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Theory of Cosmic-Ray Mesons

J. HAMILTON, W. HEITLER, AND H. W. PENG
Dublin Institute for Advanced Studies, Dublin, Ireland

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The quantum theory of damping developed by two of us (Heitler and Peng) is applied to the production of mesons by proton-proton collisions. For this purpose the modification of the meson theory proposed by Møller and Rosenfeld is used. A primary radiation consisting of protons with a suitable energy spectrum is assumed, and it is shown that the rate of meson production is so high that nearly all mesons are produced in a top layer of the atmosphere of thickness 15–30 cm H_2O. The variation of the meson intensity with energy, height, and geomagnetic latitude is found to be in good agreement with the experiments. The transverse mesons, which have a very short lifetime, are seen to give, by decay, a satisfactory account of the soft component in the high atmosphere. A number of other effects (meson showers, transformation into neutretto's) are discussed in Sections VI and VII.

I. INTRODUCTION

UNTIL recently it has not been possible to apply Yukawa's meson theory of the nuclear forces to cosmic-ray mesons and thus to establish the identity of the particles predicted by Yukawa with the cosmic-ray mesons. The reason for this deep-rooted difficulty is the following: The interaction between a meson and a nuclear particle is, in contrast to the electron-light interaction, a strong one, and becomes increasingly stronger at high energies. This makes a proper treatment of the reaction forces exerted by the meson field on the nuclear particles imperative. However, as is well known, a treatment of the radiation reaction is intimately connected with the divergence difficulties occurring in every quantized field theory. To remove this difficulty two distinctly different sets of ideas have been put forward recently. Their difference can best be understood by remembering Lorentz's expansion of the reaction force which a light wave emitted by an electron exerts on the electron. This reaction force can be expanded according to powers of the electronic radius r: The first term is proportional to the acceleration and to r^{-1} thus diverging for a point particle. This term is usually thought to be included in the inertia of the particle. The second term, the usual damping term, is independent of r and proportional to the time derivative of the acceleration. Higher terms are proportional to positive powers of r and are, as a rule, neglected. In the meson case it is the charge-and-spin degrees of freedom of the nuclear particle which are coupled strongly with the meson field. We expect, therefore, that the reaction force will produce a twofold effect: (i) a large inertia to be attributed to these degrees of freedom, (ii) a large damping. In the first set of theories[1] mentioned above attention is concentrated on the first effect. It is clear that in order to make the inertia term finite, a finite particle radius has to be introduced which makes a relativistic treatment of the nuclear particle so far impossible. In the second kind of theory no physical reality is attributed to the first term of the reaction force at all. The particle is strictly considered as a point particle. By suitable subtraction the diverging inertia term of the reaction force is made to vanish (and so are the other diverging integrals occurring in the theory). The only finite part of the reaction force is then the damping term. Along this line Dirac's[2] new quantum-electrodynamics (confined so far to the electromagnetic field) is based. Independently of Dirac but in the same spirit, though less general, two of us (Heitler and Peng)[3] have made an attempt at "guessing" the correct equations of

[1] The so-called "strong coupling theory" put forward by Wentzel [Helv. Phys. Acta. **13**, 269 (1940)], Oppenheimer and Schwinger [Phys. Rev. **60**, 150 (1941)], and recently Pauli and Dancoff [Phys. Rev. **62**, 85 (1942)]. This theory is closely connected with former suggestions made by Bhabha, Ma, and Heitler, in which it was assumed that a proton and neutron can exist in excited charge and spin states.

[2] Dirac, Proc. Roy. Soc. **180**, 1 (1942) and lectures given at the Dublin Institute for Advanced Studies, to appear shortly in the Communication of the Dublin Institute for Advance Studies.

[3] Heitler and Peng, Proc. Camb. Phil. Soc. **38**, 296 (1942).

quantum electrodynamics by omitting consistently and systematically all the diverging features from the quantized field theories. A simple new set of equations has been obtained which, however, can so far only be used for non-static problems (transition probabilities) whilst the static field appears to be a problem of a higher order of difficulty. This paper[3] will, in the following, be referred to as *I*. In contrast to the older expansion method used previously for the calculation of transition probabilities the new theory includes a damping force. In cases where a classical analogue exists it can be shown that the theory is equivalent to a treatment of the second classical damping term. The transition probabilities are to be calculated from a set of simultaneous inhomogeneous integral equations. The application to multiple processes in *I* shows that the results are at least "reasonable." We hope to be able to show that our theory can, at any rate as a good approximation, be derived from Dirac's quantum-electrodynamics.[4]

It should be possible to decide between the two kinds of theories experimentally. The cross section for the scattering of mesons by a proton depends upon the energy in a different way in the two theories. More precise measurements of the scattering cross section especially as a function of the energy are therefore very desirable. An experimental decision would virtually decide whether the elementary particles have a finite radius or are point particles (*cum grano salis*).

Further applications of our theory to the creation of mesons by light quanta and through proton-proton collisions are made in two papers,[5] the results of which are the basis of the present work (referred to as *II* and *III*, respectively). They are summarized as far as they are needed in Section II. The aim of this paper is to show that the theory gives a satisfactory account—as far as we can see—of all the chief cosmic-ray phenomena connected with mesons, including their creation, their diffusion through the atmosphere, meson showers, and the transformation into neutrettos.

We shall assume that mesons are created by a primary radiation consisting of protons, in accordance with the geomagnetic evidence.

It will be seen that the rate of production of mesons is so great that practically all the mesons are produced in a very thin top layer of the atmosphere. This is indeed what recent experiments[6] have shown to be the case.

A certain ambiguity arises from the fact that at present it is not quite known which of the various modifications of the meson theory is to be used. As we endeavor to give a *connected account of cosmic-ray and nuclear phenomena* we choose that form of the meson theory which gives the best account of the nuclear forces: This is undoubtedly the form of the theory proposed by Møller and Rosenfeld.[7] Thus we assume that vector and pseudoscalar mesons exist (with equal coupling constants for transverse and pseudoscalar mesons) and that neutrettos (neutral mesons) also exist, whose coupling constants are half of those of charged mesons.

In accordance with this theory and with the fact that only pseudoscalar mesons are found at sea level we assume that vector mesons have a much smaller lifetime than pseudoscalar mesons. A lifetime of 10^{-8} sec. at rest suffices to explain that all vector mesons decay practically at the point where they are created. The meson producing layer of the atmosphere will be seen to have a thickness of less than 15–30 cm H_2O. Most of the electrons arising from the decay of the transverse mesons are, therefore, also produced in a very thin top layer of the atmosphere. We shall see in Section V that the number of these decay electrons and their energy spectrum is just of the right magnitude and type to produce, by cascade multiplication, the soft component in the high atmosphere. The observed intensity curve is in good agreement with the calculated one. Thus the *soft component can be explained as tertiary products of the incoming primary protons.*

II. PRODUCTION OF MESONS BY PROTON-PROTON COLLISIONS. RANGE OF FAST PROTONS

If a fast proton collides with another nuclear particle it may emit a meson in analogy to the

[4] This does not mean that our results should be regarded as a proof for Dirac's theory, even if our equations should prove to be in quantitative agreement with the experiments. Indeed many of the special features of Dirac's theory only become apparent when static problems are treated.

[5] Hamilton and Peng; Heitler and Peng, to appear shortly in the Proceedings of the Royal Irish Academy.

[6] Schein, Jesse, and Wollan, Phys. Rev. **59**, 615 (1941).

[7] Møller and Rosenfeld, Kgl. Danske Vid. Sels. Math.-Fys. Medd. **17** (1940).

"bremsstrahlung" emitted when a fast charged particle is deflected in a Coulomb field. The rate of production has been calculated in *III* by applying the method of Weizsäcker-Williams which has proved useful in the electromagnetic case: The field of the fast proton is expanded into a Fourier series and is, at sufficiently high energies, equivalent (i) to a beam of light quanta arising from the expansion of the Lorentz-transformed Coulomb field and (ii) to a beam of mesons arising from the Lorentz-transformed nuclear field. Both types of quanta interact with the nuclear particle at rest and can produce a meson, (i) by the process,

$$h\nu + P \rightarrow N + Y^{+} \tag{1}$$

and (ii) by scattering. It has been shown in *III* that the contribution of (i) is entirely negligible compared with that of (ii) except if the proton has an energy $>10^{12}$ ev which is not of great interest in the present work.

Thus the cross section for meson production is obtained by multiplying the number of mesons of a given energy occurring in the spectrum of the transformed nuclear field by the cross section of scattering of mesons by the nuclear particle at rest.

