	16.4	17.1	17.7	19.4	20.7	21.6	22.2	32.2
$L\gamma_2$	N_{II}	11.0	11.6	12.1	13.2	14.0	14.6	—	18.1	19.3	19.9	39.4
$L\gamma_3$	N_{III}	10.2	10.0	11.0	12.0	12.6	13.3	15.6	16.4	17.8	19.4	32.4
$L\beta_1$	$L_{\mathrm{II}}M_{\mathrm{IV}}$	2.4	6.6	6.5	6.5	6.6	6.8	8.3	8.5	9.0	8.4	14.3
$L\gamma_1$	N_{IV}	3.95	9.5	9.3	9.5	9.4	9.6	11.4	11.5	11.7	11.4	15.9
$L\beta_2$	$L_{\mathrm{III}}N_{\mathrm{V}}$	3.72	9.8	9.8	9.8	9.9	9.7	11.1	11.2	11.5	11.8	16.1

[a] See reference 6 of text. [b] See reference 5 of text. [c] See reference 9 of text. [d] See reference 7 of text.

TABLE III. Ratios of line widths.

Ratios	Ag(47)	Ta(73)	W(74)	Rh(75)	Os(76)	Ir(77)	Pt(78)	Au(79)	Tl(81)	Pb(82)	U(92)
$L\beta_3/L\beta_2$	1.77	1.20	1.26	1.39	1.49	1.54	1.59	1.72	1.80	1.55	1.17
$L\beta_4/L\beta_2$	1.59	1.44	1.54	1.67	1.73	1.83	1.82	1.96	2.00	1.88	2.00
$L\gamma_2/L\beta_2$	2.96	1.18	1.24	1.35	1.41	1.51	—	1.66	1.71	1.69	2.45
$L\gamma_3/L\beta_2$	2.74	1.02	1.12	1.23	1.27	1.37	1.40	1.49	1.58	1.64	2.02
$L\beta_2/L\beta_1$	1.55	1.49	1.51	1.51	1.50	1.43	1.40	1.35	1.33	1.40	1.12
$L\beta_2/L\gamma_1$	0.94	1.03	1.05	1.03	1.05	1.01	1.00	0.99	1.02	1.04	1.01

this work is summarized in Table I, which lists the transition and the atomic numbers for which it is allowed. It will be observed that certain ones are allowed above one atomic number and below another, while they are forbidden for elements in between.

The atomic number of the element for which the transition is first allowed was obtained by calculating the energy difference between the initial and final states. (A transition is obviously forbidden if the difference is negative.) Values of ν/R for the various states were taken from Siegbahn's *Spectroscopie der Rontgenstrahlen, Zweite Auflage.* The assumption was made that the energy of the final state of double ionization of element Z was equal to the sum of the energy of the inner level for element Z and the energy of the outer level for element $Z+1$. The energy so obtained is a good approximation, although not exactly correct. The uncertainty in the energy of the state of double ionization leads to an uncertainty in the atomic number at which a given Auger transition becomes possible. In the worst cases this atomic number may be incorrect by as much as three or four.

III. WIDTH VARIATIONS AND AUGER TRANSITIONS

From Table I it is clear that there exist many Auger transitions which are allowed for some elements and forbidden for others. Certain of these transitions unquestionably play an important role in determining the widths of x-ray energy levels, while others are doubtlessly negligible. It is not immediately obvious which of the transitions become highly probable, although it would be possible to find out either by lengthy calculations or by detailed experiment. That there already exist sufficient line width data to establish the importance of certain of these transitions will be shown in this paper.

There have been fairly complete studies of the widths of L series lines for several different elements. An exhaustive investigation of the entire L series of silver (47) was made by Parratt.[6] Measurements of the widths of the L lines of uranium (92) are reported by Williams.[7] The gold L series was measured by Richtmyer, Barnes, and Ramberg,[8] while Shrader[9] made a similar study of lead (82). Widths of selected L series lines for elements $73 \leq Z \leq 81$ have been measured by the author.[5] These data reveal many irregular changes in the widths of levels with atomic number. Most of these variations are readily explained in terms of Auger transitions listed in Table I.

[6] L. G. Parratt, Phys. Rev. **54**, 99 (1938).
[7] J. H. Williams, Phys. Rev. **37**, 1431 (1931).
[8] Richtmyer, Barnes, and Ramberg, Phys. Rev. **46**, 843 (1934).
[9] R. E. Shrader, Cornell thesis (1936).

