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Neutron Polarization and Ferromagnetic Saturation

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The transmission of thermal neutrons through magnetized iron has been measured in its dependence upon the percentage deviation ϵ from saturation and upon the thickness d of the sample. In agreement with the theory of Halpern and Holstein it was found that the percentage increase of transmission caused by magnetization is given by $(n^2p^2d^2/2)f(\lambda/\epsilon d)$, where n is the number of iron atoms per unit volume. Writing for the scattering cross section of neutrons with parallel or antiparallel orientation of their spin with respect to the field, respectively $\sigma_0 \pm p$, we find $p = 2.0 \pm 0.1 \times 10^{-24}$ cm². From the determined value of the length $\lambda = 3.2 \pm 0.3 \times 10^{-3}$ cm, the linear dimensions of the microcrystals can be determined to be $\delta = 1.4 \times 10^{-4}$ cm. For a thickness $d = 3.8$ cm a transmission effect of almost 8 percent was observed if the magnetization was brought to within 2.5 per mille of its saturation value; more than twice this effect can be expected from the same thickness at complete saturation.

I. INTRODUCTION

It is well known that the scattering cross section of a slow neutron in a magnetized substance depends upon the orientation of its spin with respect to the magnetization. This is due to the interaction of the magnetic moment of the neutron with that of the atoms in the substance.[1] A rigorous theory of the phenomenon is complicated by various factors: It requires a basic assumption concerning the magnetic interaction between a neutron and an electron and a good knowledge of the form factor for the magnetic shells in the atom. It is also necessary to take into account the interference of the neutron waves, scattered from different atoms.

For the purpose of the present paper it is sufficient, however, to note that the cross section for a neutron of given velocity v with its spin parallel or antiparallel to the direction of magnetization can always be written in the form

$$\sigma^+ = \sigma_0 + p, \quad \sigma^- = \sigma_0 - p, \tag{1}$$

respectively, where σ_0 stands for its value in the unmagnetized substance and where it is merely the theoretical evaluation of p in its dependence on v which is affected by the previously mentioned complications.

Consider now a monochromatic collimated beam of unpolarized neutrons with fixed primary intensity, which has passed through a layer of substance with thickness d: Let I be its intensity if the substance is unmagnetized, $I + \Delta I$ if the substance is homogeneously magnetized. An elementary consideration shows then that

$$\Delta I / I = \cosh npd - 1, \tag{2}$$

where n is the number of atoms per unit volume.

[1] F. Bloch, Phys. Rev. **50**, 259 (1936); **51**, 994 (1937). J. Schwinger, Phys. Rev. **51**, 544 (1937). O. Halpern and M. H. Johnson, Phys. Rev. **55**, 898 (1939). O. Halpern, M. Hamermesh, and M. H. Johnson, Phys. Rev. **59**, 981 (1941).

For $npd \ll 1$ this simplifies to

$$\Delta I/I \cong \tfrac{1}{2}n^2p^2d^2. \tag{2a}$$

Because of the smallness of p, Eq. (2a) represents for all practical thicknesses d a sufficiently good approximation; it seems desirable to verify experimentally the quadratic dependence of the "single transmission effect" $\Delta I/I$ upon d and thus to determine the interesting quantity p. This, however, meets with a characteristic difficulty. All neutron polarization experiments, including the ones reported here, have so far been carried out on polycrystalline iron where the condition of validity of (2) and (2a), that the magnetization of the substance be homogeneous, can never be reached ideally. Because of magnetic anisotropy the direction of magnetization will be slightly different in the various microcrystals of a polycrystalline sample, depending upon the orientation of their crystal axis. It would in principle require an infinite magnetizing field to turn the magnetization of each microcrystal completely into the direction of the field and thus to reach perfect saturation and thereby magnetic homogeneity. The first experimental evidence of a strong dependence of the polarization effects upon the degree of saturation of the iron was found by Powers.[2]

It was recently pointed out by Halpern and Holstein[3] that even small deviations from saturation can be understood to cause appreciable deviations from the ideal effect, described by (2). The magnetic moment of a neutron, passing through the polycrystal, will undergo a rapid precession around the direction of the magnetic induction B, which varies slightly from microcrystal to microcrystal, and on the average this will result in a depolarization, counteracting the polarization effect described by (2). It is convenient to introduce the percentage deviation from saturation

$$\epsilon = (M_\infty - M)/M_\infty, \tag{3}$$

where M is the total magnetization of the polycrystal and M_∞ the value which it would have if the magnetization of all the constituent single crystals were perfectly parallel to the magnetizing field. The whole transition from complete depolarization to full polarization will be found to occur in a state of almost complete saturation, and it is therefore sufficient to consider only the situation where $\epsilon \ll 1$. In the limiting case $\epsilon \ll 1$ and $npd \ll 1$, which is the only one of practical importance, the theory of Halpern and Holstein yields instead of (2a) the formula

