frequency could be due to this phenomenon. The other explanation[5] is that spin 1 mesotrons are present at higher altitudes in addition to spin $0-\frac{1}{2}$ mesotrons, and these give rise to the observed bursts. One assumes that the spin 1 mesotrons are of the fast decaying type, and only the highest energy ones reach sea level.

[1] Phys. Rev. **64**, 254 (1943).
[2] R. F. Christy and S. Kusaka, Phys. Rev. **59**, 414 (1941).
[3] This point is discussed in an accompanying Letter to the Editor by S. Kusaka, Phys. Rev. **64**, 256 (1943).
[4] M. Schein and P. S. Gill, Rev. Mod. Phys. **11**, 267 (1939).
[5] H. Snyder, Phys. Rev. **59**, 1043 (1941). R. F. Christy and J. R. Oppenheimer, Phys. Rev. **60**, 159 (1941).

The Effect of Radiation Damping on Burst Production

SHUICHI KUSAKA
Department of Physics, Smith College, Northampton, Massachusetts
September 20, 1943

In a recent paper Chakrabarty[1] has repeated the calculation of Christy and the present author[2] on the frequency of burst production by mesons and concludes that the comparison with the experimental data shows that the meson has spin 1, in contradiction to our result that the spin is 0 or 1/2. He states that the reasons for the different conclusion obtained are (1) the difference in the form of the fluctuation assumed, (2) the rough values used for the average number of particles produced in a cascade shower by an energetic electron or photon, and (3) the consideration of the effect of radiation damping on the cross section for bremsstrahlung of spin 1 mesons.

Chakrabarty used the Poisson distribution in preference to the fluctuation formula obtained on the Furry model which we used, and the recent work of Scott and Uhlenbeck[3] indicates that the former is nearer the truth. However, this point does not introduce any appreciable difference since we found that the effect of the fluctuation on the frequency of burst production on our model is to give just about twice as many bursts as calculations based on the assumption of no fluctuation. The Poisson distribution gives smaller fluctuations than the Furry model, and hence it would give results intermediate between the two. Moreover we recognized the fact that the Furry model gave too great a value for the fluctuation and corrected for this by reducing the burst production probability by a factor of $\sqrt{2}$. Thus the difference introduced by the use of the Poisson distribution should at most be a factor of about $\sqrt{2}$.

The second difference is due to the fact that we used Serber's[4] calculation on the cascade theory while Chakrabarty used the recent results obtained by Bhabha and himself.[5] The latter gives for the average number of particles a value smaller than the former by a factor ranging from 1.5 to 2.1 for initial energy between 10^{10} and 10^{12} ev.

Most of the deviation between the two calculations arises from the third difference. Chakrabarty based his calculation on the formula for bremsstrahlung cross section of spin 1 meson with radiation damping obtained by Wilson,[6] but it has now been found that this formula is in error, and hence his conclusions are invalid.

Formula (52) in Wilson's paper for the cross section for scattering of light quantum by a meson with spin 1 initially at rest is incorrect and should read instead[7]

$$\varphi_0=\frac{5\pi}{36}\left(\frac{e^2}{\mu}\right)^2\frac{k_0}{\mu}\;(k_0>>\mu),$$

where k_0 is the initial energy of the light quantum. When the calculation of the cross section for bremsstrahlung is carried out in the same way by using the method of the virtual quanta and Wilson's approximate way of taking into account the effect of radiation damping, the result is

$$\varphi(\epsilon)d\epsilon=\frac{5Z^2\alpha}{144}\left(\frac{e^2}{\mu}\right)^2(2-2\epsilon+7\epsilon^2)Gd\epsilon,$$

where $\alpha=1/137$ is the fine structure constant, ϵ is the fraction of the initial energy, E_0, of the meson emitted in the form of a light quantum, and

$$G=\int_1^{\mu R}\frac{dy}{y}\int_{y\mu/E_0}^{\infty}\frac{dx}{x+(5/288)\alpha^2},$$