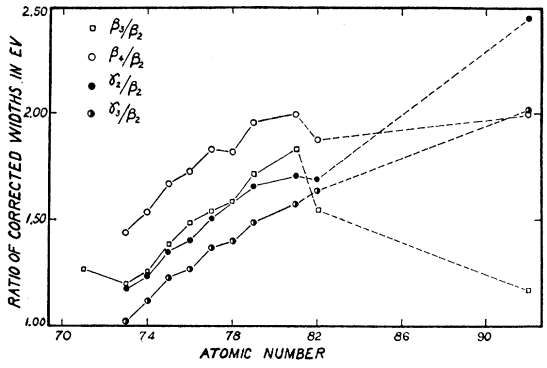

FIG. 1. Ratios of the widths of L_I lines to the width of $L\beta_2(L_{III}\rightarrow N_V)$.

In studying sudden variations in the width of a line as a function of atomic number one must remember that, in general, the width (in electron volts) increases with atomic number. This fact becomes of particular importance when one is considering elements that differ appreciably in atomic number. For example, the L series lines of gold (79) are all wider than the corresponding lines for silver (47), in most cases by a factor of two or three. This width increase for most lines is associated with the normal change with atomic number. One way of treating data which eliminates this problem is to deal with width ratios rather than widths. This essentially amounts to assuming that the "normal" width increase with atomic number is such as to leave the ratios unchanged. Available data indicate that this is a good approximation.

Width ratios have one other great advantage when one is dealing with data taken by several observers, each of whom used different crystals on different spectrometers and then corrected for crystal diffraction patterns on fundamentally different assumptions. That advantage is that the ratios are far less sensitive to crystals, spectrometers, and corrections than are the actual widths themselves. To be sure, the ratios are not independent of corrections for the finite resolving power of the spectrometers, but the percentage variation in the ratios is much less than that in the widths.

In Table II are given the corrected widths in electron volts of seven L series lines for each of eleven elements, and in Table III the ratio of the width of each of these lines to the width of $L\beta_2$ is recorded. $L\beta_2$ is chosen as the reference line because it arises from an L_{III} transition and the width of the L_{III} level is probably the least influenced by Auger transitions. The data on silver are by Parratt,[6] those on lead by Shrader,[9] and those on uranium come from Williams'[7] paper. The line widths of the other elements were measured by the author.[5] It should be mentioned that the type of corrections applied by each of these observers was different. Williams' data were corrected on the assumption that shapes were given by the Gaussian error function and his corrections were much smaller than those applied by the other observers.

In Fig. 1 the ratios of the widths of $L\beta_3$, $L\beta_4$, $L\gamma_2$, and $L\gamma_3$ to $L\beta_2$ are plotted as a function of atomic number. The first four lines all arise from transitions from the L_I state, while $L\beta_2$ comes from an $L_{III}\rightarrow N_V$ transition. There is every reason to believe that the width of $L\beta_2$ is behaving "normally" in this region; that is, neither the initial nor the final states are changing much in width. If this is true, it is clear that the width of the L_I level must be increasing between tantalum (73) and lead (82) since all the lines arising from L_I transitions show large and roughly equal increases in width in this range of atomic numbers. These width increases have been shown[5] to be associated with the rapidly increasing probabilities of the Auger transitions $L_I\rightarrow L_{III}M_V$ (possible above $Z=73$) and $L_I \rightarrow L_{III}M_{IV}$ (possible above $Z=77$).

Since all the lines (except $L\beta_2$) have the same initial state, the four curves should behave similarly unless the width of the final state is varying irregularly with atomic number. The curves do not behave alike above $Z=81$, so the conclusion is that here the widths of the final states must also be varying. In several cases the changes in width ratios are so great that the importance of certain Auger transitions are well established, while in other cases the variations are only of the order of the experimental errors. In such cases the variations may be due to Auger transitions and have been attributed to them, although it is possible that they are due to a fortuitous combination of possible errors.

The line $L\beta_3$ narrows greatly between thallium (81) and uranium (92). Since $L\beta_3$ arises from an $L_I\rightarrow M_{III}$ transition, this means that the M_{III}

level must be decreasing in width. Table I reveals that the Auger transitions $M_{III}\rightarrow M_V N_{IV}$ and $M_{III}\rightarrow M_V N_V$ become possible for elements of atomic number below 90 and are forbidden for elements of higher atomic number. Also the transitions $M_{III}\rightarrow M_{IV}N_{IV}$ and $M_{III}\rightarrow M_{IV}N_V$ are forbidden for uranium and allowed for the other elements. (See Fig. 2, reference 10 and Fig. 1, reference 11.) Judging by the great drop in the relative width of $L\beta_3$ these transitions must account for much of the width of the M_{III} level for elements around $Z=81$.

