11.5×10^9 ev and to a depth slightly beyond the maximum of the multiplication curve $\bar{N}(E_0, x)$ as given by (37).[11] Table III summarizes the result and Fig. 5 shows the integrands as functions of ξ for the two terms considered. The previous calculations referred to in Table III are results reported at the Washington meeting of 1940. At that time we did not know that it was important to include the factors $1/s$ in (35) and $1/st$ in (43) in the rapidly varying part of the integrand. Just as in I we found as a result far too great a value for the fluctuation. The accuracy of the present calculations is hard to estimate and the limits given are more or less a guess. However, it seems sure that also in the cosmic-ray case the fluctuations are much smaller than the Furry value (2) and of the order of a few times the Poisson value (2a).

One of us wishes to acknowledge the assistance of the Horace H. Rackham fund, without which much of this work would not have been possible.

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[11] The maximum occurs at $x \cong 4.8$ and has the value 17.5.

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Relation of the Cosmic Radiation to Geomagnetic and Heliophysical Activities[1]

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Relations between 28-day fluctuations of intensity of the cosmic radiation and both terrestrial magnetic activity and sunspot areas were investigated. Definite pulses, both in the magnetic character and in sunspot areas, were found to be associated with the primary pulses in the cosmic radiation at Boulder, obtained by Chree's "superposed-epoch" method. They were in general phase opposition to the cosmic-ray pulses, but the tip of the magnetic-character pulse preceded the tip of the opposite cosmic-ray pulse by one day; the lead was three or four days in the case of the opposed sunspot pulses. Similar relations were not found among secondary pulses, although a 34-day periodicity in sunspot-area pulses referred to days selected on the basis of cosmic-ray intensity was displayed. Direct application of Chree's method to the magnetic character and sunspot areas, individually, indicated a 27-day periodicity in the former and a 34-day periodicity in the latter. A second method of investigation, used by Graziadei, Kolhörster, and others, was also employed. This yielded results in some respects contradictory to the first. In particular, it indicated 27-day fluctuations in sunspot areas in phase with the cosmic-ray fluctuations and out of phase with changes in magnetic character. However, it also indicated the 34-day periodicity in sunspot areas for the period of the investigation was more pronounced than the 27-day periodicity. Among other possibilities, the possible effects of sunspots through the agency of their magnetic fields were considered.

INTRODUCTION

In a recent paper[2] the author reported a statistical investigation of cosmic-ray intensity fluctuations at Boulder (lat. 40° N; long. 105°16′ W; alt. 5440 ft.) by Chree's "superposed-epoch" method of analysis. This provided evidence for the existence of secondary pulses at about 28-day intervals both preceding and subsequent to the primary cosmic-ray pulses. These secondary pulses represented deviations from the mean amounting to about 0.2 percent of the general average cosmic-ray ionization rate (corrected[3] for barometric variations) of 38.19 ions per cc per sec. in a heavily shielded, high pressure chamber.

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[1] Preliminary reports on some portions of this investigation were made in a Letter to the Editor, Phys. Rev. **59**, 678 (1941), and at: the Lubbock, Texas, meeting of the Southwestern Division of the A.A.A.S., April 29, 1941; the Golden, Colorado, meeting of the Colorado-Wyoming Acad. Sci., Nov. 8, 1941; and the Detroit, Michigan, meeting of the Am. Phys. Soc., Feb. 20, 1942.

[2] J. W. Broxon, Phys. Rev. **59**, 773 (1941).

[3] In the paper of reference 2, this general average was incorrectly given as 38.16. It is not supposed that the difference exceeds the error of measurement of the absolute value of the ionization. However, 38.19 is nearer the average of the values used in the statistical investigation. Consequently, the upper pairs of curves in Figs. 1 and 2 of reference 2, and the cosmic-ray curves of Figs. 1, 2, 6, and 7 of this paper, are drawn 0.09 percent too high. Confidence that few errors in the statistical work have gone undiscovered is due to the fact that all tabulations have been checked, and each step in the computations has been performed at least twice except in the case of a few of the probable error computations.

The purpose of the present investigation was to seek an explanation of the recurrence pulses in the cosmic-ray intensity. Because of the magnitude of the period indicated by the pulses, and in view of the present state of knowledge of the cosmic radiation, one is particularly inclined to suspect an association between these fluctuations and variations in the earth's magnetic field and perhaps in solar conditions. Evidence of such relations has been provided both by theoretical and by experimental researches of other investigators, some of which will be discussed later. Chree[4] devised his method of analysis for the purpose of investigating 27-day recurrences in terrestrial magnetic activity.

Suspecting a relation between the magnetic pulses and solar activity, Chree proceeded to establish a relation between sunspot areas and the primary pulse in terrestrial magnetic variables. His procedure has been adopted (and extended somewhat) in the investigation of the suspected relations in the present instance.