R being $a_0Z^{-\frac{1}{3}}=(\alpha mZ^{\frac{1}{3}})^{-1}$ and where we are using units in which $\hbar=c=1$. The effect of the radiation damping appears in the term $(5/288)\alpha^2$ in the denominator. It is negligible for energies such that $(E_0/\mu)^2<<(288/5\alpha^2)$, and its effect becomes appreciable only for energies of the order $E_0/\mu\sim 10/\alpha=1370$. In the limit of very high energies, $E_0/\mu>>10\mu/\alpha^2mZ^{\frac{1}{3}}$, the cross section becomes

$$\varphi(\epsilon)d\epsilon=\frac{\pi}{12}\left(\frac{5}{2}\right)^{\frac{1}{2}}Z^2\left(\frac{e^2}{\mu}\right)^2(2-2\epsilon+7\epsilon^2)\log\left(\frac{137\mu}{Z^{\frac{1}{3}}m}\right)d\epsilon.$$

It must be noted here that there is an uncertainty in the numerical coefficient of the order unity due to the averaging over the angles which was necessary in order to compute the effect of the damping. The above result is essentially the same as that obtained by Gora,[8] and agrees with the general conclusions of Landau[9] and Oppenheimer[10] that the cross section obtained by the perturbation theory should be valid up to energies of the order 137μ. Hence our calculation in which we cut off the frequency integral at 137μ should at least give the lower limit of the burst production by spin 1 mesons.

The improvements in the treatment of the fluctuation and in the cascade theory mentioned above should change our results at most by a factor of 3, and our conclusion that the spin of the meson can be 0 or 1/2, but not 1 is still valid. Comparison of Chakrabarty's results for spin 0 and 1/2 with ours shows that his theoretical burst frequencies are smaller by a factor of about 5. We both used the same value for the meson mass, 177 m, but Chakrabarty fails to mention the value he used for β, the critical energy in the cascade theory, and the additional difference may be due to this.

It should be emphasized again that these calculations for the burst production give the *minimum* estimates in which only the electromagnetic interaction of the meson with the atomic nuclei is considered. Hence it is possible to rule out particles for which our calculations give burst production frequencies greater than the observed values, but it is not possible to do the same for particles which give smaller burst production.

It is a pleasure to thank Professor W. Pauli for valuable discussions on this subject.

[1] S. K. Chakrabarty, Ind. J. Phys. **16**, 377 (1942).
[2] R. F. Christy and S. Kusaka, Phys. Rev. **59**, 414 (1941).
[3] W. T. Scott and G. E. Uhlenbeck, Phys. Rev. **62**, 497 (1942).
[4] R. Serber, Phys. Rev. **54**, 317 (1938).
[5] H. J. Bhabha and S. K. Chakrabarty, Proc. Ind. Acad. Sci. **A15**, 464 (1942).
[6] A. H. Wilson, Proc. Camb. Phil. Soc. **37**, 301 (1941).
[7] Cf. formula in row IVA of Table IIIA on p. 230 of W. Pauli, Rev. Mod. Phys. **13**, 203 (1941).
[8] E. Gora, Zeits. f. Physik **120**, 121 (1943).
[9] L. Landau, J. Phys. USSR **2**, 483 (1940).
[10] J. R. Oppenheimer, Phys. Rev. **59**, 462L (1941).

The Multiple Production of Penetrating Particles by Cosmic-Ray Protons and Neutrons

WAYNE E. HAZEN
Department of Physics, University of California, Berkeley, California
September 21, 1943

IN a recent paper, Hamilton, Heitler, and Peng[1] have presented the results of calculations for the probability of mesotron production in close nuclear encounters by protons or neutrons. They predict a cascade process for the production of mesotrons in which the incident proton loses energy very rapidly until its energy has fallen to 10^9 ev. For example, a proton with energy 10^{10} ev should produce 3.3 mesotrons in 5 cm of lead, and a proton with energy 10^{11} ev should produce 11 mesotrons in 20 cm of lead. In other words, one mesotron should be produced in roughly every two centimeters of lead. Previously, it has been customary to hypothesize simultaneous production of several mesotrons.