$$\Delta I/I = \tfrac{1}{2}\cdot n^2p^2d^2\cdot f(\lambda/\epsilon d), \tag{4}$$

where λ is a length, correlated to the linear dimensions of the single microcrystals and where

$$f(x) = 2x^2\left(e^{-(1/x)} + \frac{1}{x} - 1\right). \tag{5}$$

f is a monotonic function of its argument with the property $f(0)=0$, $f(\infty)=1$; it represents the reduction factor of the single transmission effect due to depolarization and reaches unity only asymptotically as ϵ vanishes, i.e., according to (3), as complete saturation is reached.

In the relation between λ and the linear dimensions of the microcrystals there enters the distance which a neutron travels during one full Larmor revolution. Let l be that distance, divided by 2π, i.e., the length of path of a neutron while its magnetic moment undergoes a precession by one radian. It is given by

$$l = v/Bg, \tag{6}$$

where g is the ratio of the magnetic moment of the neutron to its angular momentum, B the magnetic induction inside a microcrystal, and v the velocity of the neutron. Assuming v to be the thermal velocity at room temperature, i.e., $v = 2.5\times10^5$ cm/sec., taking $B = 20{,}000$ gauss and the measured[4] value $g = 1.86\times10^{+4}$ c.g.s. one obtains from (6)

$$l = 6.7\times10^{-4}\ \text{cm}. \tag{7}$$

According to Halpern and Holstein one obtains a simple relation between λ and a length δ of the order of the linear dimensions of the microcrystals in the two limiting cases where δ is small or large compared to l. One has

$$\text{for } \delta \ll l: \quad \lambda = l^2/\delta, \tag{8}$$

$$\text{for } \delta \gg l: \quad \lambda = \delta/2, \tag{8a}$$

[2] P. Powers, Phys. Rev. 54, 827 (1938).
[3] O. Halpern and T. Holstein, Phys. Rev. 59, 960 (1941).
[4] L. Alvarez and F. Bloch, Phys. Rev. 57, 111 (1940).

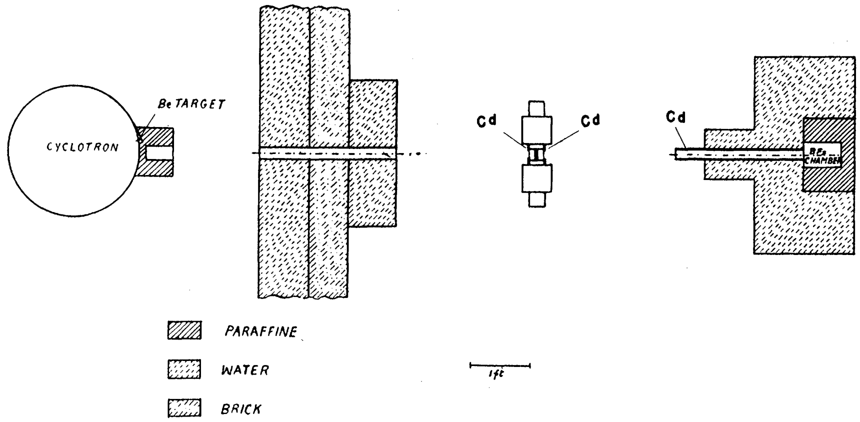

FIG. 1. Arrangement of apparatus.

and for $\delta \leqq l$: $\lambda \leqq \delta$. (8b)[5]

It is evident, from what has been said before, that a quantitative study of the single transmission effect requires not only the measurement of the percentage change of neutron transmission $\Delta I/I$ upon magnetization of the sample, but an equally accurate measurement of the percentage deviation from saturation, reached in the magnetized state, i.e., of the quantity ϵ, entering in Eq. (4). In the work presented here, we have determined $\Delta I/I$ for a series of values of ϵ and for two different thicknesses d of the same sample of hot rolled steel with the threefold objective:

1. To verify the depolarization theory of Halpern and Holstein.
2. To determine the "polarization cross section" p, defined by (1).
3. To obtain information about the characteristic length λ, entering in (4) and thereby through (8) about the linear dimension δ of the microcrystals in the sample under consideration.

