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The Spatial Asymmetry of Cerenkov Radiation as a Function of Electron Energy*

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The properties of the radiation discovered in 1934 by P. A. Cerenkov when high speed electrons traverse transparent media has been studied as a function of the energy of the bombarding particle. It is found that the direction of emission of the radiation for mica over the range from 240 to 815 kv is given by the relation proposed by Frank and Tamm, where $\cos\theta = 1/\beta n$, θ being the angle of progression of the radiation with the electron beam and n, the index of refraction of the mica used as the target. In this range β changes markedly, providing a significant test of this relation as a function of the energy of the bombarding particle. The experiments were carried out with the use of a transformer-rectifier source of potential and an electron tube giving a well-collimated beam of electrons.

INTRODUCTION

THE emission of a visible radiation with remarkable asymmetry properties when high speed electrons bombarded transparent materials was first discovered and investigated by Cerenkov[1] in 1934. This radiation, continuous in character, and appearing as a bluish white light proceeds from the point of impact of the electrons at an angle of the order of 30° with the direction of the electron beam. The nature of this radiation was formulated by Frank and Tamm[2] who recognized that the electrons causing this radiation are travelling through the media with a velocity greater than the velocity of light which they produce. A simple Huygens construction showing the formation of such a wave front when a high energy electron enters a transparent material is illustrated in Fig. 1. A cone of visible radiation appears in the medium with apex at the point of entrance of the elec-

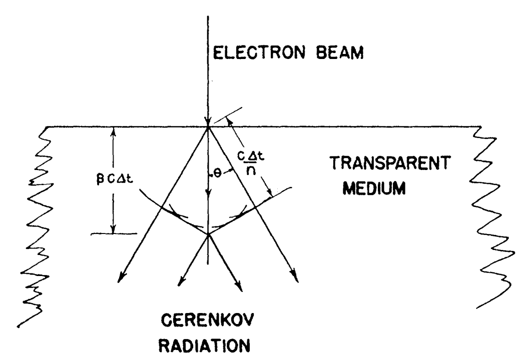

FIG. 1. Simple Huygens construction illustrating the formation of the wave fronts in the production of Cerenkov radiation. The electrons producing the radiation travel faster in the transparent medium than does the radiation which they generate. The radiation is symmetrical about the electron beam.

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[1] P. A. Cerenkov, Comptes rendus Acad. Sci., U.S.S.R. **2**, 451 (1934); **3**, 414 (1936); **14**, 101 (1937); **14**, 105 (1937). Phys. Rev. **52**, 378 (1937).

[2] I. Frank and Ig. Tamm, Comptes rendus Acad. Sci., U.S.S.R. **14**, 109 (1937).

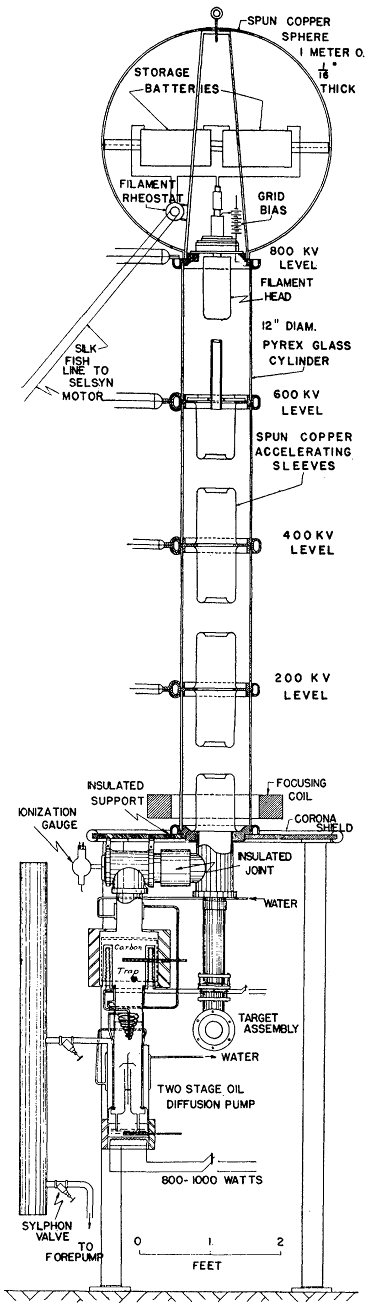


FIG. 2. A schematic diagram of the tube in which the high energy electrons are generated.

trons. From this construction it is evident that:

$$\cos\theta = 1/\beta n, \qquad (1)$$

where β is the ratio of the velocity of the electron in the medium to the velocity of light in a vacuum and n is the index of refraction of the medium for the visible light observed.