It is presumed that the data and the explanation of the procedure and the technical terminology employed in the work of reference 2 are available. However, a very brief review of some of these seems necessary. The cosmic-ray data consisted of the average (corrected) cosmic-ray ionization during the (Greenwich) day for the interval from May 25, 1938, to December 1, 1939, inclusive. There were no cosmic-ray data for 14 of the 556 days of this period. For the positive-pulse curves, the selected zero days were the five days in each full calendar month of the interval, during which the average cosmic-ray intensity was greatest. For the negative-pulse curves, the zero days were the five with least average cosmic-ray intensity in each month. The primary pulses were the largest pulses, occurring of necessity at day number zero in the final curves.

RESULTS

The terrestrial magnetic data employed in the present investigation were the world-wide or international magnetic character numbers computed by van Dijk[5] from data supplied by 40 to 56 magnetic observatories. Van Dijk also selected the five days in each month which were magnetically most calm, and the five which were magnetically most disturbed. Some indication of a relation between the cosmic-ray variations and the character numbers (representing magnetic activity) was provided by these selected days. For the 18 months of the investigation, there were 90 (magnetically) calm days and 90 disturbed days, and also 90 selected zero days of high cosmic-ray intensity and 90 of low cosmic-ray intensity. Among these, 20 magnetically calm days were identical with selected days of high cosmic-ray intensity, and 31 magnetically disturbed days were identical with selected days of low cosmic-ray intensity, appreciably higher numbers of coincidences than one would expect on the basis of random and unrelated distributions. In fact, for two different months, 3 of the 5 calm days were identical with high C-R intensity days, and for five different months, 3 of the 5 disturbed days were identical with low C-R intensity days. On the other hand, there were only 6 coincidences of (magnetically) calm days with selected days of low C-R intensity, and 5 coincidences of disturbed days with high C-R intensity days. One instance of two coincidences in a single month occurred for each of these two types.

In order to find whether geomagnetic activity might be associated definitely with the cosmic-ray pulses, new tables were formed in the same manner as in the investigation of reference 2, the only difference being that the character numbers were now employed instead of cosmic-ray intensities. The zero days were the *identical* days selected in the earlier investigation on the basis of cosmic-ray intensity. Character numbers for all the 556 days were used.

The primary, positive, cosmic-ray pulse obtained with selected days of greatest intensity for all 18 months is shown by the broken-line curve of Fig. 1. The continuous curve represents the corresponding magnetic character-number values. This shows that a negative character pulse, descending 27 percent below the general average value of 0.753, is associated with the positive cosmic-ray pulse. The magnitude of this average of the 556 character numbers shows that

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[4] C. Chree, Phil. Trans. Roy. Soc. **A212**, 75 (1913); **A213**, 245 (1914).

[5] G. van Dijk, Terr. Mag. **44**, 391 (1939); **45**, 351 (1940).

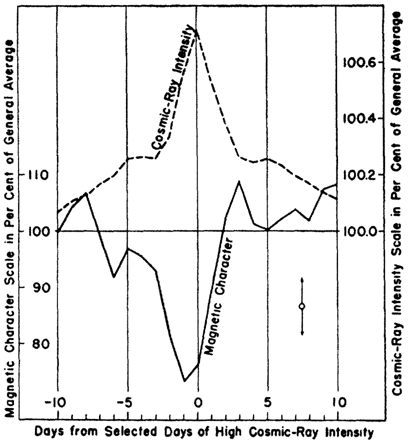

FIG. 1. Primary, positive, cosmic-ray pulse, together with corresponding pulse in world-wide magnetic character. The selected zero-days are the five of *greatest* average *cosmic-ray intensity* in each of the eighteen months, June, 1938, through November, 1939. The arrows represent the largest of the probable errors of the average values of magnetic character designated in this diagram, for an arbitrarily chosen group of six *n* days.

there was considerable magnetic activity during the period. The average recorded by Chree and Stagg[6] for all days of the twenty years, 1906 to 1925, was 0.619. The lowest tip of the magnetic-character curve occurs at day, −1, or one day preceding the representative day of greatest C-R intensity. The shape of the curve at the tip indicates the minimum character value might have preceded the C-R peak by less than one day if a unit of time less than one day had been employed.

Figure 2 shows the primary, negative, cosmic-ray pulse obtained with selected days of least intensity for all 18 months, together with the corresponding magnetic-character-number curve. The latter displays a well-defined peak at day, −1, rising 44 percent above the general average.

The curves of Figs. 1 and 2 indicate a rather intimate inverse relation between the primary, cosmic-ray intensity pulses and terrestrial magnetic activity. To be sure, the character pulses do not represent as great deviations from the mean as do the primary, magnetic-character pulses obtained by Chree with zero days selected on the basis of magnetic calm or disturbance. However, they are about as large as his first secondary pulses observed at 27 days before or after the primary, and are larger than his secondaries of higher order.

In view of this relation and the nearly equal periodicities displayed by the cosmic-ray pulses and Chree's magnetic pulses, it was thought that the extension of the procedure to larger day numbers might show variations of magnetic character corresponding to the secondary cosmic-ray pulses. Accordingly, magnetic character-number tables were composed and averages computed, for day numbers from $n=-5$ to 135 for zero days selected from the first fifteen months of the period, and for day numbers from $n=5$ to -135 for zero days selected from the last fifteen months. These were the identical zero days used in obtaining the cosmic-ray, subsequent-pulse, and previous-pulse curves of reference 2. Consequently, any magnetic-character pulses yielded should be directly comparable with the cosmic-ray pulses of Figs. 1 and 2 in that paper.