[5] It is clear that Eqs. (8) and (8a) are somewhat qualitative, insofar as δ, although doubtlessly of the order of the linear dimensions of the microcrystals, actually has to be obtained as a very complicated average value, not only over the sizes, but also over the varying geometrical shapes of the microcrystals; furthermore, as pointed out by Halpern and Holstein, there enters in the rigorous calculation the demagnetizing field from the surfaces of the microcrystals. It may be noted that the rigorous and general expression of the length λ entering in (4) can actually be written down as the average value of a certain double integral over the volume of a single crystal; assuming random orientation of the microcrystals it then appears that, due to the demagnetizing field, λ depends also upon the angle θ between the direction of the traversing neutrons and that of the magnetization. In the case corresponding to (8) one obtains

$$\lambda \sim (1+\tfrac{3}{8}\sin^4\theta)^{-1}.$$

In the case corresponding to (8a), the length λ is simply correlated to the total area of the surfaces of all microcrystals. In fact, if we denote by a the mean value of this area per unit volume of the sample, one obtains in this case

$$\lambda = \frac{1}{8a}\left(9+2\cos^2\theta - \frac{\cos^4\theta}{3}\right)^{-1}.$$

In view of the fact, however, that in our arrangement we had always $\theta=\pi/2$ and that the shape and size of the microcrystals are not known the simple formulae (8) and (8a) are quite sufficient for our purposes.

II. MEASUREMENT OF THE SINGLE TRANSMISSION EFFECT

The method and arrangement to determine the single transmission effect were essentially the same as used by Alvarez and Bloch[4] in their determination of the neutron moment. The slow neutrons (see Fig. 1) emerged from a paraffin howitzer directly in front of the target chamber of the Stanford cyclotron[6] where a Be target was bombarded with 2.5-Mev deuterons. In a double screening wall of water-filled tin cans for the absorption of neutrons and of bricks for the absorption of γ-rays, there was in front of the howitzer a circular hole of 2-in. diameter to obtain a collimated neutron beam. This beam passed through a square plate of hot rolled steel between the poles of an electromagnet at right angles to the magnetizing field and from there through a Cd tube of 2-in. diameter and $26\frac{1}{2}$-in. length which provided further collimation to the detecting BF_3 filled ionization chamber. In order

[6] The construction of this $27\frac{3}{4}$-in. cyclotron by Bloch, Bradbury, Staub, and Stephens was made possible by grants from the Rockefeller Foundation, the Research Corporation, and from various private sources. It was completed in the autumn of 1941, giving under normal operation a beam of about 10 μamp. of 2.5-Mev deuterons.

to prevent neutrons, scattered on the material of the magnet from entering the chamber, Cd diaphragms were placed across the neutron beam both on the front side and on the back side of the magnet. Except for a circular hole of 2-in. diameter in front, the ionization chamber was completely shielded with Cd and embedded in a large block of paraffin, surrounded by a layer of water-filled tin cans, leaving only its front side open. All these precautions were necessary in order to obtain conditions where the majority of the recorded neutrons were of thermal energy and had actually passed through the strongly magnetized region of the steel sample; even with a thickness of steel traversed of as much as 3.8 cm we found that 67 percent of the neutrons recorded were thermal and had passed through the steel. This was ascertained by shielding just the steel sample with Cd and counting as background the remaining neutrons, partly coming from elsewhere and partly consisting of those collimated neutrons above the thermal region which were still able to pass the Cd shield.

The boron trifluoride ionization chamber which was similar to the one used by Powers[2] had an active volume of 2-in. diameter and 6-in. length. It was filled with 2 atmos. of BF_3. The aluminum electrodes consisted of three concentric cylinders. The most inner and outer electrodes were connected to a potential of 3500 v and the intermediate one to the grid of a 38 tube. Porcelain was used as an insulator and during a full year did not seem to cause any trouble. Both leads were brought out of the chamber by spark plugs. The bottom of the chamber was sealed by an ordinary rubber gasket.

The boron-disintegration pulses in the ionization chamber were amplified in a Wynn-Williams type amplifier and, through a scale of 8 or 16 scaling circuit, were recorded by a Cenco counter. Since the collection time for the ions is by no means very small compared to the characteristic time of the amplifier, the number of pulses recorded would strongly depend on the voltage applied to the chamber. This effect can be reduced if one sets the bias of the recording device so that it is below the size of the great majority of the pulses which, due to the imperfect collection, consist of a rather broad group tailing off toward lower energies. The runs were taken with good steady conditions of the cyclotron, and we always operated the beam at such an intensity as to give us a rate of about one count per second on the Cenco counter. In order to be independent of fluctuations of the neutron intensity, we used the monitor of Fryer and Staub,[7] consisting of another small BF_3 ionization chamber and an electronic circuit which periodically recorded intervals of an equal amount of charge having been collected in this chamber. The monitor chamber was surrounded with paraffin and placed at a distance of about one foot from the Be target of the cyclotron. If the monitor would be ideal and no changes in the recording apparatus would occur, one should obviously record the same number of neutrons per monitor interval, independent of the intensity of the cyclotron beam. Actually, we found in the course of our measurements that this number fluctuated by about 1 percent; this may have been due to a slight dependence of the monitor performance upon the position of the target spot or to variations of the line voltage and, therefore, of the collecting voltage of the ionization chamber (which would affect the rate of recorded pulses). In order to eliminate the influence of these slow variations upon the accuracy of our measurements, the magnetizing current was turned on and off in alternate monitor intervals, and the percentage difference of the total number of counts, obtained under the two different conditions was then taken as a measure of the transmission effect.