A classical calculation of the energy to be expected in such a process has also been given by Frank and Tamm and later by Fermi.[3] A quantitative experimental investigation of this radiation has been made by Collins and Reiling[4] who have demonstrated that Eq. (1) holds within their experimental limits for mica, Cellophane, and glass targets as far as the dependence of the angle of emission upon the index of refraction is concerned. Their work can scarcely be regarded as a complete verification of this equation as one value of β only was used, namely, that corresponding to 1.9 Mev. In this region β changes very little with the energy of the impinging electron. The present experiment was designed primarily to investigate the dependence of the angle of emission upon the energy of the impinging electron in a region where β changes sufficiently to provide a further test of Eq. (1) as far as the dependence of θ upon β is concerned. Specifically this involved the measurement of the angle of emission of the radiation caused by a well-collimated beam of high energy electrons impinging on a thin target of mica.

APPARATUS AND PROCEDURE

A schematic diagram of the tube in which the high energy electrons were generated and the auxiliary apparatus is shown in Fig. 2. The tube, which was designed for operation at potentials up to 800 kv, consists of four sections to which the voltage was applied in successive steps of up to 200 kv each. The circuit design and voltage wave form of this particular Villard-type supply may be found elsewhere.[5] This supply is part of the super-voltage x-ray equipment installed by the General Electric X-Ray Corporation at the Swedish Hospital in Seattle. The bronze castings holding the accelerating sleeves were separated with Pyrex cylinders 1 foot in diameter and 2 feet long. At the top of the tube a one-meter sphere of spun copper serves as a corona shield for the filament and grid supply. The target assembly, pumping

[3] E. Fermi, Phys. Rev. **57**, 485 (1940).
[4] G. B. Collins and V. G. Reiling, Phys. Rev. **54**, 499 (1938).
[5] J. E. Henderson, J. E. Rose, and W. H. Goss, Rev. Sci. Inst. **6**, 63 (1935).

system, and magnetic focusing coil are located at the base of the tube.

The pumping system consisted of a carbon trap used in conjunction with a two-stage oil diffusion pump[6] having a speed of approximately 100 liters per second. The carbon trap was used primarily to eliminate oil vapors from the tube to obtain a maximum filament life in both the gauges and cathode. Although not necessary for operation of the pump, a fore-vacuum of less than two microns was maintained as measured by an oil McLeod gauge. This permitted a tube vacuum of better than 2×10^{-6} mm Hg which rose to about twice this value during operation. The entire pumping system is insulated from the tube proper by a mica and wax joint to facilitate the measurement of tube currents.

The cathode head is shown in Fig. 3. It faces a small acceleration electrode which may be screwed in or out of the casting at 600 kv for adjusting the distance between the electrodes in the top section. Since each accelerating sleeve is really an electrostatic lens this distance of separation is important and particularly so in the first section. The filament and focusing cup are insulated from each other and from the filament housing. As the voltage supply is pulsating d.c., this makes possible the placing of a negative potential on the cup so that appreciable current passes through the tube only near the peak of the cycle. These parts may also be individually adjusted vertically by means of the sylphons and adjusting nuts without breaking the vacuum. A double eccentric similar to that used by Tuve[7] and Taylor[8] permits the lateral adjustment of the whole filament with an amplitude of about 1 cm in every direction from the axis of the tube. An insulating band midway along the top section supports a permanent magnet whose distance from the axis of the tube is adjustable. Adjustment of these variables aids materially in the final focusing of the tube.