Throughout this paper we shall use "natural meson" units, putting $\hbar = c = \mu$ (meson mass) $= 1$. Energies are then measured in units of $\mu c^2 \sim 90$ Mev and cross sections in units of $(\hbar/\mu c)^2 = 4.3 \times 10^{-26}$ cm^2. Furthermore, we introduce a unit for the thickness of matter traversed, namely, that thickness which a fast particle with unit charge has to traverse in order to lose an energy by ionization equal to $\mu c^2 = 1$. This unit thickness is about 45 cm H_2O. Thicknesses measured in these units are denoted by x. The coupling constants of longitudinal, transverse, and pseudoscalar mesons with a nuclear particle are denoted by g^2, f^2, f'^2 (dimensionless), respectively, and their numerical values taken from the theory of nuclear forces (cf. reference 7):

$$g^2 = 0.054, \quad f^2 = f'^2 = 0.13. \tag{2}$$

The coupling constants for neutrettos are halves of these values ($f_0^2 = f_0'^2 = 0.065$, $g_0^2 = 0.027$). For the meson mass we assume the value $\frac{1}{10}$ of the proton mass.

In the "equivalent spectrum" of the field of the fast proton with energy E, charged and neutral mesons of all polarizations occur. The number of longitudinal mesons, however, is negligible. The number of transverse and pseudoscalar mesons with energy between ϵ and $\epsilon + d\epsilon$ was found in *III* to be

$$q_{tr}^{(\epsilon)d\epsilon} = \frac{d\epsilon}{\pi\epsilon} f^2 D_{tr}, \quad q_{ps}^{(\epsilon)d\epsilon} = \frac{d\epsilon}{\pi\epsilon} f^2 D_{ps}. \tag{3}$$

Here the D's are rather complicated functions of ϵ/E, involving Hankel functions, but when plotted against ϵ/E it turns out that they can quite well be represented by the following simple functions (with errors less than 30 percent)

$$\begin{aligned} &D_{tr} = 165, \qquad D_{ps} = 50(\epsilon/E)^{\frac{1}{2}}, \\ &D_{tr} + D_{ps} = 200, \quad \tfrac{1}{2}D_{tr} + D_{ps} = 115. \end{aligned} \tag{4}$$

Only these combinations of the D's will occur. D_{tr} comprises both transverse polarizations. The number of neutrettos occurring in the spectrum is half of (3).

The above method and therefore the expressions (3) are only accurate if: (i) $\epsilon \gg 1$, (ii) $E \gg M$ (proton mass). For smaller proton energies the Weizsäcker-Williams method fails completely. (For a more detailed discussion of the validity of the method cf. *III*.) In the following we shall only be interested in meson energies $\epsilon > 1/f \backsimeq 3$ for which (i) is fairly well satisfied. We shall use (3) for energies E down to values $\sim M$. This is certainly crude, but cannot involve very large errors because the slower protons are not very effective in producing mesons, on account of their small energy. If $E < M$ a proton may still produce mesons. A guidance as to the order of magnitude of this effect can be obtained in the following way: If the damping is neglected altogether and if $E \ll M$ the rate of meson production can quite easily be calculated directly by the old methods.[8] The result can be compared with that obtained by using the Weizsäcker-Williams method, also, of course, neglecting the damping, which means $\epsilon < 1/f \backsimeq 3$. The result is that the actual rate of meson production is about 10 times smaller than that obtained by using (3). We therefore expect that the *rate of meson production drops rapidly for*

[8] Cf. for example: Nordheim and Nordheim, Phys. Rev. **54**, 254 (1938).

$E<M$, and we shall neglect the contribution from protons with $E<M$.

The cross section for scattering of a meson by a nuclear particle has also been derived in *III*.[9] Since the exact expressions would be very complicated, we use only their asymptotic forms for $\epsilon \gg 1/f$. ϵ may then still be either $<M$ ("non-relativistic case") or $>M$ ("extreme relativistic case"). Since a meson with given charge and polarization may be "scattered" into a meson with different charge (a charged meson may be transformed into a neutretto and vice versa) and different polarization, we write the result in the form of a matrix attributing rows to the primary and columns to the secondary particles. Attributing rows and columns to the various polarizations as indicated in the formulae we found for the scattering cross sections:

$$\Phi=\frac{4\pi}{\epsilon^2}\begin{pmatrix} 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \frac{1}{3} & \frac{1}{3} & \cdot & \cdot & \cdot \\ \cdot & \frac{1}{3} & \frac{1}{3} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \frac{1}{3} & \frac{1}{3} & \cdot \\ \cdot & \cdot & \cdot & \frac{1}{3} & \frac{1}{3} & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & 1 \end{pmatrix} \begin{matrix} \text{long.} \\ \text{transv.} \\ \text{pseud.} \\ \text{pseud.} \\ \text{transv.} \\ \text{long.} \end{matrix} \begin{matrix} \\ \left.\begin{matrix} \\ \\ \end{matrix}\right\}\text{charged} \\ \\ \left.\begin{matrix} \\ \\ \end{matrix}\right\}\text{neutral} \end{matrix} \quad (\epsilon \ll M) \qquad (5a)$$

$$\Phi=\frac{16\pi}{\epsilon M}\begin{pmatrix} 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \frac{1}{4} & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \frac{1}{4} & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & 1 \end{pmatrix} \begin{matrix} \text{long.} \\ \text{transv.} \\ \text{pseud.} \\ \text{pseud.} \\ \text{transv.} \\ \text{long.} \end{matrix} \begin{matrix} \\ \left.\begin{matrix} \\ \\ \end{matrix}\right\}\text{charged} \\ \\ \left.\begin{matrix} \\ \\ \end{matrix}\right\}\text{neutral} \end{matrix} \quad (\epsilon \gg M) \qquad (5b)$$

The rows and columns marked transv. are to be understood as giving the transition probabilities into *any one* of the two transverse polarizations of the secondary meson and thus have actually to be understood as a submatrix: for instance $\begin{pmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ instead of $\frac{1}{4}$ in (5b). At the places marked by dots in (5) expressions occur which are of a smaller order of magnitude containing high negative powers of ϵ. We shall use (5a) and (5b) for energies $\epsilon \lesssim M$ and $\epsilon \gtrsim M$, respectively.

In the rows and columns marked transv. of (5b) an additional diagonal term has been omitted which increases with ϵ but is multiplied by such a small factor that it becomes only appreciable for $\epsilon \gtrsim 1000$. There are also other processes, not considered in this paper at all, which become important for such extremely high energies. Throughout this paper it must be kept in mind that our formulae are no longer valid whenever such high energies are involved.

The most remarkable feature of (5) is the occurrence of *selection rules*. A meson has the tendency to conserve its charge as well as—to some extent—its polarization, and this becomes a strict law in the limit of high energies. The occurrence of these selection rules is a special feature of our theory and entirely due to the damping. No such selection rules occur if damping is neglected (which is permissible only for $\epsilon \lesssim 1/f$) and our matrices (5) would be filled everywhere with expressions of the same order of magnitude. For the interpretation of cosmic rays it will be of particular importance that charged mesons cannot be transformed into neutrettos in a collision with a nuclear particle, except at very small energies ($\epsilon \sim 1/f$) (cf. Section VII).

Another remarkable fact is that the energy dependence changes from a $1/\epsilon^2$ to a $1/\epsilon$ law for $\epsilon > M$. This fact will be fundamental for our understanding of cosmic rays.

Multiplying (5) by (3) we obtain the cross section for the production of a meson of given energy and polarization. If $\epsilon < M$ it is seen from (5a) that the pseudoscalar and the transverse parts of the equivalent spectrum combine to produce either transverse or pseudoscalar mesons. Thus the cross sections for the production of a meson with energy ϵ become

$$\Phi_{tr}(\epsilon)d\epsilon=\frac{8}{3}f^2\frac{d\epsilon}{\epsilon^3}(D_{tr}+D_{ps}), \qquad (6a)$$

$$(\epsilon<M)$$

$$\Phi_{ps}(\epsilon)d\epsilon=\frac{4}{3}f^2\frac{d\epsilon}{\epsilon^3}(D_{tr}+D_{ps}). \qquad (6b)$$

Equation (6a) is already summed over both transverse polarizations. If $\epsilon > M$ the pseudoscalar part of the equivalent spectrum produces

[9] Earlier work assuming charged mesons only: Heitler, Proc. Camb. Phil. Soc. **37**, 291 (1941), and Wilson, *ibid.*, 301. (In these two papers only the non-relativistic case is treated.) Exact relativistic expression (for charged vector mesons only) by S. T. Ma, *ibid.*, in the press. Cf. also Fierz, Helv. Phys. Acta. **14**, 105 (1941); Landau, J. Phys. U. S. S. R. **2**, 483 (1940).

only pseudoscalar mesons and the transverse part only transverse mesons. Thus

$$\Phi_{tr}(\epsilon)d\epsilon=(8/M)f^2\frac{d\epsilon}{\epsilon^2}D_{tr}, \quad (\epsilon>M) \tag{6c}$$

$$\Phi_{ps}(\epsilon)d\epsilon=(16/M)f^2\frac{d\epsilon}{\epsilon^2}D_{ps}. \tag{6d}$$