By a suitable choice of the condenser, discharged by the ionization current in the monitor chamber, the monitor intervals were made of an approximate duration of $1\frac{1}{2}$ min. At the end of each monitor interval conditions were automatically changed through a telephone selector relay, which at the same time switched alternatingly over from one to the other of the two Cenco counters, thus allowing one counter to read all counts taken with and the other counter all those without magnetizing current. Measurements were performed on two samples of hot rolled steel of identical material. They were made of machined square plates of 5×5 cm, one sample having a total thickness of 1.91 cm, the

[7] E. Fryer and H. Staub, Rev. Sci. Inst. **13**, 187 (1942).

other of 3.81 cm. Two opposite sides of the square were clamped between the poles of the magnet, its face perpendicular to the neutron beam and carefully centered with respect to it. The runs were taken by setting the magnetizing current to a certain value and then taking counts with this current alternatingly on and off; the setting of the current was frequently changed to ascertain that independent runs with the same value of the current gave statistically consistent results. A sufficiently large number of counts was taken for each setting to obtain the difference of the neutron intensity with and without magnetizing current to within an accuracy of about $\frac{1}{4}$ percent and $\frac{1}{2}$ percent of the intensity for the thin and the thick sample, respectively. The following tabulation gives the results for the two samples and for various values of the magnetizing current up to the maximum value of 15 amperes which we were allowed to pass permanently through our magnet. The transmission effect $\Delta I/I$ and its probable error tabulated here were obtained by dividing the directly observed difference of intensity by $I(1-b)$, where b is the percentage background, taking into account that the background is independent of the presence or absence of the magnetizing current.

Thin sample (1.91 cm). Percentage background $b=6.95\pm0.1$ percent.

Current (amperes)	0.50	0.70	1.0	2.0	3.0
$\Delta I/I$ (percent)	$0.43\pm.13$	$1.76\pm.25$	$1.08\pm.19$	$1.65\pm.24$	$2.78\pm.26$
Current (amperes)	5.0	9.0	15.0		
$\Delta I/I$ (percent)	$3.33\pm.25$	$3.27\pm.25$	$3.67\pm.25$		

Thick sample (3.81 cm). Percentage background $b=32.8\pm0.81$ percent.

Current (amperes)	1.0	1.2	2.0	3.0	4.0
$\Delta I/I$ (percent)	$0.80\pm.76$	$-0.06\pm.72$	$2.79\pm.69$	$3.24\pm.76$	$3.27\pm.77$
Current (amperes)	6.9	9.0	11.25	15.0	
$\Delta I/I$ (percent)	$4.51\pm.69$	$6.39\pm.71$	$7.22\pm.75$	$7.70\pm.69$	

It is of course important, when dealing with such small effects, that any small source of error be eliminated. Both before and between our runs various tests and blank runs were undertaken to ascertain carefully the reality of the observed effects. The test probably most severe consisted of runs where the iron sample was substituted by a brass sample with a thickness so as to transmit the same number of neutrons as the iron and under otherwise identical geometrical and physical conditions; the fact that we thus obtained well within the accuracy of our transmission effects no difference in the recorded counts, taken with and without the magnetizing current, shows particularly that the stray field of the magnet does not disturb the recording apparatus since it would be even stronger with brass than with iron between the poles.

III. MEASUREMENT OF THE MAGNETIZATION

As pointed out in Section I, it is not the dependence of the single transmission effect upon the magnetizing current, which is of physical interest, but its dependence upon the magnetization of the sample. After carrying out the measurements, described in the previous paragraph. we thus had to devise methods by which the small deviations from saturation reached in our experiments could be determined with fairly good accuracy.