The Cerenkov radiation under investigation in this experiment was also used as an aid in focusing the tube. A sheet of mica was mounted at the bottom of the top accelerating sleeve in the section at 600 kv and the normal potential was applied across the top section of the tube. By adjustment of the focusing variables the electron beam was brought to the tube axis and its size adjusted. This focused condition was shown by the appearance of a bright spot of Cerenkov radiation on the axis at the tube at the point of impact of the electrons on the mica. The same procedure was repeated for each section of the tube. During the initial focusing of the tube it was found necessary to install a short length of 8″ diameter Pyrex tubing in the section between the 600- and 400-kv levels coaxial with the 12″ diameter tubing to eliminate puncture of the main Pyrex cylinders. The beam as it emerges from the bottom of the tube is diverging. The magnetic focusing coil at the bottom of the tube permitted compensation so that the beam was sensibly parallel. The current required for this depended upon the energy of

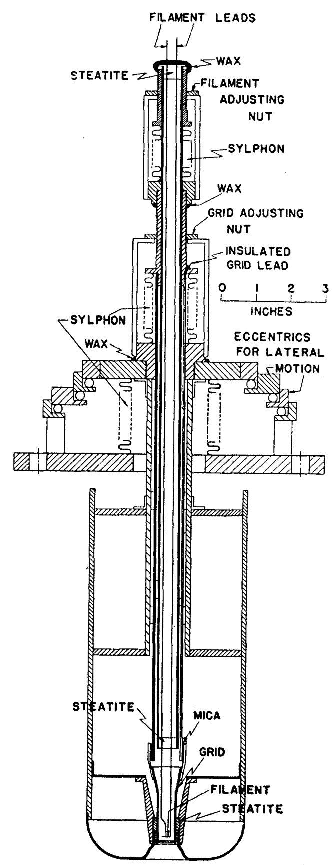


FIG. 3. The cathode assembly. The double eccentric permits a lateral adjustment of the whole cathode.

[6] J. E. Henderson, Phys. Rev. **50**, 388 (1936).
[7] M. A. Tuve, Phys. Rev. **48**, 241 (1935).
[8] L. S. Taylor, J. Research Nat. Bur. Stand. **21**, 19 (1938).

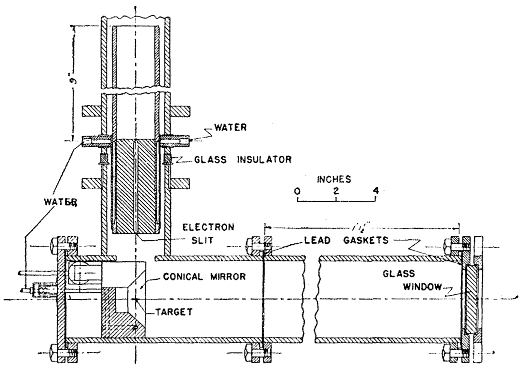


FIG. 4. The target and electron slit assembly. The electrons strike the mica target located on the axis of a 45° conical mirror. Radiation proceeding in a plane perpendicular to this axis is reflected parallel to the axis.

the electrons and provided a means of selecting electrons of a particular energy for the experiment.

The target and electron slit assembly are shown in Fig. 4. A water-cooled double slit of $\frac{1}{16}''$ diameter defines the electron beam at the bottom of the tube so that it has a maximum divergence of about $\frac{3}{4}°$. This collimated beam falls on a mica target in which it produces the Cerenkov radiation. The surface of the mica intersects the axis of an insulated and water-cooled conical mirror of silver with a $\frac{1}{2}$ angle of 45° so that all radiation produced in the mica which is traveling in a plane perpendicular to the axis of the cone is reflected by the mirror into rays parallel to this axis. By this arrangement rays emerging from the mica in that plane will, after reflection from the mirror, appear along the arc of a circle. The position along this circle gives the angle of the ray with the electron beam. These rays then pass from the vacuum through a thick glass plate at the end of the tube housing the mirror. It was necessary to place this window at considerable distance from the target to avoid discoloration from x-rays and electrons necessarily present there. From here the rays passed through a hole in a $2\frac{1}{2}$-foot concrete wall to a lens system and camera where they were focused on a strip of photographic film. The wall was necessary for the protection of both the film and observers from the extremely penetrating x-rays which were also produced by the tube. In order to obtain the radiation through the surface of the target without total internal reflection the surface of the mica is tilted at an angle with the electron beam. The mica was mounted so that it could be tilted at an angle of 20° in either direction by the mechanism shown in Fig. 5, where the target is fastened to an axle through the axis of the cone which may be rotated by air pressure through a total of 40°. Radiation should therefore appear at equal angles with the beam when the axle is rotated to its two extreme positions. The target was adjusted until these readings were equal within the accuracy of measurement. The mica used as targets in the final experiments was 0.001 inch and 0.00005 inch thick and came from two independent sources. With these thicknesses the average energy loss in the mica is small and sharp images were obtained. With extremely thin layers of mica the intensity of the radiation is small. The thicknesses chosen were really a compromise between these two factors.