In (6c, d) ϵ is not the energy of the meson produced but the energy lost by the moving proton, i.e., the energy of the meson produced plus the recoil energy of the nuclear particle originally at rest. The probability for the meson to take up an energy ϵ' leaving the energy $\epsilon-\epsilon'$ to the recoil particle is simply $d\epsilon'/\epsilon$. This is true if, as in fact is the case, the angular distribution of scattering is uniform in a Lorentz system where the meson and nuclear particle are colliding with opposite and equal momenta; thus the cross section for producing a meson of energy ϵ' is in the extreme relativistic case:

$$d\epsilon'\int_{\epsilon'}^{E}\Phi_{tr}(\epsilon)\frac{d\epsilon}{\epsilon}\backsimeq\tfrac{1}{2}\Phi_{tr}(\epsilon')d\epsilon', \tag{6'c}$$

$$d\epsilon'\int_{\epsilon'}^{E}\Phi_{ps}(\epsilon)\frac{d\epsilon}{\epsilon}\backsimeq\tfrac{2}{3}\Phi_{ps}(\epsilon')d\epsilon'. \quad (\text{for } E\ll\epsilon') \tag{6'd}$$

The energy distribution remains the same but the number of mesons is only $\frac{1}{2}$ or $\frac{2}{3}$ of what it would be if all the energy would be taken up by the meson. The rest of the energy is taken up by the recoil particles. Their energy distribution is also given by (6c, d) and their number is (6c, d) multiplied by $\frac{1}{2}$ and $\frac{1}{3}$, respectively. These recoil particles are further capable of producing mesons and recoil particles. The process repeats itself until the energy has degenerated to a sufficiently low value to make further meson production impossible. The process very much resembles the *cascade multiplication* of electrons but is not so pronounced because the energy distribution (6) favors low energies more strongly than in the case of the bremsstrahlung emitted by a fast electron. Thus the energy degenerates more quickly than in the electromagnetic case. A detailed treatment of this cascade process lies outside the scope of this paper which is only intended to give a first orientation. We can take account of it in a crude way by using (6c, d) instead of (6'c, d) also for the number of mesons produced. By doing so we represent the energy loss of the primary proton correctly. Also the total energy content of the charged mesons is correct because the energy given by the recoil particles to neutrettos is compensated by the energy of those charged mesons which are produced by the recoil particles from neutrettos. Using (6c, d), we overrate the number of fast mesons by a factor 2 or $\frac{3}{2}$, respectively, but we also underrate the number of slower mesons. Thus the error committed by using (6c, d) is a slight distortion of the energy spectrum in favor of high energies, whilst the total number of mesons will be somewhat bigger than what we obtain in this way.

We multiply (6) by the number of nuclear particles contained in a cylinder of unit length, measured in our x units, and with cross section equal to our unit cross section. For water or air this figure is 1.18, but is not very different for other materials (for Pb it is about 1.3).

We then obtain the number of mesons with energy ϵ produced by a proton in traveling the distance dx (using (2) and (4)):

$$\left.\begin{aligned}\Phi_{tr}(\epsilon)d\epsilon dx&=82\frac{d\epsilon}{\epsilon^3}dx \qquad (7a)\\ \Phi_{ps}(\epsilon)d\epsilon dx&=41\frac{d\epsilon}{\epsilon^3}dx \qquad (7b)\end{aligned}\right\}\ (\epsilon<M),$$

$$\left.\begin{aligned}\Phi_{tr}(\epsilon)d\epsilon dx&=21\frac{d\epsilon}{\epsilon^2}dx \qquad (7c)\\ \Phi_{ps}(\epsilon)d\epsilon dx&=12.3\left(\frac{\epsilon}{E}\right)^{\frac{1}{2}}\frac{d\epsilon}{\epsilon^2}dx \qquad (7d)\end{aligned}\right\}\ (\epsilon>M).$$

For neutrettos the above figures are to be halved. All these formulae are only valid for $\epsilon>1/f\backsimeq 3$.

From (7) it is seen immediately that a very fast proton produces more mesons with energy between $1/f$ and M than with $\epsilon>M$, but far the greater part of the *energy* is contained in the fast mesons.

The mesons with $\epsilon>M$ are all emitted in the forward direction, within a very small angle. This is not so for the mesons with $\epsilon<M$. The latter play a not very important role, except in

TABLE I. Range of fast proton.

E_0	20	50	100	300	1000	10,000
$x_{E_0,M}$	0.18	0.45	0.83	2.0	4.1	28

the top part of the atmosphere. For the calculation of intensities, the error will not be *very* large if we disregard any angular dependence and assume that *all particles* are always *emitted in the forward direction.* This we shall do throughout this paper. For this reason, and because we use in many places asymptotic laws instead of exact ones, and finally because we have only taken a crude account of the cascade process mentioned above, we must not expect too high an accuracy for our calculations. On the average, errors will be of the order of magnitude of, say, a factor 2.

From (7) the energy loss of a proton can be obtained immediately. Multiplying by $\frac{3}{2}$ to account for the energy lost by producing also neutral particles and summing over-all polarizations we find (for $E>M$, of course)

$$-\frac{dE}{dx}=\sum_{\text{polar.}}\int_{1/f}^{E}\epsilon\Phi(\epsilon)d\epsilon=43\log 0.3E. \quad (8)$$

The energy loss is very high. It depends in a similar way on E to the ordinary energy loss by ionization but is roughly a hundred times larger. Per cm Pb (about $\frac{1}{5}$th of our x units) a proton with an energy of 3×10^9 ev loses an energy of 2×10^9 ev.

Accordingly the *range of a fast proton* is very small. Of course, (8) is only valid for $E>M$; therefore, we can calculate only the distance a proton travels until it is slowed down to an energy $\sim M$. We find

$$x_{E_0,M}=\int_M^{E_0}dE\Big/\left(-\frac{dE}{dx}\right)=\frac{1}{13}\{\text{li}(0.3E)-\text{li}(0.3M)\} \quad (9)$$

where "*li*" is the integral-logarithm. For practical purposes *lix* can well be replaced by $x/\log x$ (this is exact for large x). For $x_{E_0,M}$ we thus find for the range the values given in Table I. In our units the thickness of the atmosphere is 22. Thus a proton needs an energy of more than 7000, i.e., 7×10^{11} ev in order to penetrate through the whole atmosphere and still retain an energy M. The majority of the protons entering the atmosphere at a latitude of 50° ($E\sim22-50$) lose the effective part of their energy in distances of 0.2–0.5 (9–23 cm H_2O).

The theory does not tell us how quickly a proton loses its energy *after* having been slowed down to an energy M. Although meson production is then negligible compared with its rate at higher energies, the energy loss may still be much greater than that due to ionization. If our above estimate is correct (rate of meson production $\frac{1}{10}$ of that at higher energies), the energy loss would be 10 times greater than that due to ionization and the range of a proton with $E\sim M$ about 2 of our units (1 m H_2O).

Below we shall need the distance a proton travels in order to lose energy from E_0 to E:

$$x_{E_0,E}=\frac{E_0}{43\log 0.3E_0}-\frac{E}{43\log 0.3E} \quad (10)$$

in which we have replaced the $li(x)$ function by $x/\log x$.

III. PRODUCTION AND DIFFUSION OF PSEUDOSCALAR MESONS IN THE ATMOSPHERE

Since the primary protons have an extremely short range the majority of the mesons is produced in a thin top layer of the atmosphere. The transverse mesons are expected to decay almost at once, and only the pseudoscalar mesons will travel through the atmosphere. Their absorption is due to two factors: (i) energy loss by ionization (ii) β-decay. The latter depends upon the distance the meson travels but not on the amount of matter traversed. We assume for simplicity that the density of the atmosphere at a depth x below the top is proportional to x. The probability of a meson at a depth x and with energy ϵ decaying while traveling the distance dx is therefore $dxb/\epsilon x$, where b is inversely proportional to the lifetime τ of a meson at rest. We choose τ from the results of Rossi and Hall,[10] who measured the ratio τ/μ. For $\mu=185$, τ becomes $2\times7\times10^{-6}$ sec. b is then

$$b=\frac{1}{c\tau}\times(\text{distance in cm corresponding to one } x \text{ unit at sea level})\times(\text{height of atmosphere in } x \text{ units})$$

or $b=13$.

[10] Rossi and Hall, Phys. Rev. **59**, 223 (1941).