We define the magnetization by

$$M=(B-H)/4\pi, \tag{9}$$

whereby its determination rests upon that of the difference of the magnetic induction B and the magnetic field H in a region where one can expect both to be uniform. The geometrical shape of the square plates used in the transmission measurements is rather inconvenient for such a determination; besides, since their linear dimensions are comparable to those of the poles of the magnet, B and H cannot be expected to be very uniform all over the sample. We had therefore first to determine ϵ in its dependence upon H on a sample of the identical material, but of such size and shape as to give more favorable conditions for the measurement of the magnetization. Such conditions were found in the so-called "isthmus method" which we have used, where a long and thin cylindrical piece of the material under test, the "isthmus," is mounted across the poles of the magnet and where B and H are measured in its central region. The arrangement is schematically indicated in Fig. 2. The cylindrical isthmus is wound with a winding w_1, consisting of a certain number of turns. On a coaxial brass casing is wound a second winding w_2, consisting of the same number of turns. If we assume B and H to be parallel to the cylinder axis within the cylindrical sections, enclosed by w_1 and w_2, their changes ΔB and ΔH upon a sudden change of the magnetizing current can be determined from the ballistic throw of a galvanometer G, connected in series with the windings. Indeed, let A be the wound area of the isthmus, a_1 and a_2 the wound areas of the air space surrounded by the windings w_1 and w_2, respectively; let, further, D_1 be the ballistic deflection of the galvanometer if connected in series with w_1 and D_2 the deflection if G, w_1, and w_2 are all three connected in series, the latter two such that their induced electromotive forces oppose each other. Taking the galvanometer readings upon variation of the magnetizing current between the same two limits, one has then

$$\Delta H=\frac{D_2}{k(a_2-A-a_1)}, \tag{10}$$

$$\Delta B=\frac{1}{kA}\left(D_1-\frac{a_1}{a_2-A-a_1}D_2\right). \tag{10a}$$

The constant k depends upon the characteristics of the galvanometer and other elements of the circuit; we determined it in the usual way by calibration with an air coil. Its actual value, however, does not enter in the ultimate determination of ϵ.

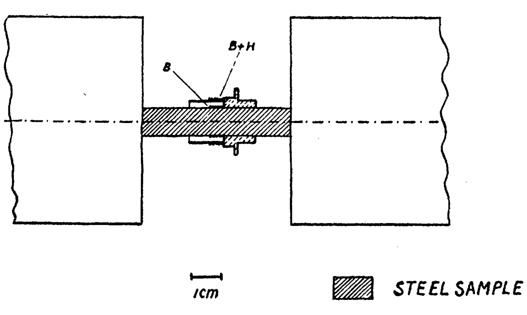

FIG. 2. Arrangement of windings and sample for the determination of H and B.

The requirement of accuracy in the determination of the quantities (10) and (10a) is evidently quite stringent since their difference $\Delta B-\Delta H$, which determines the change of magnetization, becomes increasingly smaller upon approach to saturation. Changing, for example, the magnetic H field in the neighborhood of 3000 gauss by 300 gauss, we found the corresponding change of $B-H$ to be about 10 gauss, or $\frac{1}{2}$ per mille of its saturation value. In order to ascertain this change to within 5 gauss or $\frac{1}{4}$ per mille of saturation, it requires therefore a determination of the deflections D_1 and D_2 and of the wound areas A and a_2 such that in the difference of (10) and (10a) they do not involve an error of more than 5/300 or about $1\frac{1}{2}$ percent, i.e., each of these quantities must be known with an accuracy of about 1 percent; a_1 which itself represents only a small correction, due to the finite thickness of wire and insulation of winding w_1, has to be known with correspondingly less accuracy. While it is true that the determination of these quantities with the required accuracy demands great care, it is not prohibitive, and we feel confident that we have succeeded. The determination of the wound areas requires merely well-machined cylindrical surfaces, the diameter of which can be easily measured with a micrometer; the correction for wire thickness and insulation must, of course, be applied after remeasuring the diameter when the wire has been wound on. A sufficient accuracy of the galvanometer reading can be ascertained by patient repetition.

The only remaining source of error can arise if B and H are appreciably non-uniform in the investigated central region of the isthmus. While it is true that in the absence of the isthmus the magnetic field in this region is sufficiently uniform and that its distortions by the isthmus become negligible if the isthmus is thin enough, one has nevertheless to make sure that this uniformity is maintained under the conditions of the experiment. We have considered as criterion for sufficient thinness of the isthmus that making the isthmus still thinner does not change the result. After carrying out the measurements with an isthmus of 5.1-cm length when its own diameter was 0.961 cm and that of the brass casing 1.430 cm, we have repeated them once more with another isthmus of the same length and the two