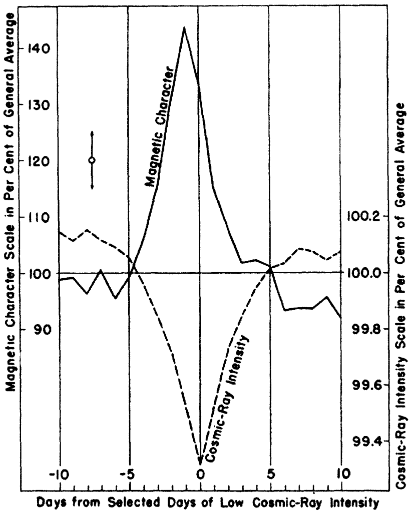

FIG. 2. Primary, negative, cosmic-ray pulse, together with corresponding pulse in world-wide magnetic character. The selected zero days are the five of *least* average *cosmic-ray intensity* in each of the eighteen months, June, 1938 through November, 1939. The arrows represent probable errors of magnetic character values as in Fig. 1.

The magnetic-character-number curves obtained in this manner are shown in Figs. 3 and 4. Those corresponding to the cosmic-ray, subsequent-pulse curves are shown in Fig. 3, and those corresponding to the cosmic-ray, previous-pulse curves, in Fig. 4. In either case, curve *A* cor-

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[6] C. Chree and J. M. Stagg, Phil. Trans. Roy. Soc. **A227**, 21 (1928).

responds to the cosmic-ray, positive-pulse curve; that is, the zero days for the curves *A* are those of high cosmic-ray intensity. The curves marked *B* correspond to the cosmic-ray, negative-pulse curves. In either case, the "difference" is obtained by subtracting the ordinates of curve *B* from those of curve *A* for the same day number.

The curves do not provide any clear indication of magnetic pulses corresponding to the secondary cosmic-ray pulses. There are quite large fluctuations, in some instances comparable in magnitude to those at $n=-1$. However, the pulsations in the difference curves are smaller and quite irregular, in general, although two rather large pulses appear in the subsequent difference curve at large day numbers. Certainly the pulses do not display the striking decrease of magnitude with order of displacement from the primary, characteristic of Chree's magnetic pulses.

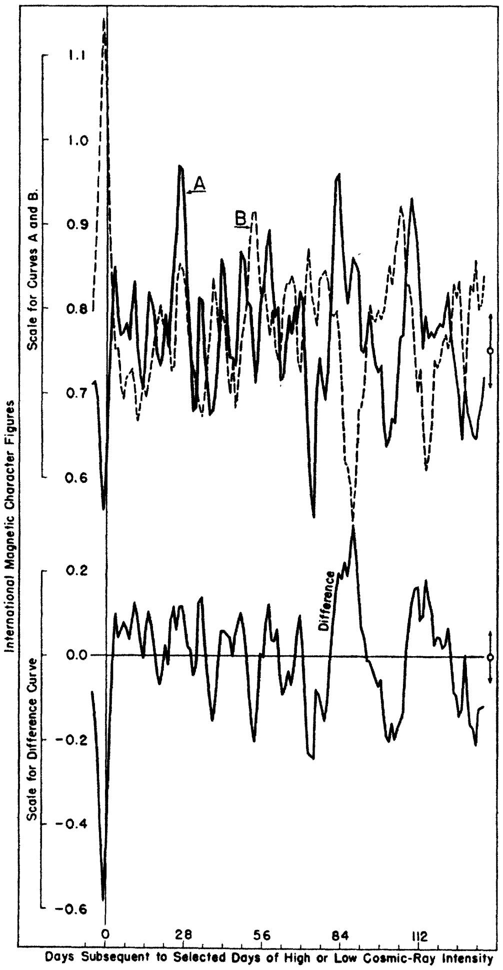

FIG. 3. Primary and subsequent magnetic-character pulses as related to primary cosmic-ray pulses. For curve *A* the zero days are the five of *greatest* average *cosmic-ray intensity* in each of the first fifteen months of the period specified in Fig. 1. For curve *B*, the zero days are the five of *least* average *cosmic-ray intensity* in each of the same fifteen months. The arrows represent probable errors as in Figs. 1 and 2, but in this case the largest for twelve arbitrarily chosen n days is represented.