Let $f(\epsilon, x)d\epsilon$ be the number of mesons with energy ϵ at depth x and $S(\epsilon, x)dx$ the number of mesons produced at depth x in the distance dx. Then f satisfies the diffusion equation

$$\frac{\partial f}{\partial x}=\frac{\partial f}{\partial \epsilon}-\frac{b}{\epsilon x}f+S. \tag{11}$$

This differs from the diffusion equation considered and solved previously by Euler and Heisenberg[11] by the inclusion of the source function S (which is determined by our theory). The term $\partial f/\partial\epsilon$ accounts for the energy loss by ionization. Equation (11) can be solved by introducing new variables

$$\eta=\epsilon+x, x$$

instead of ϵ and x. The solution with the boundary condition $f=0$ for $x=0$ is

$$f(\eta, x)=\left(\frac{\eta-x}{x}\right)^{b/\eta}\int_0^x S(\eta, \xi)\left(\frac{\eta-\xi}{\xi}\right)^{-b/\eta}d\xi,$$

$$\eta=\epsilon+x. \tag{12}$$

S is determined by our theory if we know the number of protons of each energy and at each depth x. The latter depends, of course, on the energy spectrum of the protons falling onto the top of the atmosphere. From the measurements at great depths it will be seen below that the primary spectrum is within certain comparatively wide energy regions a simple power law. For our purpose it is convenient to include also the logarithmic term occurring in (8) in the expression for the primary spectrum; thus we assume that the number of primary protons with energy larger than E is of the form $A\{E/43\log 0.3E\}^{-\alpha}$. There is, of course, not the slightest reason why α should be exactly a constant. The experiments only show that α does not vary much if E changes by a factor 10 or so. Indeed, the measurements at extreme depths indicate an increase of α with increasing E. We shall determine α from underground measurements and shall find $\alpha=2.2$ for E between 100 and 1000. For larger E, α is bigger. Consequently we expect α to be smaller for $E<100$. The latter energy region will be of importance for the upper regions of the atmosphere and for the intensity curve of the soft component. α can be determined then from the shape of the Regener-Pfotzer-curve. We found $\alpha=1.3$ satisfactory for $E<100$. Thus the chief phenomena of cosmic radiation will be explained by the following crude but simple primary differential spectrum:

$$dF(E)=\frac{\partial}{\partial E}\frac{A}{(E/43\log 0.3E)^{2.2}}\quad(\text{for } E>100),$$
$$dF(E)=\frac{\partial}{\partial E}\frac{B}{(E/43\log 0.3E)^{1.3}}\quad(\text{for } E<100), \tag{13}$$
$$B=\frac{2.2}{1.3}\left(\frac{43\log 30}{100}\right)^{0.9}A=2.4A,$$

the connection between B and A being determined by the continuity of the primary spectrum. In reality α will change gradually from smaller to larger values as E increases. For any depth x larger than 2 only the high energy part of (13) will be important. The number of protons with energy larger than E at depth x is, according to (10) and (13):

$$F(E, x)=A\left(\frac{E}{43\log 0.3E}+x\right)^{-2.2},$$
$$F(E, x)=B\left(\frac{E}{43\log 0.3E}+x\right)^{-1.3}$$
$$-B\left(\frac{100}{43\log 30}+x\right)^{-1.3}$$
$$+A\left(\frac{100}{43\log 30}+x\right)^{-2.2}, \tag{14}$$

according to whether

$$\frac{E}{43\log 0.3E}+x\gtrless\frac{100}{43\log 30}.$$

The source function $S(\epsilon, x)$ for pseudoscalar mesons is then, by (7b, d),

$$S(\epsilon, x)=41\frac{1}{\epsilon^3}F(M, x)\quad(\epsilon<M), \tag{15a}$$

$$S(\epsilon, x)=12.3\frac{1}{\epsilon^2}\int_\epsilon^\infty\left(\frac{\epsilon}{E}\right)^{\frac{1}{2}}\frac{\partial F(E, x)}{\partial E}dE$$
$$(\epsilon>M). \tag{15b}$$

[11] Euler and Heisenberg, Ergeb. d. exakt. Naturwiss. **17** (1938).

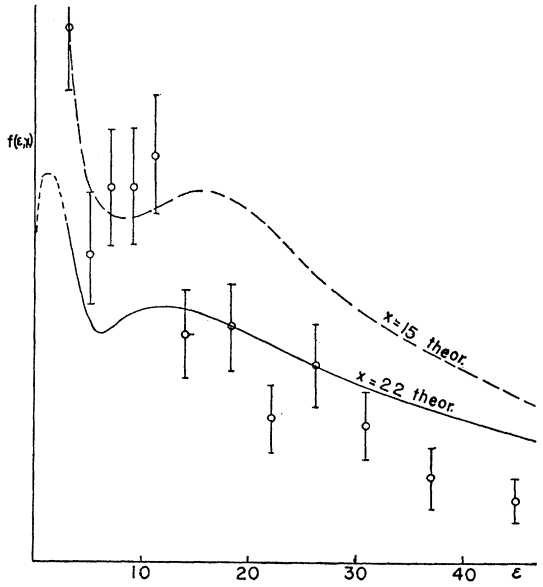

FIG. 1. Energy spectrum of pseudoscalar mesons at sea level, theoretical and experimental. Dotted curve: theoretical spectrum at a height of 4000 m (50 cm Hg).

Thus $f(\epsilon, x)$ becomes:

$$f(\epsilon, x)=41\left(\frac{\eta-x}{x}\right)^{b/\eta}\int_0^x \frac{\xi^{b/\eta}}{(\eta-\xi)^{3+b/\eta}}F(M, \xi)d\xi \qquad (\eta-\xi<M), \quad (16a)$$

$$f(\epsilon, x)=12.3\left(\frac{\eta-x}{x}\right)^{b/\eta}\int_0^x d\xi\int_{\eta-\xi}^{\infty} dE\left[\frac{(\eta-\xi)}{E}\right]^{\frac{1}{2}} \times\frac{\xi^{b/\eta}}{(\eta-\xi)^{2+b/\eta}}\frac{\partial F(E, \xi)}{\partial E} \qquad (\eta-\xi>M), \quad (16b)$$

$$\eta=\epsilon+x.$$

In (16) the limits 0, x have to be replaced: in (16a) by $\eta-M\cdots x$ and in (16b) by $0\cdots\eta-M$ if $\eta-M$ lies between 0 and x. After the integration η is again to be replaced by $\epsilon+x$. In (16) the expression (14) for $F(E, X)$ is to be inserted. Actually each integral (16) consists of two parts with two different values of α. It turns out, however, that for most values of ϵ, x, only one part of (14) is predominant, the other leading only to minor corrections; in particular for any depth $x\geqslant 2$ only that part of F with $\alpha=2.2$ is important.

The integrals (16) cannot be worked out exactly in closed form, but for most values of ϵ, x good approximations can be found. Equation (16a) is needed only for a few values of x and ϵ ($\epsilon<M$) and has been worked out numerically. In (16b) the logarithm occurring in F may be replaced by an average value. The chief contribution arises from small ξ (corresponding to the fact that the protons have extremely small range). For any value of $x>2$, and even with reasonably good approximation for $x=1$, ξ can be neglected against $\eta=\epsilon+x$ and the integration be extended to ∞ instead of x. The exact condition for this to be true is

$$x\gg\frac{\eta}{43\log 0.3\eta} \qquad (17)$$

which is satisfied for all but extremely small x or extremely large ϵ. Equation (16b) then becomes, if only the high energy part of F, ($\alpha=2.2$) is used

$$f(\epsilon, x)=12.3A\frac{\Gamma(1+b/\eta)\Gamma(\alpha-b/\eta)}{\Gamma(\alpha)\Gamma(\alpha-\tfrac{1}{2}-b/\eta)}\left(\frac{\epsilon}{x}\right)^{b/\eta}\eta^{-1-\alpha} \times(43\log 0.3\eta)^{\alpha-1-b/\eta} \quad \eta=\epsilon+x. \quad (18)$$

For $x<1$ the contribution of (16b) has only been estimated but here (16a) is much more important than (16b).

From (18) the asymptotic laws can immediately be read off:

(i) *Tail end of the energy spectrum.* Consider $\epsilon\gg x$ which for the lower part of the atmosphere also implies $\epsilon\gg b=13$.

$$f(\epsilon, x)d\epsilon\propto d\epsilon\cdot\epsilon^{-1-\alpha}(\log 0.3\epsilon)^{\alpha-1}. \qquad (19)$$

Since $\alpha=2.2$ the energy spectrum falls off a little less rapidly (because of the logarithmic term) than $\epsilon^{-3.2}$.

(ii) *Great depths.* For underground measurements in dense materials the decay constant b should be put equal to zero. $f(\epsilon, x)$ is then a function of $\epsilon+x$ only. Integrating over ϵ we find for the total number of mesons:

$$I(x)=\int_{1/f}^{\infty} f(\epsilon, x)d\epsilon\propto x^{-\alpha}(\log 0.3x)^{\alpha-1}. \qquad (20)$$

Thus the total intensity decreases like $x^{-\alpha}$, again apart from a logarithmic modification, and it is in this way that $\alpha=2.2$ was determined.