The current to the target was indicated by the deflection of an oscilloscope which was placed across a high resistance in series with the lead from the target to ground. A background which existed on the screen of the oscillograph due to the current induced in the target-ground leads by the potential on the tube gave a means of determining the phase of the target current. An electronic switch which was used in the initial experiments was found unnecessary because of this effect.

The final focusing of the tube was attained for the different voltages by means of the large electromagnetic focusing coil around the lowest section of the tube. The current to this coil was adjusted so that the target current peak appeared superimposed on the peak of the voltage wave. In general this current peak was not more than 20 electrical degrees in width. A current of 10^{-6} ampere was sufficient to give intensities

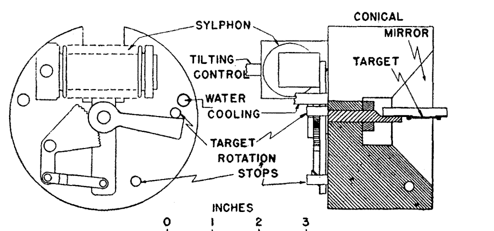
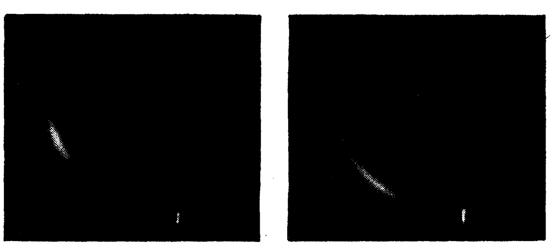


FIG. 5. Target tilting mechanism. The target must be tilted to avoid total reflection at the surface.

strong enough for recording. The voltage indicated by a sphere gap which was previously calibrated against a rotary voltmeter[5] was therefore a measure of the energy of the electrons producing the radiation.

RESULTS

A series of pictures of the radiation was taken over the voltage range from 240 to 815 kv. Typical pictures are shown in Fig. 6. The bright spot at the bottom is the image of the filament. One exposure was made with a variable voltage and with both positions of the mica so that the two spots were spread into a large arc on each side of the filament image. An enlarger was used to focus the image of this negative on the periphery of a small protractor. The angles of all of the other exposures were then measured by the protractor for the same setting of the enlarger. Actually, due to the finite width of slits and range of energies possible, the image is not a point but a narrow band. The mean of the upper and lower limits of angle in the exposed arc measured from the filament image appearing at the lower edge of the mirror was taken as the true angle. This averaging was justified by calculations which showed, at least for short exposures, that the angular width of the radiation could be attributed to the finite resolving power of the electron slit. Longer exposures showed wider spreads in this angular width which was probably due to scattering of electrons in the thin mica target and could therefore be assumed to produce equal effects on each side of the mean.



800 kv 258 kv

FIG. 6. Typical pictures of the radiation with the mica tilted 20° to the electron beam. After reflection from the conical mirror the radiation appears along the arc of a circle. The angle measured along the arc from the filament image at the bottom gives the angle of emergence of the radiation from the mica. The shift in angle of emission of the radiation with energy of the electron beam is readily visible.

In all cases the total angular spread was so small that the rays at the average angle inside the mica were also at the average angle outside the mica after refraction within the error of measurement of angles.