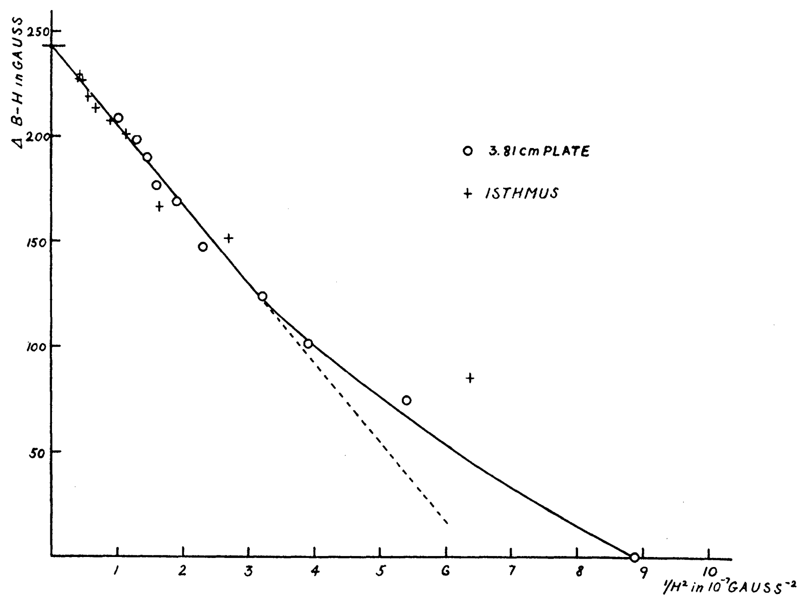

FIG. 3. Change of $B-H$ *versus* $1/H^2$. The actual value of $B-H$ is obtained by adding to the value of $\Delta(B-H)$ the constant value 19,800 gauss.

diameters, respectively 0.638 cm and 0.946 cm. Since the results, obtained for the approach to saturation, agreed in the two cases within the experimental error, it seems safe to assume that the error involved from non-uniformity was negligible.

There is another independent and rather severe test for the correct determination of the wound areas and for the uniformity of H and B. If the magnetizing current is more and more increased, and the corresponding increases of B and H are computed from (10) and (10a), it must turn out that while both B and H keep on increasing, their difference must converge towards a finite value, which through (9) determines the saturation value of the magnetization M. Both non-uniformities and errors in the wound areas would manifest themselves in an unlimited increase or decrease of the computed value of M. Unless there is an accidental cancellation of these errors, the very fact of the computed value of M converging towards a finite saturation value indicates the correctness of the performed measurement.

From the theoretical side one can predict the law of approach to saturation as long as one assumes that in the magnetization process of a macroscopic sample the magnetic moment of each microcrystal is changed only in its orientation but not in its magnitude. One obtains then for strong fields

$$M=M_\infty-a/H^2, \tag{11}$$

where a depends upon the material[8] and is a slowly varying function of H.

This law has been verified with great accuracy by Czerlinsky;[9] the small deviations which he has observed for very strong magnetic fields indicate indeed a slow increase of a with H. Within the experimental error we were able for high magnetic fields to represent our results by a law of the form (11) with a constant a, indicating that for the material and the fields used by us, the above-mentioned deviations were of no importance. In evaluating the percentage deviation ϵ of Eq. (3), we have used the deviation of the magnetization from that value, obtained by (11), upon extrapolation to $H=\infty$ with constant a.[10]

[8] R. Becker and W. Doring, *Ferromagnetismus* (Springer, Berlin, 1939), p. 167. T. Holstein and H. Primakoff, Phys. Rev. **59**, 388 (1941).

[9] E. Czerlinsky, Ann. d. Physik **13**, 89 (1932).

[10] It should perhaps be mentioned that it is essential to use (11) for extrapolation; the increase of magnitude of the microcrystalline magnetization, which will actually occur in very strong fields and which is omitted in that formula, has no effect upon the depolarization and on the value of ϵ, to be used in the sense of the theory. Our

In Fig. 3 we have plotted $\Delta(B-H)$ and its percentage deviation from saturation ϵ in their dependence on the magnetic field H, as obtained by the isthmus method for hot rolled steel. Although deviations of the order of one-tenth of a percent are to be considered significant in this plot, we do not claim nearly such an accuracy for the absolute determination of the extrapolated value of the magnetic saturation.

Once we have established the relationship between ϵ and H by the isthmus method, the problem of assigning the proper value of ϵ to a given magnetization current in the single transmission experiments, described in Section II, becomes comparatively simple. It reduces to the determination of the magnetizing field H for a given value of the current, the percentage error in ϵ thus involved being merely of the order of that of H. To determine the field in the central region of the plate, through which the neutrons pass in the transmission experiment, we used a thin rectangular search coil, placed right alongside the face of the plate. Its wound area was again effectively determined by calibrating the ballistic throw of the galvanometer against that produced by the known field at the center of a current solenoid. The position of the search coil was not very critical since, moving it over the whole area through which neutron passage occurs, we did not find the field to vary by more than about 3 percent; with the same accuracy we found the field along the entrance and exit face of the plate to be the same. In addition, we had to convince ourselves that between these two faces there were no appreciable deviations of H; since the thicker sample consisted actually of three, the thinner of two separate plates, this could be done by spacing these plates just enough to insert other rectangular search coils between them. It was thus easy to determine any possible difference of H between the plates and along their outside faces. Having found no such difference within the accuracy of our measurements of a few percent, we see no reason to doubt that the field H was sufficiently uniform throughout the passage region of the neutrons, and that for a given magnetizing current we had that percentage deviation from saturation which from the previous isthmus measurements corresponded to the field measured for this current.