It was thought that any possible association between the secondary cosmic-ray and magnetic-character pulses obtained in this manner might be displayed by a combination difference curve similar to that in Fig. 3 of reference 2. The solid-line curve of Fig. 5 was thus obtained, its ordinate at any day number n being the average of that of the difference curve of Fig. 3 at day number n and that of the difference curve of Fig. 4 at day number $-n$, but in the combination

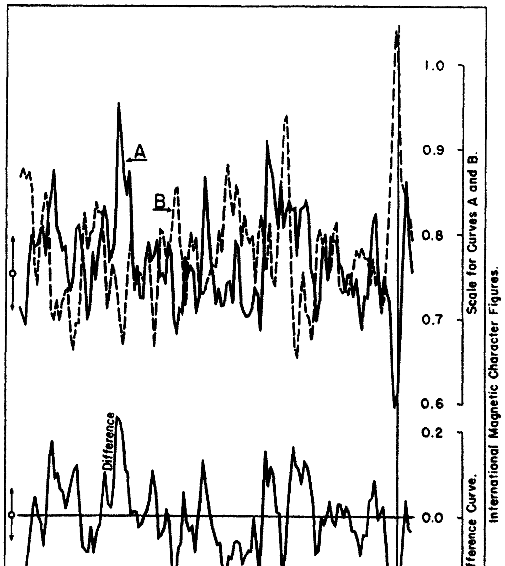

FIG. 4. Primary and previous magnetic-character pulses as related to primary cosmic-ray pulses. As in Fig. 3, the zero days for curve *A* are the selected days of *greatest* average *cosmic-ray intensity*, while those for curve *B* are the selected days of *least* average *cosmic-ray intensity*, but in this instance zero days for the last fifteen months of the eighteen-months period are used. The arrows represent probable errors as in Fig. 3.

difference curve this average is expressed in percent of the general average. The corresponding cosmic-ray, combination difference curve is represented by the broken line. Although there still appears a large magnetic pulse nearly opposite in phase to the primary cosmic-ray pulse, these combination curves do not show such a relation persisting among the secondaries. In fact, there is some hint of pulses in phase opposition at the lower day numbers, with a gradual shift resulting in pulses in phase at day number, 112, or the fourth cosmic-ray secondary pulse.

The next step was to find whether a relation between the cosmic-ray intensity and solar variables might be brought out by this method. Of the several solar variables regularly observed and recorded by astronomers, it is doubtful whether any of them affords a proper measure of the heliophysical activities one might consider capable of affecting more or less directly the intensity of the cosmic radiation at the earth's surface. If one knew the instantaneous rates of discharge of electricity from the sun or the variations in the solar magnetic moment, he might more optimistically undertake an investigation of such correlations. In this case, sunspot areas were chosen. The values employed were those supplied by the U. S. Naval Observatory in cooperation with the Harvard and Mount Wilson observatories, in the *Monthly Weather Review.*[7] The numbers listed there and used here represent sunspot areas, corrected for foreshortening, in terms of one-millionth of the sun's visible hemisphere as unit. The areas listed for individual spots and groups for a particular day were added to give the total sunspot area, irrespective of the positions of the spots. These total sunspot areas have been employed in this paper as indicators of the heliophysical activity. Solar data for 21 days of the period of the cosmic-ray measurements were missing. Dr. Seth B. Nicholson of Mount Wilson Observatory kindly supplied data for 8 of these 21 days from visual observations. Consequently, there were only 13 of the 556 days which were left blank. Dr. Nicholson has written that the time of maximum sunspot activity was 1937.4. Hence the period of the present investigation was during a time of decreasing sunspot activity, beginning just about one year after the maximum.

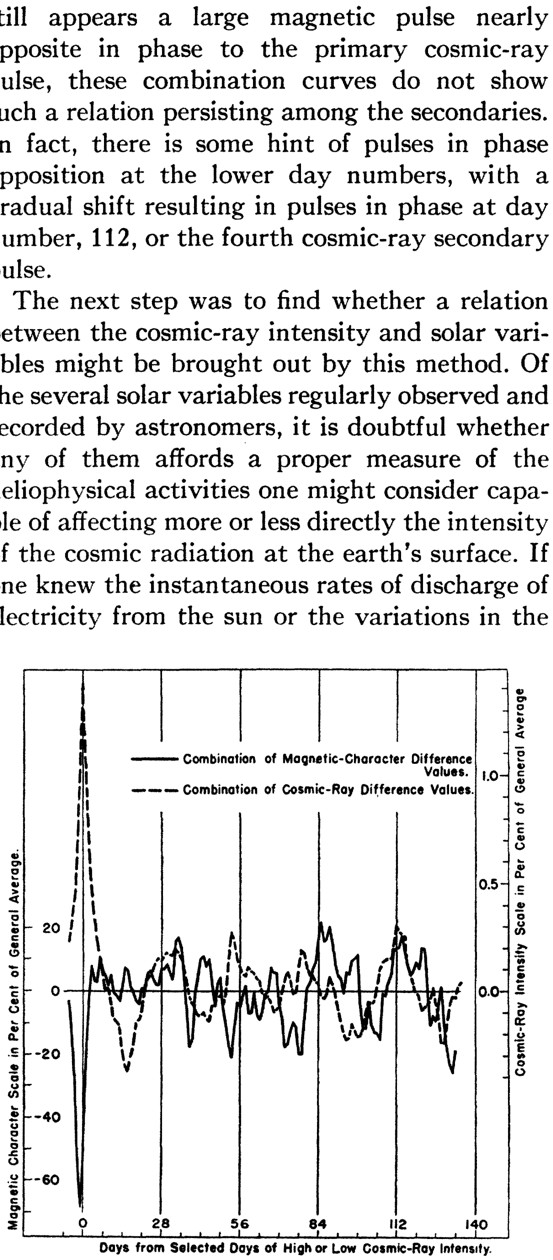

FIG. 5. The solid-line curve represents a combination of the "difference" curves of Fig. 3 and Fig. 4. The ordinate for any day number n, is the average of the "difference" value for day number n, in Fig. 3, and that for day number $-n$, in Fig. 4, expressed in percent of the general average character number. The broken-line curve is the curve of Fig. 3 of reference 2, and represents the similar combination of cosmic-ray difference pulses.