In Fig. 1 the theoretical energy spectrum is

compared with the measurements by Blackett[12] for small and medium energies. It is seen that the shape of the theoretical curve is nearly the same as the experimental one. On the whole the theoretical spectrum falls off less rapidly than the experimental one. This was to be expected because of the crude way in which we have accounted for the cascade production of mesons as explained in Section II. The experimental minimum at $\epsilon=20$ does not appear in the theoretical

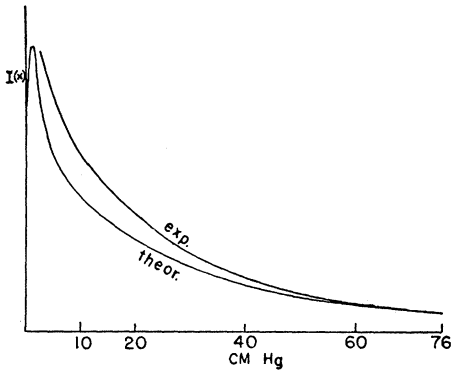

FIG. 2. Theoretical number of pseudoscalar mesons and experimental number of penetrating particles as function of height.

curve and is probably due to a special absorption process as was supposed by Blackett (production of proton pairs?). The maximum at $\epsilon=10$ is clearly indicated in the experiments as well as in the theoretical curve. It is difficult to trace the origin of this maximum as many factors combine to produce it. This maximum did not appear, however, in the calculations of Heisenberg and Euler, in which the exact source function was not used. For the low energy end ($\epsilon\sim3$) of $f(\epsilon, x)$ also the contribution from those very few very energetic protons which penetrate to the lower parts of the atmosphere and produce a large number of slow mesons is important. Hence the rise of the spectrum for small ϵ. Actually the curve will bend down to zero for $\epsilon<3$, as indicated in the figure, but in order to reproduce this part of the curve correctly it would be necessary to use the exact cross section for scattering instead of the asymptotic one (5).

There is another reason why we must expect to find few slow mesons for $\epsilon<3$, say. It will be seen in Section VII that a meson, once having reached an energy $\epsilon<4$, has a probability of 79 percent to be transformed into a neutretto before reaching the end of its range. This would remove a large fraction of mesons from the lower end of the spectrum.

The high energy tail end of the spectrum, i.e., the decrease of $f(\epsilon, x)\propto\epsilon^{-3.2}$ (modified by the logarithmic term) has already been adequately discussed by Euler and Heisenberg[11] and was found to agree with the measurements. In order to see how the energy spectrum varies with height we have also plotted the theoretical spectrum for $x=15$ (50 cm Hg). Slow mesons become relatively more predominant.

In Fig. 2 the total number of mesons

$$I(x)=\int_{1/f}^{\infty}f(\epsilon, x)d\epsilon$$

is plotted against height and compared with the recent measurements by Schein, Jesse, and Wollan.[13] Here also the agreement is as good as can be expected. The maximum occurs at an extreme height of $x=0.5$ corresponding to only 22 cm H_2O or 1.8 cm Hg. In the measurements at extreme heights presumably some of the primary protons also are included as "penetrating particles," but it is difficult to say how many protons have to be classed as "penetrating" because we do not know exactly the range of a proton with energy less than M. We estimate the range to be about 2 x units (=9 cm Pb) for $E=M$ and accordingly smaller for $E<M$. The theoretical curve of Fig. 2 refers to pseudoscalar mesons only. The two curves are normalized so as to agree at sea level.

IV. LATITUDE EFFECT

In the calculations of Section III no account has been taken yet of the fact that the primary energy spectrum is cut off for energies smaller than E_ϑ depending on the geomagnetic latitude ϑ. $E_\vartheta=22$ for $\vartheta=50°$ and $E_\vartheta=150$ for $\vartheta=0°$ (equator). The correction can easily be made. From (16) the contribution from those protons which, at the top of the atmosphere had an

[12] Blackett, Proc. Roy. Soc. **159**, 1 (1937), **165**, 11 (1938). Compare also: Jones, Rev. Mod. Phys. **11**, 235 (1939); Hughes, Phys. Rev. **57**, 592 (1940).

[13] Schein, Jesse, and Wollan, Phys. Rev. **59**, 615 (1941).

energy $<E_\vartheta$ has to be subtracted, i.e., for which

$$\xi+E/43\log 0.3E<E_\vartheta/43\log 0.3E_\vartheta.$$

It has turned out that for "normal latitudes" 50°, say, the correction is negligible throughout the atmosphere. This means that *no increase of intensity is to be expected for more northern latitudes*, not only at sea level but also at any height above sea level, except the very top of the atmosphere, and even there the increase is small. This explains the well known *knee of the latitude effect* at 50°.

The reason is easily to be understood. Protons stop producing mesons for $E<M=10$.

Even the contribution from protons with energy between 10 and 22 is very small on account of their smaller energy (in fact even smaller than assumed in this paper where the asymptotic law for meson production for $E\gg 10$ has been extended to $E=10$). Thus the reason for the knee of the latitude effect is simply the fact that protons of energy less than $E_{50°}$ cease to produce, or produce very few mesons. The same applies to the soft component. We shall see in Section V, that the soft component is probably due to the decay of transverse mesons in the high atmosphere, the former being also produced by the same primary protons. Thus the same knee of the latitude effect is to be expected for electrons, which is also found experimentally.[14] The fact that the soft component shows the same latitude knee as the hard component supports the assumption that both are produced by the same primary radiation, i.e., by protons, as explained by our theory.

TABLE II. Latitude effect. $(I_{50°}-I_{\text{equ}})/I_{50°}$.

	mesons				soft component
x	22 (sea level)	15	2	1	$x=2$
theor.	0.13	0.13	0.18	0.33	0.62
exp.	0.1	—	—	—	0.75

For latitudes $<50°$ an appreciable latitude effect is to be expected. We calculate the *latitude defect* $\Delta_\vartheta f(\epsilon,x)$, i.e., the difference of $f(\epsilon,x)$ between the North Pole and the latitude ϑ. As mentioned above $\Delta_\vartheta f(\epsilon,x)$ is negligible for $\vartheta>50°$. From (16) those contributions have to be subtracted for which $\xi+E/43\log 0.3E<E_\vartheta/43\log 0.3E_\vartheta$. Thus $\Delta_\vartheta f(\epsilon,x)$ is given by (for $\eta-\xi>M$):

$$\Delta_\vartheta f(\epsilon,x)=12.3\left(\frac{\eta-x}{x}\right)^{b/\eta}\int_0^{\xi_\vartheta}d\xi\int_{\eta-\xi}^{E_{\vartheta\xi}}dE \times\left[\frac{(\eta-\xi)}{E}\right]^{\frac{1}{2}}\frac{\xi^{b/\eta}}{(\eta-\xi)^{2+b/\eta}}\frac{\partial F(E,\xi)}{\partial E}, \quad (21)$$

with

$$\frac{E_{\vartheta\xi}}{43\log 0.3E_{\vartheta\xi}}+\xi=\frac{E_\vartheta}{43\log 0.3E_\vartheta},$$

$$\xi_\vartheta=\frac{E_\vartheta}{43\log 0.3E_\vartheta}-\frac{\eta}{43\log 0.3\eta}$$

$$\eta=\epsilon+x.$$

(In the condition for ξ_ϑ, ξ has been neglected compared with η). $\Delta_\vartheta f$ is, of course, zero whenever in (21) an upper limit is smaller than the lower limit. A similar integral holds for $\eta-\xi<M$.

We have worked out the integrals numerically. Here the change of α at $E=100$ is important. Integrating over ϵ we find the latitude effect for the total number of mesons:

$$I_{50°}-I_\vartheta=\int_{1/f}^{\infty}\Delta_\vartheta f(\epsilon,x)d\epsilon.$$

The results are given in Table II. The latitude effect is fairly constant for low levels and only increases for $x<2$. The figure for sea level is in good agreement with the experiments, whilst at greater elevations no measurements seem to have been made yet for the hard component. For the figure concerning the soft component see Section V.

V. THE TRANSVERSE MESONS AND THE SOFT COMPONENT

In addition to pseudoscalar mesons the primary protons produce a large number of transverse mesons which we assume to decay at once. If the mesons have energy $\epsilon\gg 1$—which is nearly always the case—the decay electrons are emitted in the forward direction and take up with equal probability, any amount of energy between 0 and ϵ. Thus a large number of slow and fast electrons are produced. The fast ones multiply by cascade multiplication and it will be seen that a satisfactory account of the soft component can be

[14] Carmichael and Dymond, Proc. Roy. Soc. **171**, 321 (1939).