Since the probability of the transitions will decrease as the energy of the ejected electron becomes too great, the width of the M_{III} level might be expected to be less for Ta(73) than for Tl(81). This argument is supported by the observation that the actual width increase of $L\beta_3$ between these elements is greater than that of any of the other L_I lines. This would be expected if the Auger transitions $M_{III}\rightarrow M_V N_{IV}$ and $M_{III}\rightarrow M_V N_V$ had a maximum probability in the neighborhood of $Z=81$.

Further support for this explanation can be found in the work on $M\alpha$-satellites by Hirsh,[10] who has shown that these satellities have a maximum relative intensity near lead (82). Since these satellites arise from an $M_V N_{IV,V}$ initial state, they will obviously increase in intensity as the Auger transitions $M_{III}\rightarrow M_V N_{IV,V}$ become more probable. Thus the importance of these radiationless transitions is established by independent observations.

The width of $L\beta_4(L_I\rightarrow M_{II})$ appears to be relatively stable between thallium and uranium although data on intermediate elements might indicate width variations associated with the Auger transition $M_{II}\rightarrow M_{IV}N_I$ which is possible[11] for elements of atomic number below 88.

Both $L\gamma_2(L_I\rightarrow N_{II})$ and $L\gamma_3(L_I\rightarrow N_{III})$ are much wider for uranium than might have been expected, showing that the N_{II}, and *particularly* the N_{III}, levels have become very wide. There is nothing in Table I which would predict such an increase. The addition of ten electrons in outer levels between lead and uranium will make many new Auger transitions possible and it is conceivable that one of them might be very probable.

[10] F. R. Hirsh, Phys. Rev. **57**, 662 (1940).
[11] F. R. Hirsh, Phys. Rev. **59**, 766 (1941), Fig. 1.

Figure 2 shows the width ratios $L\beta_2/L\beta_1$ and $L\beta_2/L\gamma_1$. $L\beta_2$ arises from the transition $L_{III}\rightarrow N_V$, while $L\beta_1$ and $L\gamma_1$ have L_{II} as an initial state and their final states are M_{IV} and N_{IV}, respectively. It is immediately clear that these ratios are much more constant than those involving L_I lines. The width ratio $L\beta_2/L\gamma_1$ is almost constant for all the elements listed, but the ratio $L\beta_2/L\beta_1$ decreases with atomic number. There are a number of Auger transitions which may play a part in this. First, $L\beta_1$ may be wider at uranium because the Auger transition $L_{II}\rightarrow L_{III}M_V$ becomes possible above $Z=90$ and may contribute to the width of the L_{II} level. If this is true, the width of $L\gamma_1$ should be similarly influenced, and in order to explain its constancy, it is possible to point out that the N_{IV} level may have narrowed because the transitions $N_{IV}\rightarrow N_V O_{IV,V}$ are forbidden for elements above 81. Part of the decrease for the width ratio $L\beta_2/L\beta_1$ may be due to a narrowing of the N_V level for uranium, since the transitions $N_V\rightarrow N_{VI}N_{VI,VII}$ and $N_V\rightarrow N_{VII}N_{VI,III}$ are impossible for that element. Another factor which contributes to the small ratio for uranium is the smaller correction subtracted from the widths in this case. (Obviously, too small a correction would tend to make this ratio nearer to unity.)

Some interesting changes in the widths of levels occur between elements 73 and 47. For silver the lines $L\gamma_2$ and $L\gamma_3$ are nearly twice as wide as $L\beta_3$ and $L\beta_4$ which indicates that the N_{II} and N_{III} levels are much wider than the M_{II} and M_{III} levels. For elements 73 to 82 the M_{II}