The statistical work was carried out for the sunspot areas just as for the cosmic-ray intensities and the magnetic character numbers, the zero days still being the *identical* days selected on the basis of cosmic-ray intensity. The results obtained for sunspot areas in this manner are shown in Figs. 6–10, inclusive, which correspond to the magnetic-character curves in Figs. 1–5, inclusive, in order, and also in Fig. 11. In all of these the sunspot areas are expressed in percent of the general average for all days, which was 1889.32 units.

As in the case of magnetic character, Figs. 6 and 7 show quite large pulses in sunspot area in approximate phase opposition to the primary, cosmic-ray pulses obtained with zero days for all 18 months. The tip of the negative sunspot-area pulse corresponding to the positive, primary, cosmic-ray pulse occurs at day number $n=-4$, and is 14 percent below the average. The peak of the positive sunspot-area pulse corresponding to the negative, primary, cosmic-ray pulse, occurs

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[7] Monthly Weather Review **66** (1938); **67** (1939); see tables for individual months.

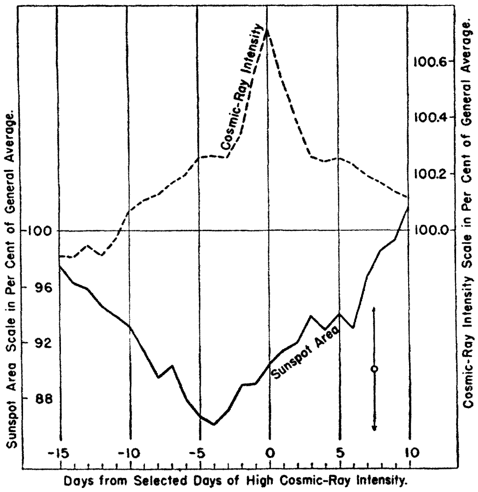

FIG. 6. Primary, positive, cosmic-ray pulse, together with corresponding pulse in total sunspot area. The selected zero days are the five of *greatest* average *cosmic-ray intensity* in each of the eighteen months, June, 1938 through November, 1939. The arrows represent the largest of the probable errors of the average values of sunspot area designated in this diagram, for an arbitrarily chosen group of six *n* days.

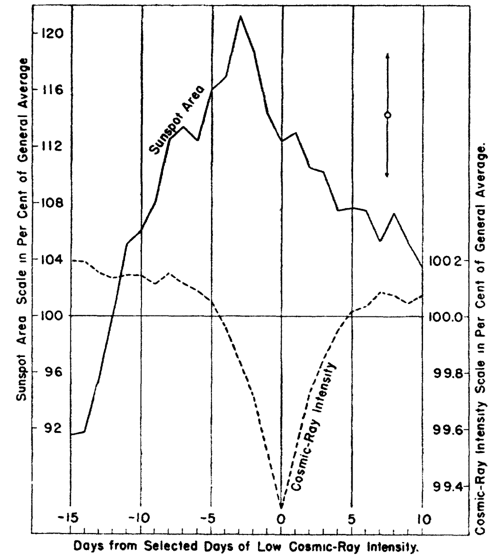

FIG. 7. Primary, negative, cosmic-ray pulse, together with corresponding pulse in total sunspot area. The selected zero days are the five of *least* average *cosmic-ray intensity* in each of the eighteen months, June, 1938 through November, 1939. The arrows represent probable errors of sunspot-area values as in Fig. 6.

at day number $n=-3$, and is 21 percent above the average. These sunspot-area pulses are much broader than the corresponding magnetic pulses of Figs. 1 and 2, in general agreement with Chree.[4] (In comparing widths of the cosmic-ray pulses in Figs. 1, 2, 6, and 7, it should be kept in mind that the cosmic-ray curves in all of these are drawn 0.09 percent too high.[3])

Rather surprisingly, the secondary sunspot-area pulses of Figs. 8 and 9 are in general larger and more clearly defined in relation to the primary than are the corresponding secondary magnetic pulses of Figs. 3 and 4. They do not, however, show the 28-day periodicity of the cosmic-ray secondaries, as may be seen by comparison with the abscissae numbered in the diagrams. There does seem to be an indication of a longer period. This is brought out more clearly in Fig. 10, where the lower curve shows the combination of the sunspot-area difference curves obtained in the usual manner, but inverted in order to facilitate comparison with the cosmic-ray, combination, difference curve, shown above it. The secondary sunspot-area pulses in this appear to be about as well defined as do the cosmic-ray pulses, but as indicated in the diagram they display a periodicity of about 34 days.