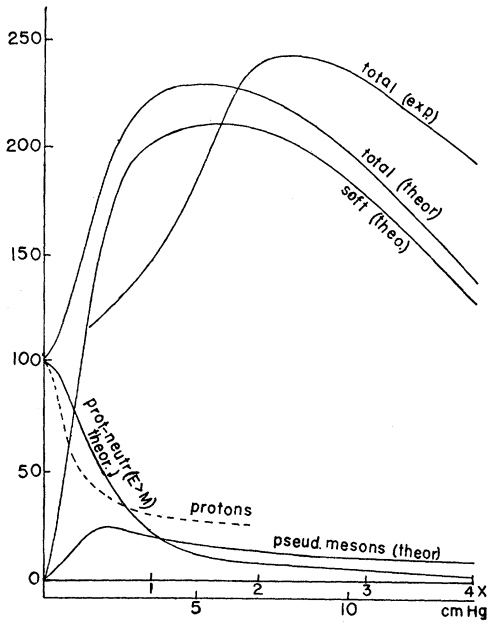

FIG. 3. Cosmic-ray intensities in the top part of the atmosphere for 100 primary protons. Soft component (theor.), pseudoscalar mesons (theor.), primary protons and neutrons (theor.), and total intensity (theor. and exp.).

given in this way. Let $F(\epsilon, x)$ be the number of protons at depth x with energy larger than ϵ. [F is given by (14).] Then the number of electrons produced at x in the energy interval $d\mathcal{E}$ is

$$d\mathcal{E}\int_{\mathcal{E}}^{\infty} F(\epsilon, x)\Phi_{tr}(\epsilon)\frac{d\epsilon}{\epsilon}.$$

Let now $C(\mathcal{E}, \xi)$ be the cascade multiplication function (i.e., the number of electrons produced in a depth ξ by one primary electron of energy $\mathcal{E}$), the total number of electrons at depth x is

$$Z_{tr}(x)=\int_0^x d\xi\int_0^{\infty} d\mathcal{E}\int_{\mathcal{E}}^{\infty}\frac{d\epsilon}{\epsilon}\Phi_{tr}(\epsilon) \times F(\epsilon, \xi)C(\mathcal{E}, x-\xi). \quad (22)$$

Equation (22) can be worked out partly by making suitable approximations and partly numerically. For $\Phi_{tr}(\epsilon)$ the expressions (7a, c) and for F (14) are to be used. For the high atmosphere ($x<4$, say) practically only the low energy part of $F(\alpha=1.3)$ is important. For C we have used the figures given by Arley.[15]

To obtain an idea of the effect to be expected we consider large values of x. ($x>5$, say.) We can then make the following approximations: $F(\epsilon, \xi)$ decreases rapidly with ξ for any ϵ of importance. $C(\mathcal{E}, x-\xi)$ can be replaced by $C(\mathcal{E}, x)$ and the integral over ξ extended to infinity instead of to x. This amounts to assuming that all the mesons produced in the atmosphere are practically produced at the very top and, decaying at once, produce a "primary" electron spectrum $N(\mathcal{E})d\mathcal{E}$ say, which later produces cascade effects. $N(\mathcal{E})$ is given by

$$N(\mathcal{E})=\int_{\mathcal{E}}^{\infty}\Phi_{tr}(\epsilon)\frac{d\epsilon}{\epsilon}\int_0^{\infty}F(\epsilon, \xi)d\xi. \quad (23)$$

We divide the range of energy $\mathcal{E}$ into 4 regions, and find for $N(\mathcal{E})$ (considering logarithmic terms as constants, and using the low energy part of F only):

$$\begin{aligned} N(\mathcal{E})d\mathcal{E} &= 33B\frac{d\mathcal{E}}{\mathcal{E}^2}\left(\frac{43\log 0.3\mathcal{E}}{\mathcal{E}}\right)^{0.3} && (\mathcal{E}>E_\vartheta=22)\\ &= Bd\mathcal{E}\left\{\frac{61}{\mathcal{E}^2}-\frac{99}{\mathcal{E}43\log 0.3\mathcal{E}}+0.1\right\} && (E_\vartheta>\mathcal{E}>M=10)\\ &= Bd\mathcal{E}\left\{\frac{142}{\mathcal{E}^3}+0.34\right\} && (10>\mathcal{E}>1/f=3)\\ &= 5.6Bd\mathcal{E} && (\mathcal{E}<3). \end{aligned} \quad (24)$$

Equation (24) plays the role of the primary electronic energy spectrum responsible for the soft component. $N(\mathcal{E})$ decreases approximately like $1/\mathcal{E}^2$, at any rate in the high energy region. But there are also a large number of soft electrons present. Apart from the latter $N(\mathcal{E})$ is indeed very similar to the primary electron spectrum deduced first by Nordheim and Heitler[16] from the intensity curve of the soft component on grounds of the cascade theory and later found by Bowen, Millikan, and Neher[17] by latitude measurements. We therefore expect that (24) should give rise to an intensity curve for the soft component very similar to the observed one. The effect of the low energy electrons with small range also present in

[15] Arley, Proc. Roy. Soc. **A168**, 519 (1938). In the cascade theory different units for the thickness of matter traversed are usually used. For air or water one cascade unit is about $\frac{2}{3}$ of our x units.

[16] Nordheim, Phys. Rev. **51**, 1110 (1937); Heitler, Proc. Roy. Soc. **161**, 261 (1937).

[17] Bowen, Millikan, and Neher, Phys. Rev. **52**, 80 (1937) and **53**, 855 (1938).

(24) will be that the intensity curve does not fall down very steeply near the top but becomes in fact rather flat.

For the most interesting part of the atmosphere ($x=2, 3$, maximum of the Regener-Hotzer-curve) the use of (24) is too crude an approximation. In fact quite a number of low energy mesons are still produced at levels $x>2$. We have therefore used (22) directly and evaluated the integrals. We find that on the average one primary proton produces about 1.5 electrons at $x=2$ or 3 in this way.

In addition to the electrons produced by the decay of transverse mesons there are also electrons produced by the decaying pseudoscalar mesons. Their number is smaller but not negligible compared with the number of electrons due to transverse mesons. We easily find:

$$Z_{ps}=\int_0^x d\xi\int_0^\infty d\beth\int_\beth^\infty \frac{d\epsilon}{\epsilon}\frac{b}{\xi}f(\epsilon,\xi)C(\beth,x-\xi), \quad (25)$$

where $f(\epsilon, \xi)$ is the energy spectrum of pseudoscalar mesons as calculated in Section III. The result is for instance the following for $x=2$ and for one primary proton: $Z_{tr}=1.5$, $Z_{ps}=0.6$, $Z_{tr}+Z_{ps}=2.1$. Thus one primary proton produces in the average altogether about 2 electrons at the Regener-maximum, roughly $\frac{1}{3}$ of which are due to pseudoscalar and $\frac{2}{3}$ to transverse mesons. This is in very good agreement with the experiments. In Fig. 3 we have plotted the intensities of all the cosmic-ray components for the top part of the atmosphere (normalized for 100 incident protons). The agreement with the Regener-Hotzer-curve for the total intensity is as good as can be expected. The position of the maximum is, however, slightly shifted towards greater heights, than is found experimentally. The reason is probably that we have been overrating the production of slow mesons (and therefore slow electrons) by protons of energy just above M.

The latitude effect of the soft component is very large. We have calculated it for $x=2$ and included the result in Table III. The agreement with the measurements by Bowen, Millikan, and Neher[17] is good. The latter measurements refer to the total intensities, of which, however, the soft component is far the strongest.

There is one experiment which is apparently in contradiction to our assumption that the soft component originates from primary protons. Whilst the mesons in the lower atmosphere show an east-west asymmetry as big as the latitude effect proving that they originate from positive primaries no such east-west asymmetry has been found for the soft component in the high atmosphere although the latter is also—and very strongly—latitude sensitive.[18] If this result is taken at its face value it would mean that the soft component originates from equal numbers of positive and negative primary particles. We must remember, however, that the soft component is, if our picture is correct, created by the primary protons in a rather indirect way: The protons first produce mesons which decay into electrons, which then multiply by means of the cascade process. An east-west asymmetry can only be expected if in each of these three processes the particles are always strictly emitted in the forward direction with no appreciable angular straggling. This is surely the case if the secondary mesons have energies large compared with M, and if these mesons then decay while still having an energy large compared with one.

In the high atmosphere, however, low energy particles are predominant. Indeed, as it appears from the calculations of this section, most of the electrons in the high atmosphere originate from mesons with energy less or not much greater than M. These mesons are bound to be emitted in all directions with only a slight favoring of forward directions. Even for the more energetic particles the predominance of forward directions will be impaired to some extent by the many processes which have to take place to produce the soft component. It might, therefore, well be that the east-west effect is completely masked by the large angular straggling of the soft component.

TABLE III. Cosmic-ray intensities (100 primary particles).*

	soft comp. $x=2$	soft+hard $x=2$	mesons sea level	energetic protons and neutrons sea level
theor.	208	220	1.7	1/500
exp.	—	225	4	1/2000

* The column soft+hard ($x=2$) does not include the primary protons at this depth. Their number should be added to the theoretical figure 220 but is probably less than 20 (cf. Section VI).

[18] Johnson, Phys. Rev. **56**, 219 (1939).