In a cloud chamber containing eight 0.7-cm lead plates[2] operated at 10,000 feet, no events were observed of the cascade type mentioned in the preceding paragraph. On the other hand, four photographs were obtained that showed multiple production, apparently in a single event, of high velocity penetrating particles. The expected number of high energy protons, which are presumed to produce the events, can be estimated from the information given by Hamilton, Heitler, and Peng.[1] Since the number of high energy protons varies as the inverse cube of the depth in the atmosphere, there should be three times as many energetic protons at 10,000 feet as at sea level. The number of penetrating particles increases by a factor of two and hence the relative abundance of energetic protons should increase by a factor of 1.5. Now since 1/12,000 of the rays at sea level produce showers of penetrating particles,[3] 1/8,000 of the particles at 10,000 feet should produce such showers. In the present experiment 13,000 tracks of penetrating particles were observed, and one would therefore expect to observe one or two showers of penetrating particles. As previously mentioned, there did occur four events in which the multiple production in a single event of penetrating particles with range greater than four lead plates was observed. These particles were still traveling with velocity $\backsimeq c$ when they passed out of the chamber, and thus it is possible that they had sufficient energy to produce the penetrating type of shower observed by Janossy.[3]

The showers of penetrating particles were produced both by ionizing and by non-ionizing rays just as in the observations of Janossy. None of the four pictures showed electron showers associated with the penetrating shower. However, this fact does not necessarily indicate lack of association with an Auger shower since there is a probability of at least 1/4 that the cloud chamber was located in a region surrounded by shower particles but itself untouched by shower particles.[4] Janossy[3] concluded that all the penetrating showers observed with no absorber above the first counters were parts of Auger showers, whereas less than 1/3 were associated with Auger showers when 1.8 cm of lead was placed above the first counters; the latter arrangement was comparable with that used by the author.

There does not appear to be any direct experimental evidence for a cascade production of penetrating particles. On the other hand, several cases of multiple production of energetic penetrating particles in a single event (or a cascade confined to a few millimeters thickness of lead) have been photographed both by the author and by other observers.

[1] Hamilton, Heitler, and Peng, Phys. Rev. **64**, 78 (1943).
[2] W. E. Hazen, Phys. Rev. **64**, 7 (1943).
[3] L. Janossy, Proc. Roy. Soc. **179**, 361 (1942).
[4] See L. Janossy and A. C. B. Lovell, Nature **142**, 761 (1938) as an example.

The Diffraction of X-Rays by Binary Alloys

R. SMOLUCHOWSKI
Research Laboratory, General Electric Company, Schenectady, New York
October 1, 1943

IT is generally assumed that binary alloys, apart from those which show a definite periodicity of atomic arrangement, have the two kinds of atoms distributed among the lattice points at random.

In connection with a study of the various types of deviations of the atomic distributions from the purely statistical one, the effect of atomic arrangement on the x-ray diffraction was investigated. It is assumed that no long range order exists so that the periodicity of the lattice is unchanged. However, the average neighborhood of an atom A is in general different from that of a B atom. This statistical preference of like or opposite atoms to be near to each other can be expressed in various ways. Let us consider an A–B alloy in which there are more B atoms than A atoms and let $\alpha(r)$ be the probability that one of the neighbors at a distance r from an atom A is also an A atom and $\beta(r)$ the probability that one of the neighbors at a distance r from an atom B is an A atom. We define the distribution factor $s=s(r)=\beta(r)-\alpha(r)$; it is equal to zero in a random crystal at all concentrations. There are altogether N atoms, and the concentration of atoms A is c_a and of atoms B is c_b. Then the mathematical expectation of the scattering factor of an atom n at the distance r from an atom A is

$$F_n^{\mathrm{A}}=[c_a-c_b s(r_n)]F_{\mathrm{A}}+[c_b+c_b s(r_n)]F_{\mathrm{B}},$$

and similarly

$$F_n^{\mathrm{B}}=[c_a+c_a s(r_n)]F_{\mathrm{A}}+[c_b-c_a s(r_n)]F_{\mathrm{B}}$$