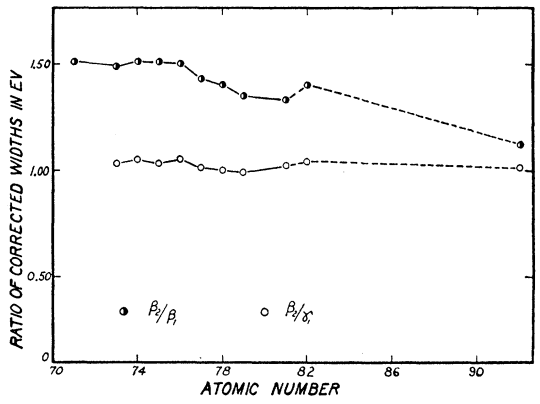

FIG. 2. The width ratios $L\beta_2/L\beta_1$ and $L\beta_2/L\gamma_1$ as a function of atomic number. (Transitions: $L\beta_2$, $L_{III}\rightarrow N_V$; $L\beta_1$, $L_{II}\rightarrow M_{IV}$; $L\gamma_1$, $L_{II}\rightarrow N_{IV}$.)

TABLE IV. Widths of $M\zeta$ in electron volts (after Kiessig).

Element	Width	Element	Width	Element	Width
Sr(38)	1.99	Rh(45)	4.57	Sb(51)	18.5
Zr(40)	2.06	Pd(46)	6.74	Te(52)	18.5
Nb(41)	3.44	Ag(47)	13.6	Ba(56)	7.0
Mo(42)	3.05	Cd(48)	14.8	La(57)	9.1
Ru(44)	3.79	Sn(50)	16.4	Ce(58)	10.6

and M_{III} levels are wider than the corresponding N levels, but for uranium the N levels are again wider. This can be partially explained as follows: (1) For elements of atomic number 70 to 82 the M_{III} level should be wide because of the high probabilities of the transitions $M_{III}\rightarrow M_V N_{IV,V}$ and (possibly) $M_{III}\rightarrow M_{IV}N_{IV,V}$ which have already been mentioned in connection with the small width of $UL\beta_3$. Also, for silver the Auger transitions $M_{III}\rightarrow M_{IV}N_I$ and $M_{III}\rightarrow M_V N_{I,II}$ are impossible, whereas they should contribute to the width of the M_{III} level for elements of higher atomic number. (2) The M_{II} level may be expected to be wide for elements not too far above 70 because of the Auger transitions $M_{II}\rightarrow M_{III}N_{IV}$ and $M_{II}\rightarrow M_{III}N_V$. These transitions are forbidden for elements between (about) 65 and 48. Whether or not they are allowed for silver is doubtful, but judging by the small width of the Ag M_{II} level it seems certain that these transitions are very improbable for silver even if they are possible.

While these Auger transitions may serve to explain why the M levels are wider than the N levels for elements of atomic number between 73 and 82, still the widths of the N_{II} and N_{III} levels for silver are so very great that their excessive widths require further explanation. The N_{II} level is widened by the transitions $N_{II}\rightarrow N_{III}O_{II,III}$ which are possible for elements of atomic number below 55 and by $N_{II}\rightarrow N_{IV,V}N_{IV,V}$ which are allowed below $Z=60$. The great width of the N_{III} level is attributed to the transitions $N_{III}\rightarrow N_{IV,V}N_{IV,V}$ which are possible for elements below $Z=56$.

Line width data by Kiessig[12] support the contention that these transitions must be highly probable for silver (47). In Table IV are the widths of $M\zeta$ for fifteen elements as reported by Kiessig. The line $M\zeta$ arises from a transition $M_V\rightarrow N_{II,III}$. The width increases rapidly from $Z=38$ to $Z=52$, showing that the N_{II} and N_{III} levels are widening rapidly. (The M_V level is "normal" for silver and there is no reason to think that these width changes are due to the initial level.) The width has decreased tremendously at $Z=56$, just as would be expected from a consideration of Auger probabilities. Kiessig did not offer an explanation for the intensity anomaly, but pointed out that the elements at the beginning of the fifth period of the periodic table (36–54) had small $M\zeta$ widths and those at the end had great widths, while the three elements at the beginning of the sixth period again had narrow breadths. Siegbahn and Magnusson[13] had previously found that the $M\zeta$ lines of Br(35) and Rb(37) were very sharp, so that the generalization that the width increased as one went through an atomic period was not justified. Now it seems virtually certain that the above-mentioned Auger transitions provide the correct explanation for this intensity anomaly.

It is not inconceivable that the width increase between element 56 and 58 is due to the increasing probability of $N_{II}\rightarrow N_{IV,V}N_{IV,V}$ but the data do not cover a big enough atomic number range to be conclusive. If the width of $M\zeta$ were shown to be much less above $Z=60$, this explanation would be much better established.