For a judgment of this theory it must be remarked that, once the number of primary protons is normalized, and their energy spectrum given, the theory allows one to calculate the *absolute intensities* of all cosmic-ray components at each depth without further adjusting our intensities to those of the experiments. It is clear that the values of the intensities depend entirely on the constants occurring in the theory, especially on the nuclear coupling constants g, f. All these constants are either determined from nuclear facts (g, f) or by direct experiments (meson mass, decay constants). It is interesting to compare the intensities obtained by our theory with the experimental ones. Table III gives a selection of data. The absolute number of mesons at sea level is about twice of that calculated. This is quite what we had to expect as was remarked in Section II. (The last column refers to measurements by Jannossy, discussed in Section VI.)

VI. PROTONS, NEUTRONS, MESON-SHOWERS

The primary protons entering the atmosphere are very quickly slowed down to energy ~ 10. Since their energy loss is mainly due to producing charged mesons we must expect that half of these "protons" have in fact become neutrons after having travelled only a very small distance. In Fig. 3 we have plotted the number of these protons or neutrons with energy $> M$ as a function of depth for the primary energy spectrum (13), the number simply being given by the relation (14). It is seen that their intensity decreases very rapidly. For a discussion of the experiments, however, we would rather like to know the number of protons (not neutrons) with all energies, including $E < M$. This number can only be guessed at present, since we do not know at what rate protons lose their energy once E is less than M. All we can say is that the number of protons drops very quickly to half the initial value (the other half have become neutrons). We might estimate that the energy loss of a proton becomes say 10 times smaller as E becomes less than M (cf. Section II)—a figure which can only be a very crude guess. The intensity curve thus obtained is dotted in Fig. 3. The recoil protons and neutrons are not included in these curves. The curve marked "total" does not include these protons except those at the very top of the atmosphere.

After having reached an energy $E < M$ the protons and neutrons have a much larger range than the distance that a proton or neutron can travel while having an energy $> M$. It is therefore not surprising that a large number of slow protons and neutrons are found in the upper atmosphere. Fluctuations of the range are large, since energy is lost in large portions, thus a small but appreciable fraction of them may even manage to travel through a considerable part of the atmosphere. In addition there are numerous fast and slow recoil protons. The experiments show a rapid increase of slow protons and neutrons with height, though their number is still very small at a height of 4000 m compared for instance with the number of mesons. A quantitative discussion is not yet possible. A certain fraction of these slow protons and neutrons are recoil products or are produced by photoelectric nuclear disintegrations, etc.

In addition to these slow particles we must expect, in any part of the lower atmosphere, a very small number of very energetic protons and neutrons, namely, the primaries themselves which have sufficient energy to travel such large distances. Let us estimate their number, at sea level. The energy necessary to penetrate through the atmosphere is at least 7×10^{11} ev. The primary energy spectrum (13) is valid for E up to 1000 (10^{11} ev). For higher energies we know from the measurements at great depths that α increases with increasing energy. Thus for the energies in question we may assume $\alpha = 3$, for $E > 1000$, say. The number of primary protons at sea level is then given by $A' x^{-3}$, $x = 22$ where A' is determined by A [Eq. (13)] and the condition that the spectrum is continuous at $E = 1000$, thus $A' = 23$ if A, B, A' are normalized for 100 incident protons. Thus we find 1/500 energetic protons or neutrons at sea level for 100 primary protons at the top of the atmosphere.

This figure may in fact be still too large, for the following reason: As was mentioned in Section II, our formula for the energy loss of a fast proton is no longer valid for energies $> 5 \times 10^{11}$ ev, because then other modes of energy

TABLE IV. Number of mesons (with $\epsilon>10$) produced by energetic proton or neutron.

E_0	100	300	1000
$x_{E_0, M}$	0.9	2.1	4.3
$N_{ps}+N_{tr}$	3.3	6.3	11.0

loss become appreciable.[19] Thus still higher energies may be required for a proton to penetrate to sea level making their number still smaller. From measurements discussed below Janossy found that the number of protons or neutrons at sea level is about 1/12,000 of the total number of cosmic-ray particles at sea level. Putting the latter value to be about 6 (for 100 incident protons) the experiments would give the number of these energetic protons and neutrons as 1/2000 which may be considered to be in reasonable agreement with our theoretical estimate 1/500.

When these energetic particles travel through matter they will produce a number of mesons in succession thus producing a *meson-shower*. Showers consisting of penetrating particles have been observed by several authors.[20] The most extensive measurements are due to Janossy: A radiation whose intensity was found to be 1/12,000th of the total cosmic-ray intensity at sea level was found to produce showers consisting of penetrating particles which are certainly mesons with energy $\epsilon>10$. The transition curve of these showers in lead reaches saturation at about 5 cm Pb (about 1 x unit). About a third of the primary radiation producing these showers consists of neutral particles. Janossy found, however, that for showers produced by the neutral primaries saturation is only reached after 10 cm of Pb. Each shower consists of an average number of 2–6 *recorded* penetrating particles. The actual number of particles in each shower may be larger.

We can only give a very preliminary discussion of these experiments. The most obvious explanation is that the very energetic protons and neutrons expected from our theory are responsible for these showers. There should be about equal numbers of protons and neutrons. Janossy found about a third to be neutral particles. Let $E\gg M$ be the average energy of the protons and neutrons and $x_{E,M}$ their range as given in Table I (i.e., the distance travelled until the energy is M).

The number of mesons with energy >10 produced is then according to (7):

$$N_{tr}=21\int_M^{E_0}\frac{dE}{-dE/dx}\int_{10}^{E}\frac{d\epsilon}{\epsilon^2}\sim 2.1x_{E_0,\,M},$$

$$N_{ps}\sim 15.5E_0^{-\frac{1}{3}}x_{E_0,\,M}.$$

Thus the total number of mesons produced is for various energies E_0 (from Table I) given by Table IV.

These values cannot, however, be compared directly with the experiments. It has kindly been pointed out to us by Janossy, that the saturation point of the transition curve is not at all identical with the range of the primary radiation but marks the point where enough mesons are produced to be recorded as a shower. This figure is about 2–3. A further increase of the lead thickness will only increase the size of the showers but not the number of the showers. Experimentally, we find, therefore, that 2 or 3 mesons are produced in the first 5 cm Pb or in the first x unit. This is in very good agreement with Eq. (26), the theoretical number of mesons produced in $x=1$ being $2.1+15.5E_0^{-\frac{1}{3}}$. We do not know E_0 exactly, but it certainly is rather large (>100). For greater thickness of Pb, Table IV shows that the size of the showers increases very much and even large showers are quite possible. On the other hand it is difficult to understand why neutrons should give rise to showers with a larger saturation thickness, than that for showers produced by protons. The processes considered in this paper lead to no explanation of this asymmetry.

There are, however, a number of points, omitted so far in the present theory, which will have to be considered before a final discussion of the penetrating showers can be given:

(i) As was mentioned in Section I, there are further modes of energy loss by a fast proton, expected from the present theory, including also a higher rate of production of transverse mesons,

[19] The limit of validity for the energy loss formula is higher than that for the scattering cross section because the former is an integral of the latter.

[20] Wataghin, de Souza Santos, and Pompeia, Phys. Rev. **57**, 61 and 339 (1940). Janossy, Proc. Roy. Soc. **179**, 361 (1942); Janossy, McCusker, and Rochester, Nature **148**, 660 (1941).

which become important if the energy exceeds the value 5000, say. The primary protons would then hardly have a chance at all to reach sea level. The protons and neutrons responsible for the penetrating showers must be the fast recoil particles mentioned above which naturally have also smaller energies than the primaries. (If this argument holds the last column in Table III has no meaning.) Some of the processes taking place at extremely high energies depend on the Coulomb field of the proton which might perhaps indirectly account for the difference of the behavior of the protons and neutrons.

(ii) Throughout this paper we have neglected the occurrence of multiple processes. It is indeed possible that a fast proton creates mesons, not only one by one in succession, but several mesons in one elementary act. We obtain the rate of occurrence of this event by multiplying the equivalent meson spectrum of a fast proton by the cross section for the splitting up of a meson into several mesons instead of by the scattering cross section (5). The splitting up processes have been calculated in *I* for the case when all energies concerned are smaller than M. In this case it has been found, indeed, that the splitting-up cross section for such multiple processes is always at least 10 times smaller than that for ordinary scattering (which is the reason why multiple processes were neglected in this paper). It may well be—but this could only be decided by further, rather difficult, calculations—that the ratio is more in favor of multiple processes if the energy of the proton is $>M$. If this should turn out to be true, the range of the fast protons would be diminished and the number of mesons increased.

(iii) We have always assumed that the particles in the nuclei of the matter traversed act independently. It might very well be that the nuclear fields of the nuclear particles overlap and interfere (as their Coulomb field does) producing an effect which increases with a different power of the atomic weight than the first power. This might lead to some changes of our theoretical results for heavy materials but hardly for air.