Figure 11 shows a combination of the sunspot-area difference curves which does not correspond to combinations made heretofore. Because the peaks of the primary sunspot-area pulses occurred at (or about) three days preceding the cosmic-ray primaries, it was decided to reflect the difference curve of Fig. 9 horizontally at day number $n=-3$, before combining it with the difference curve of Fig. 8. The combination was then inverted to give Fig. 11. Oddly enough, not only the primary but the first couple of secondary peaks and troughs appear sharper in this than in the combination curve of Fig. 10. The curve of Fig. 11 does not provide as good an indication of a 34-day period, although the first couple of pairs of pulses fit well into a 35-day or 36-day period.

In view of the irregularities of the secondary magnetic pulses and the unexpectedly long period indicated by the sunspot pulses, it was suspected that there might have been extraordinary abnormalities in the behavior of the terrestrial magnetic field and the sunspots during the interval of this investigation. Consequently,

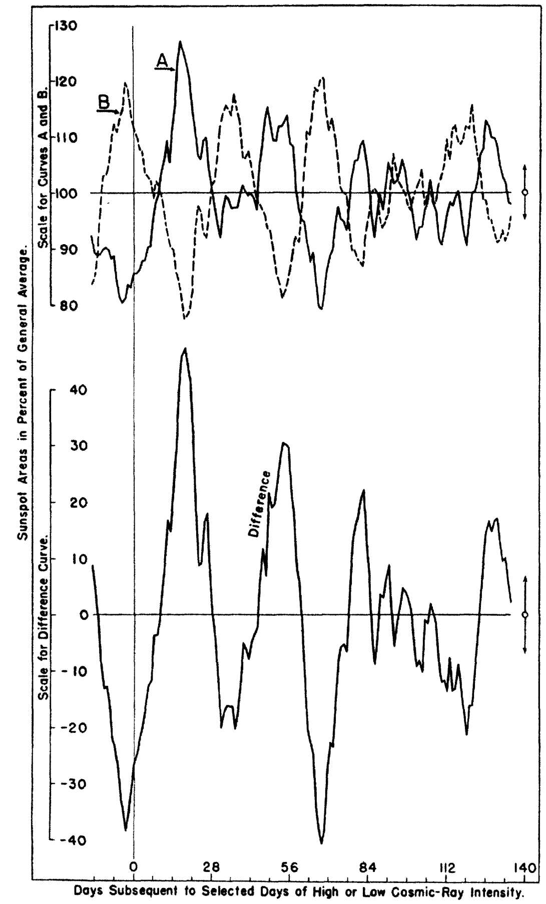

FIG. 8. Primary and subsequent sunspot-area pulses as related to primary cosmic-ray pulses. For curve *A*, the zero days are the five of ***greatest*** average ***cosmic-ray intensity*** in each of the first fifteen months of the period specified in Fig. 6. For curve *B*, the zero days are the five of ***least*** average ***cosmic-ray intensity*** in each of the same fifteen months. The arrows represent probable errors as in Figs. 6 and 7, but in this case the largest for twelve arbitrarily chosen *n* days is represented.

it was decided to subject the magnetic character for the period to precisely the same procedure employed by Chree, and then to proceed similarly with the sunspot areas. The results of this are shown in Figs. 12 and 13.

The zero days used in obtaining Fig. 12 were those selected by van Dijk.[5] For curve *A*, the zero days were the five magnetically disturbed days for each of the first fifteen months, June, 1938 through August, 1939, as usual in obtaining subsequent-pulse curves. For curve *B*, the zero days were the five magnetically calm days for each of the same months.

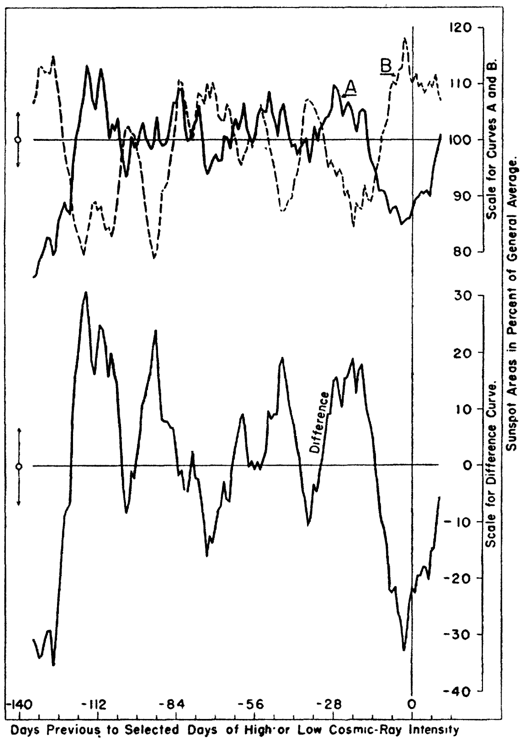

FIG. 9. Primary and previous sunspot-area pulses as related to primary cosmic-ray pulses. As in Fig. 8, the zero days for curve *A* are the selected days of ***greatest*** average ***cosmic-ray intensity***, while those for curve *B* are the selected days of ***least*** average ***cosmic-ray intensity***, but in this instance zero days for the last fifteen months of the eighteen-months period are used. The arrows represent probable errors as in Fig. 8.