TABLE I. Percentage deviation (ϵ) from saturation.

Thin sample (1.91 cm) Current (amp.)	H (gauss)	ϵ (percent)	Thick sample (3.81 cm) Current (amp.)	H (gauss)	ϵ (percent)
0.50	592	2.50	1.0	570	2.56
0.70	795	1.70	1.2	678	2.01
1.0	1100	1.16	2.0	1060	1.21
2.0	1690	0.63	3.0	1360	0.87
3.0	2140	0.41	4.0	1590	0.69
5.0	2710	0.25	6.9	2060	0.44
9.0	3370	0.17	9.0	2280	0.38
15.0	3980	0.12	11.25	2480	0.30
			15.0	2770	0.24

Table I gives the values for H and the corresponding ϵ for those currents for which the transmission effect was measured.

While measuring the magnetizing field for these samples, we measured at the same time the induction by winding the plates in the central region and observing the induced current pulse in the galvanometer. The values for the magnetization thus obtained agreed with those of the isthmus measurements for percentage deviations from saturation above about 0.5 percent. For smaller deviations we found systematic and ever-increasing differences, evidently arising from the fact that under the unfavorable geometrical conditions, presented by the square plates, the knowledge of the wound areas and the uniformity of B and H became insufficient to make the determination for the smaller deviations reliable. The agreement was, however, sufficient to indicate that there were no accidental differences in the material used for the isthmus and the square

TABLE II. Dependence of single transmission effect $\Delta I/I$ upon ϵ.

Thin sample (1.91 cm) ϵ (percent)	$\Delta I/I$ (percent)	Thick sample (3.81 cm) ϵ (percent)	$\Delta I/I$ (percent)
2.50	$0.43\pm.13$	2.56	$0.80\pm.76$
1.70	$1.76\pm.25$	2.01	$-0.06\pm.72$
1.16	$1.08\pm.19$	1.21	$2.79\pm.69$
0.63	$1.65\pm.24$	0.87	$3.24\pm.76$
0.41	$2.78\pm.26$	0.69	$3.27\pm.77$
0.25	$3.33\pm.25$	0.44	$4.51\pm.69$
0.17	$3.27\pm.25$	0.38	$6.39\pm.71$
0.12	$3.67\pm.25$	0.30	$7.22\pm.75$
		0.24	$7.70\pm.69$

extrapolation with constant a is very nearly correct, since variations would make themselves noticeable only in considerably higher fields.

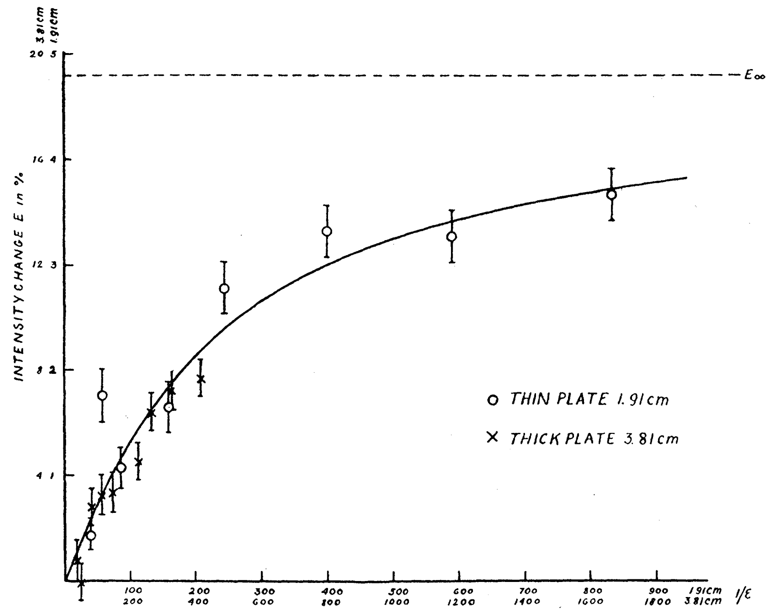

FIG. 4. Single transmission effect for both plate thicknesses as a function of deviation from saturation. Heavy line represents best fitting theoretical curve with corresponding saturation effect E_∞.

plates, and that we could indeed expect the same relation between ϵ and H for both.