The width of the N_{II} level is consistently wider than that of the N_{III} level for all elements considered here. This is in accord with the general rule that for different levels having the same total quantum number the level width decreases with increasing azimuthal quantum number. An exception to this rule is found in silver where the M_{III} level has a greater width than the M_{II} level. The unexpectedly large M_{III} width may be associated with relatively great probabilities of the transitions $M_{III}\rightarrow M_{IV}N_I$ and $M_{III}\rightarrow M_V N_I$, possible below atomic numbers 55 and 57, respectively.

It is interesting to note that the width ratios $L\beta_2/L\beta_1$ and $L\beta_2/L\gamma_1$ for silver are almost the same as for elements of atomic number 73 to 81. Since all the levels involved in these lines are associated with relatively high azimuthal quantum numbers, the Auger transitions generally

[12] H. Kiessig, Zeits. f. Physik **109**, 671 (1938), and **95**, 555 (1935).

[13] M. Siegbahn and T. Magnusson, Zeits. f. Physik **88**, 559 (1934).

contribute much less to the level widths. The relative constancy of the width ratios for these lines over so great an atomic number range supports the ratio method for handling width data for elements of vastly different atomic number.

IV. CONCLUSIONS

Since the width of a state is inversely proportional to the mean life of atoms in that state, a very probable Auger transition increases the level width. This results in a width increase for all lines which have this level as either an initial or final state. Because Auger transition probabilities frequently vary rapidly as a function of atomic number, one should expect rapid changes in line width. This prediction is verified. In this paper it has been shown that the width variation of $L\beta_3$ for elements $73 \leq Z \leq 92$ can be accounted for in terms of the Auger transitions $M_{III} \rightarrow M_V N_{IV,V}$. Also the anomalous width behavior of $M\zeta$ reported by Kiessig has been shown to be associated with the rapid changes in the Auger probabilities $N_{II} \rightarrow N_{III} O_{II,III}$, $N_{II} \rightarrow N_{IV,V} N_{IV,V}$, and $N_{III} \rightarrow N_{IV,V} N_{IV,V}$ for elements $38 \leq Z \leq 58$. Certain other irregular width variations have been shown to be explicable in terms of Auger transitions. More data on line widths would doubtless reveal other cases in which line widths are increased by Auger transitions.

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On Apertures of Transmission Type Electron Microscopes Using Magnetic Lenses

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The numerical aperture of light microscopes is larger than that of electron microscopes at their present state of development by a factor of 100 to 1000. This is due to the fact that electron lenses have higher aberrations than the highly corrected glass lens systems used in modern light microscopes. In the design of an electron microscope care has to be taken therefore that the proper numerical aperture should be used to give the optimum resolution. The majority of transmission type electron microscopes use at present magnetic lenses. For magnetic lenses of the axial field distribution $H(z)=H_0/[1+(z/a)^2]$ optimum conditions for size and location of apertures will be stated. The behavior of the condenser lens-objective lens system with respect to the angular aperture of the illuminating electron beam also will be discussed.

I. THE MAGNETIC LENS OF THE AXIAL FIELD DISTRIBUTION $H(z)=H_0/[1+(z/a)^2]$

THE small values of numerical apertures of electron microscopes can be achieved by allowing only such electrons to take part in the image formation whose paths do not pass through the lenses too far away from the optical axis. These paths are called paraxial. They can be described mathematically as the solutions $r=r(z)$ of the so-called paraxial ray differential equation:

$$\frac{d^2r(z)}{dz^2}+\frac{e}{8mV}H^2(z)r(z)=0, \qquad (1)$$

where $r=r(z)$ signifies the distance of the electron from the axis at the point z—the axial coordinate; $H(z)$ is the axial field distribution; e/m is the specific charge of the electron, and V is the accelerating potential for the electrons.

It has been shown[1] that for an axial field distribution, very closely approximated in practical magnetic lenses,

$$H(z)=H_0/[1+(z/a)^2]. \qquad (2)$$

The solution of Eq. (1) is given by

$$r(z)=a(1+(z/a)^2)^{\frac{1}{2}} \times\{C_1 \sin[(1+K^2)^{\frac{1}{2}} \operatorname{arc\,cot}(z/a)] + C_2 \cos[(1+K^2)^{\frac{1}{2}} \operatorname{arc\,cot}(z/a)]\}, \qquad (3)$$

[1] Walter Glaser, Zeits. f. Physik **117**, 285 (1941).