The difference curve is yielded by subtracting ordinates of curve *B* from the corresponding ones of curve *A*. The secondary pulses give fair indication of a period of 27 days, in agreement with Chree. Their relative irregularities might be attributable to the brevity of the time interval of this investigation, but Chree and Stagg[6] found a very high degree of regularity of the period indicated by the peaks of their pulses, even for individual years. They display a marked difference from Chree's magnetic secondaries in that all the secondaries remain about equal in magnitude instead of decreasing with displacement from the primary. The ratio of the magnitude of a secondary to the primary pulse is comparable to the corresponding ratio for Chree's secondary pulses at 54 days from the primary. This constancy of amplitude of the magnetic secondaries of Fig. 12 reminds one of the cosmic-ray secondaries. It will be noted that the secondaries are not larger than those obtained with zero days determined by cosmic-ray intensity.

To investigate further the changes in sunspot area, new zero days were selected. These consisted of the five with largest total area, and the five with least total area, in each month. In one instance the same area was found for a fifth and a sixth day. In this case the one nearer the other four of the group was selected. Among the groups of 90 days selected on the basis of sunspot area and the groups selected on the basis of cosmic-ray intensity, there did not exist as striking a relation as was found among the groups of cosmic-ray zero days and the magnetically calm and disturbed days. However, 20 selected days of large sunspot area were found to be identical with selected days of low cosmic-ray intensity, and 19 selected days of small sunspot area were identical with selected days of high cosmic-ray intensity. On the other hand, there were 9 coincidences of selected days of large sunspot area with selected days of high C-R intensity, and 11 coincidences of selected days of small sunspot area with selected days of low C-R intensity.

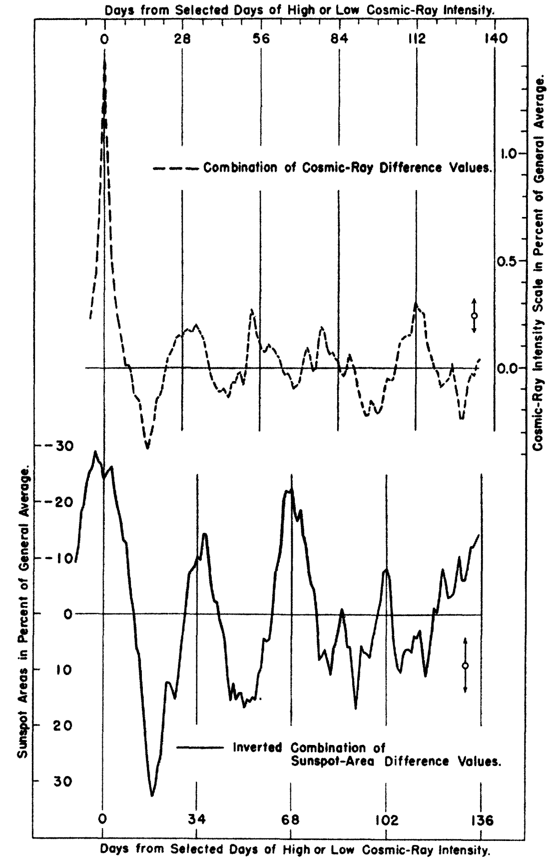

FIG. 10. The upper, broken-line curve is the cosmic-ray, combination difference curve shown in Fig. 5, and in Fig. 3 of reference 2. The lower, solid-line curve represents a combination of the "difference" curves of Fig. 8 and Fig. 9. The ordinate for any day number *n*, is the average of the "difference" value for day number *n*, in Fig. 8, and that for day number $-n$, in Fig. 9. Positive values are plotted downward and negative values upward, thus inverting the sunspot-area, combination difference curve.

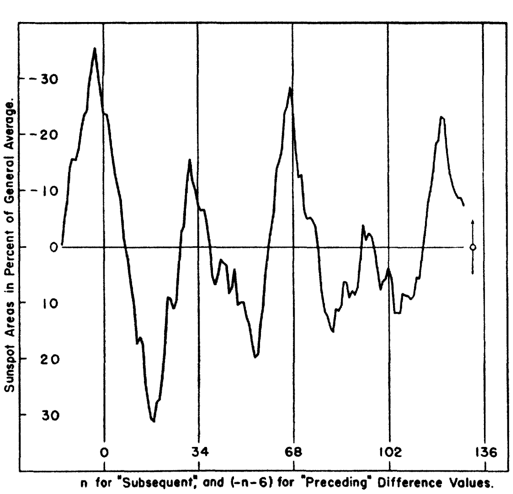

FIG. 11. Special, sunspot-area, combination difference curve, inverted. The ordinate for any day number *n*, is the average of the "difference" value for day number *n*, in Fig. 8, and that for day number $(-n-6)$, in Fig. 9. Negative differences are plotted upward.