IV. DISCUSSION OF THE RESULTS

From the measurements, reported in Section II and Section III, we are now able to give the final results in Table II, which represents the dependence of the single transmission effect $\Delta I/I$ upon the percentage deviation from saturation ϵ for our two samples.

These results are plotted in Fig. 4; the ordinates are the single transmission effects $E=\Delta I/I$, the abscissae the corresponding values of $1/\epsilon$. Since the ratio of the thicknesses d of the two samples is just about 1 : 2, we have chosen a representation in which for the thick sample the scale of the abscissae is reduced by a factor 2, that of the ordinates by a factor 4.

One can see that by this transformation the points obtained for the two samples fall within the statistical error into the same pattern; this shows that in agreement with Eq. (4) the polarization effect is proportional to the square of the thickness and that the depolarization factor f depends merely upon the product ϵd. Quite independent of the special form (5) of the function f, we consider this agreement as a first important test for the correctness of the theory of Halpern and Holstein.

The complete quantitative test of this theory is evidenced by the drawn out curve of Fig. 4. It represents the best fitting member of the two-parameter family of curves which one obtains from (4) and (5) by varying the two unknown constants p and λ. The thus experimentally determined values of these two constants are with their estimated errors

$$p=2.0\pm.1\times10^{-24}\ \text{cm}^2, \tag{12}$$

$$\lambda=3.2\pm.3\times10^{-3}\ \text{cm}. \tag{13}$$

The best theoretical evaluation of p is given by Hamermesh;[11] actually he finds for the quantity $w=np$ ($n=$ the number of iron atoms per cc) $w=0.090\ \text{cm}^{-1}$ from which it would follow with $n=8.2\times10^{22}/\text{cc}$

$$p=1.1\times10^{-24}\ \text{cm}^2; \tag{14}$$

this is smaller by a factor 1.8 than the observed value, and therefore from (4) would lead to about three times too small transmission effects. The results of Powers[2] for thin iron agree with our experimental determination of p, indicating

[11] M. Hamermesh, Phys. Rev. 61. 17 (1942).

that his iron was sufficiently magnetized to reach the saturation value.

In comparing the theoretical and experimental values of p, it has to be remembered that the calculation of the former is based upon the not-too-well-known wave functions for the $3D$-shell of Fe. A qualitative description of the disagreement in terms of the wave functions can be given by stating that all their linear dimensions would have to be contracted by a factor $1\frac{1}{2}$ in order to reestablish agreement. It is true that so big an error in the scale of wave functions, calculated by the Hartree method, is surprising and that it is by no means certain that this is the cause of the observed discrepancy. On the other hand, the neutron polarization depends on the atomic form factor of the $3D$-shell alone and represents a rather more severe experimental test on the Hartree method than the spectroscopic and x-ray measurements by which it has been tested before.

As another alternative, Halpern and Johnson[12] have suggested the possibility of the neutron having the spin $\frac{3}{2}$, instead of the usually accepted value $\frac{1}{2}$. In view of the difficulties which such an hypothesis would introduce in the interpretation of nuclear phenomena, the sole support from the above discrepancy in the single transmission of neutrons does not seem to us sufficient to make it appear probable.

[12] O. Halpern and M. H. Johnson, Phys. Rev. **57**, 160 (1940).

As mentioned in Section I, the determination (13) of λ is connected with the linear dimension δ of the microcrystals. According to (8) and (8a) two values of δ are possible for a given λ, depending upon whether δ is small or large compared to the length l, given in (7). Depending upon the choice we get

from (8) $$\delta = 1.4 \times 10^{-4} \text{ cm}, \tag{15}$$

from (8a) $$\delta = 6.4 \times 10^{-3} \text{ cm}. \tag{15a}$$

The value (15a) is far too big to be plausible, and we consider the value (15) as a fair determination of the microcrystalline dimensions by means of this method.

In view of our results we foresee no essential experimental difficulties to overcome the rather strong depolarizing influences and thus to obtain considerably higher polarization effects than the ones obtained so far. It is true that in the case of thin iron we were able to obtain almost the complete effect, but for obvious reasons we were far from doing so with the thick sample. This could be helped, no doubt, by raising the magnetizing field to the order of 10,000 gauss which would merely require a somewhat larger electromagnet than the one at our disposal. Besides, however, it may well turn out that hot rolled steel is by no means the ideal material in the sense that other grades of iron will consist of smaller microcrystals or may be found easier to saturate, both desirable features to keep the necessary magnetizing field as low as possible.