(iv) So far we have considered only protons and neutrons as primary agents. It is to be expected that mesons also could create penetrating showers themselves, by the very splitting up process mentioned in (ii). Again no quantitative discussion is possible unless the calculations are performed for energies $>M$. A crude estimate on grounds of the non-relativistic calculations shows indeed that the rate of occurrence of this process is of the observed order of magnitude.

Mesons have also been found to occur in very large cascade showers. This is easily understood. According to our theory, a large cascade shower owes its origin to a very energetic primary proton emitting an energetic transverse meson which, by decaying, produces the very energetic electron responsible for the cascade. Naturally, the primary proton will also produce numerous other mesons, including pseudoscalar ones, during its path through the atmosphere. These mesons will, of course, occur together with the cascade shower. In addition, energetic light quanta, have a small chance of creating mesons themselves by process (1) instead of producing an electron pair. The cross section for (1) was found in *II* to be $\sqrt{2}\pi^2 ef/\epsilon^2$ for pseudoscalar mesons, if $\epsilon < M$. Comparing this with the cross section for pair production $\sim 15Z^2/137$ we find that the chance for a light quantum producing a meson with $\epsilon = 10$, say, is in air 1/200. Since in a big cascade shower at the point of its maximum several thousand light quanta may occur, quite a number mesons are to be expected. The mesons accompany the cascade shower but are much more slowly absorbed; the ratio of the number of mesons to that of soft particles may therefore be quite appreciable at sea level.

VII. NEUTRETTOS

According to our theory neutrettos should be very frequent particles in cosmic radiation. The total number produced is half the total number of charged mesons produced. We know nothing, however, about their absorption and possible β-decay nor does it seem that they could produce any noticeable secondary effects. It is true, though, that in colliding with a nuclear particle a neutretto can be transformed into a charged meson. But, as mentioned in Section II, the cross section is very small at high energies decreasing like ϵ^{-6} and it is unlikely that this effect will ever be observed. Slow neutrettos must be

very rare because there is nothing that would slow down a once fast neutretto.

There is, however, one observable effect, by which the existence of neutrettos could be established experimentally, namely, the transformation of a charged meson into a neutretto. At high energies $\epsilon \gg 1/f \sim 3$ again the cross section Φ^0 for this transformation is negligible ($\Phi^0 \propto \epsilon^{-6}$) compared with the cross section for scattering ($\propto \epsilon^{-2}$). This is, indeed, an important result of our theory because it explains why all attempts at detecting this transformation[21] have failed whereas the anomalous scattering of mesons has been observed. This is no longer true for energies of the order of magnitude $\epsilon \sim 3$. The cross section becomes then appreciable and, as we shall see, big enough to lead to observable effects.

The method for calculating Φ^0 as a function of ϵ is described in detail in *III* and the calculations are carried out up to a point where numerical evaluation is possible. The exact formula would be a rather lengthy and complicated expression because no approximation can be made in this case except that we can assume ϵ to be small compared with M. We shall not give the formula here. Φ^0 is, for very small energies, proportional to $p^2=\epsilon^2-1$, rises to a maximum at $\epsilon=2$ and decreases rapidly for higher values. The value of Φ^0 at the maximum ($\epsilon=2$) is in our units $\overset{0}{\Phi}_{\max}=0.56$. This refers to the transformation of a pseudoscalar charged meson into neutrettos of all polarizations (longitudinal, transverse, or pseudoscalar).

We shall calculate the total probability for a meson to be transformed into a neutretto while traveling through matter, starting with a high initial energy $\epsilon \gg 1/f$ until it is stopped. If $-\partial\epsilon/\partial x$ is the energy loss by ionization this probability is

$$w=\frac{NA}{2}\int_1^\infty \frac{\Phi^0(\epsilon)}{-\partial\epsilon/\partial x}d\epsilon, \qquad (27)$$

where N is the number of nuclei per cm^3, A the atomic weight. The factor $\frac{1}{2}$ is due to the fact that a positive meson can only be transformed into a neutretto if it collides with a neutron and a negative meson only if it collides with a proton. The average number of "active" nuclear particles is therefore $A/2$. We have worked out (27) by numerical integration for Pb and found[22]

$$w=0.79. \qquad (28)$$

For any other materials w will not be very different because $\partial\epsilon/\partial x$ is nearly proportional to NZ which differs only slightly from $NA/2$. Practically the whole contribution to the integral (27) arises from energies between $\epsilon=\frac{3}{2}$ and $\epsilon=4$.

We see that, once a meson has reached an energy as small as 4, the chance is so great that it will be transformed into a neutretto. Only 21 percent of the mesons reach the end of their range as charged mesons. It may very well be that this is one of the reasons why so very few slow mesons with $\epsilon<2$ occur in cosmic radiation.

The length of path a meson has to travel in order to lose energy from $\epsilon=4$ to $\epsilon=\frac{3}{2}$ is about 1 km of normal air or 18 cm of Pb. While traveling through the gas of a cloud chamber the chance is so negligible that the effect could be observed. But, if mesons are stopped in a thick block of lead an appreciable fraction of the mesons should not reach the end of their range but should be transformed *earlier* into neutrettos. This is important for the type of experiments carried out by Rassetti.[23] Rassetti found that if mesons are stopped in 10 cm of aluminum (equivalent to 2.8 cm of Pb) only a certain fraction (42 ± 15 percent) give rise to decay-electrons. The effect was interpreted by Rassetti in the following way: Once a meson is stopped entirely and retains only an energy of a few electron volts it may either decay or else be captured by a nucleus giving its rest energy to the nucleus. Only negative mesons, however, can be captured in this way, since a positive meson with such a small energy could not come in contact with the nucleus. Calculations referred to by Rassetti indicate indeed, that for such slow negative mesons the capture-probability is larger than the decay probability. It may very well be, that transformation into neutrettos as well as nuclear capture is responsible for the fact that only a fraction of the mesons are really

[21] Cf., for instance, Nishina and Birus, Sci. Pap. Inst. Phys. Chem. Res. Tokyo **38**, 360 (1941).

[22] For this result the numerical values (2) for the coupling constants have been used.

[23] Rassetti, Phys. Rev. **60**, 198 (1941).

decaying. In the above experiment 12 percent of the mesons should be transformed into neutrettos before being stopped. Of the remaining 88 percent half should be captured leaving 44 percent decaying into electrons.

The experiments are not accurate enough to decide whether the number of decay electrons is actually appreciably less than 50 percent. The actual percentage should depend on the thickness of the absorber.

VIII. CONCLUSIONS

On the whole the theory was seen to give a satisfactory account of all the chief cosmic-ray phenomena. Although a number of points, especially the cascade production of mesons (cf. Section II), the east-west effect of the soft component and the penetrating showers, must await further investigation, it is probable that this theory is fundamentally correct. The conclusions that can be drawn from our results are twofold: (i) The theory of damping used as a basis for all our calculations will be a correct part of future quantum electrodynamics or at least a good approximation. (ii) Cosmic-ray mesons are in fact identical with the quanta predicted by Yukawa which are responsible for the nuclear fields. It may perhaps be too early to say that the special form of the meson theory used in this paper (the one suggested by Møller and Rosenfeld) is the correct form of the meson theory. We have carried out all the calculations also with a different form of the theory (assuming that only charged mesons exist) and found the agreement with cosmic-ray experiments less good, though this form of the theory cannot be wholly excluded. (The energy loss of fast protons would be three times smaller.) It is satisfactory that the form of the meson theory which gives the best account of the nuclear forces also agrees best with cosmic-ray facts. Especially, the assumption of both pseudoscalar and transverse mesons is strongly supported by the fact that all components of cosmic radiation can be explained as owing their origin to one kind of primary particles, i.e., protons.

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On the Adiabatic Demagnetization of Iron Alum

J. A. SAUER*
Harvard University, Cambridge, Massachusetts
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The behavior of iron alum during adiabatic demagnetization to temperatures near the absolute zero is of particular theoretical interest because of the deviations from Curie's law that arise from the perturbing actions of crystalline field and magnetic dipole-dipole coupling. The effect of these perturbations on the magnetic moment and the entropy is calculated exactly to second-order terms in the magnetic coupling and to third-order terms in the crystalline potential. The calculations are presented for crystalline fields of either cubic or axial symmetry and are valid for the case of large applied fields in which saturation effects become important. Theoretical values of the adiabatic moment are found to be in satisfactory agreement with the experimental values determined by Casimir and de Haas. A true thermodynamic scale is established that enables the temperature to be calculated at any value of the magnetic field during demagnetization. The relationship of this scale to the temperatures determined by the magnetic method of de Haas and Wiersma is discussed.

PART I: DESCRIPTIVE ANALYSIS

I. Introduction

ONE of the greatest difficulties encountered in the production of low temperatures by the method of adiabatic demagnetization is the determination of the thermodynamic temperature. Until recently, it has been the general practice of experimenters to set up a "magnetic" temperature scale based on the validity of Curie's law. Deviations from this law, however, will be

* Now at Mellon Institute of Industrial Research, University of Pittsburgh, Pittsburgh, Pennsylvania.