The solid curve of Fig. 7, where the angle of the radiation in the mica is plotted against the energy of the electron producing it, shows the theoretical curve for the two samples of mica used. These values were calculated from Eq. (1)

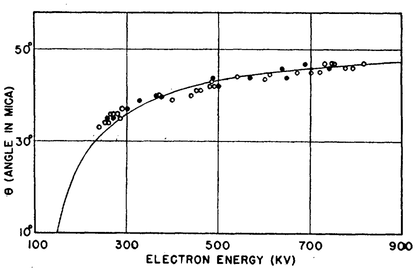


FIG. 7. The angle of progression of the radiation in mica as a function of the electron energy. The solid curve is the theoretical one obtained from Eq. (1). The open and closed circles represent experimental points obtained by two different samples of mica.

by substituting the index of refraction of 1.59 obtained for the mica film. The index of refraction of the mica was measured by the immersion method with the use of a petrographic microscope. Although mica is biaxial, two of its indices are so close together that for the experiment mica may be considered as uniaxial. The measured values of the index showed 1.59 as the proper value of the index to use in the computation. The experimental points are indicated by the small circles on these graphs. They are determined from the experimentally measured angles by taking into account the refraction of the radiation on emergence from the mica. The solid and open circles represent two different samples of mica with the same index of refraction. These experimental points for both samples all fall on the theoretical curve within our estimate of the limit of the experimental error. The functional dependence of the angle of emission of the radiation upon the velocity of the electron is that given by Eq. (1) within the range used in this experiment.

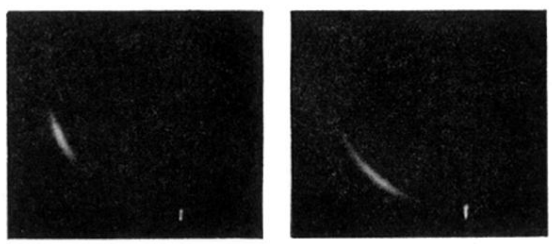
Although the deviation of the points at lower voltages from the theoretical curve is very close to our estimate of the limit of error of the experiment, all the points, even after repeated check experiments, fall above the theoretical curve suggesting that in this region a possible deviation from theory may occur. Lower energy points are very difficult to obtain with this apparatus because of the focusing difficulties with such a long electron path. Experiments are in progress to facilitate more measurements in this region with a shorter tube to see if this suggested diversion is real or apparent.

The result of this research coupled with the experiment of Collins and Reiling provides strong evidence for the soundness of the theory of Frank and Tamm. Not only is there the proper dependence of the direction of emission upon the index of refraction but also the proper dependence upon the energy over the range from 240 kv to 815 kv in which region the angle of emission changes rapidly. To this should be added the single point obtained by Collins and Reiling at 1900 kv.

Since the agreement between theory and experiment must be regarded as satisfactory it is of interest to recall the basis upon which this theory is founded. The theory is entirely classical. This satisfactory application of classical continuum theory to the problem is not surprising as the problem is essentially one of propagation of radiation in a dielectric. For the radiation to be propagated as a wave front, destructive interference must occur in all directions except that in which propagation occurs. Evidently the substance bombarded must radiate coherently over at least one wave-length of the light observed. The displacement of the dispersion centers by the electron is evidently the same all along the path of the electron. The field surrounding the moving electron acts to polarize the medium resulting in a displacement of the dispersing centers supporting the proposal of Fermi that in the energy loss of high speed electrified particles passing through matter the polarization of the medium must be taken into account.

If the explanation is valid, an experiment of this type provides a means of determining the velocity of high speed electrified particles in terms of time and distance since the angle of emission depends upon the distance the particle moves in the time for the radiation to travel one wave-length of the light observed. The values of β used in computation of the theoretical curves are based upon the relativistic expression for the energy of a particle. In this same sense this expression is substantiated.

It is with pleasure that we acknowledge our debt to the Board of Directors and the Tumor Institute of the Swedish Hospital of Seattle and particularly to Dr. N. A. Johansen, Chairman of the Board, for placing their equipment at our disposal. We are also indebted to the Seattle City Light and the Puget Sound Light and Power Company for generous donation of materials and to the Service Shop of the General Electric Company for the use of their atomic hydrogen welding equipment.



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