In Fig. 13 are shown the sunspot-area curves obtained with zero days from the first fifteen months, selected on the basis of sunspot areas. As anticipated, the primary pulses of Fig. 13 are considerably larger than those of Figs. 8 and 9. However, the secondary pulses are rather smaller and less regular than those referred to the cosmic-ray zero days. In the difference curve there seems to be some indication of the 34-day period for two intervals, but not thereafter. There is some indication of it even after three 34-day intervals in the positive-pulse curve *A*, but the irregularity of the negative-pulse curve *B* destroys it in the difference curve.

In view of the relations indicated by the foregoing analysis, it was decided to use in addition a very simple method of analysis which has been employed by T. and B. Düll,[8] Sanford,[9] Grazia-

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[8] T. and B. Düll, Virchow Arch. **293**, 272 (1934).

[9] F. Sanford, Pub. Astronom. Soc. Pac. **47**, 180 (1935).

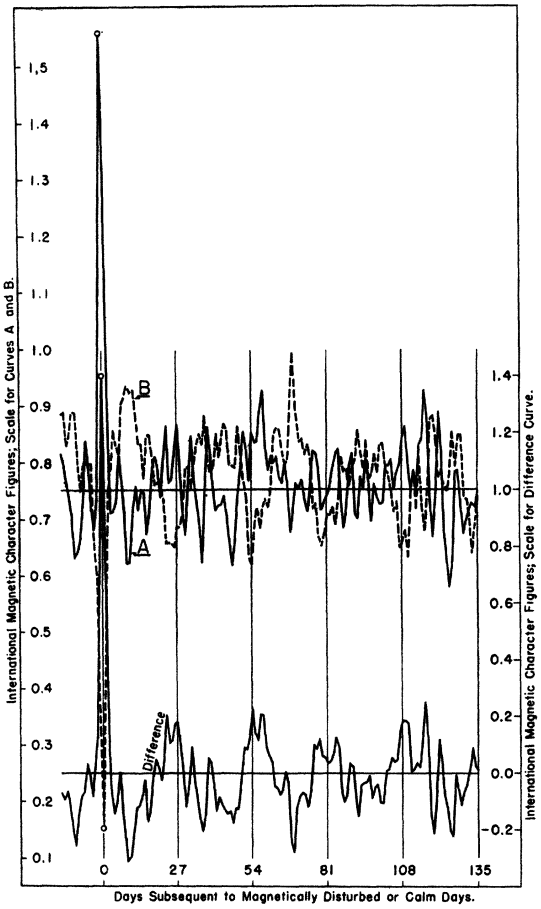

FIG. 12. Primary and subsequent magnetic-character pulses. For curve *A* the zero days are the five magnetically disturbed days selected by van Dijk for each of the first fifteen months. For curve *B* the zero days are the five magnetically calm days selected by van Dijk for each month of the same period.

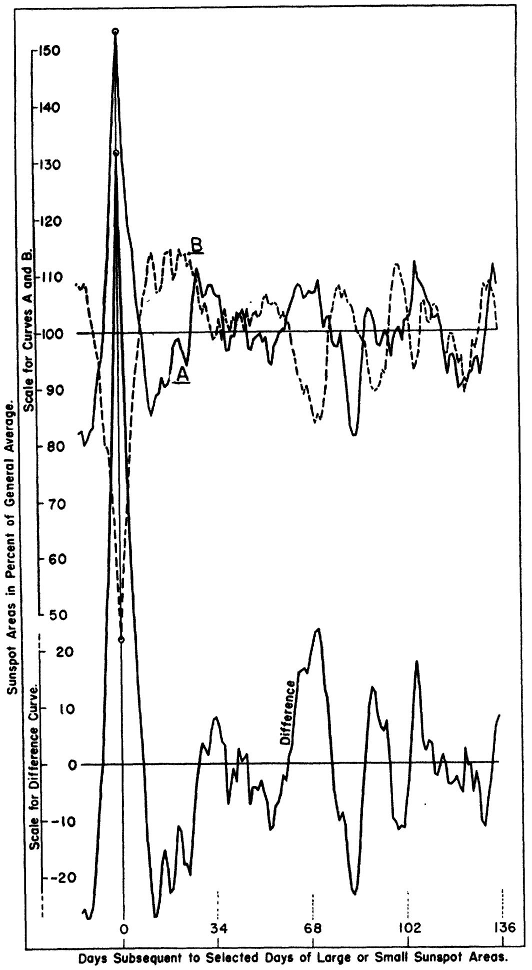

FIG. 13. Primary and subsequent sunspot-area pulses. For curve *A* the selected zero days are the five of greatest total sunspot area in each of the first fifteen months. For curve *B* the zero days are the five of least total sunspot area in each of the same months.

dei,[10] and Kolhörster,[11] for instance. In this method, if a periodicity of a certain magnitude in a certain variable is suspected (and perhaps relations among several variables), then the whole time of investigation is divided into segments of that length and a sort of average or typical curve for the variable (or variables) is obtained for the period being considered. For instance, to investigate by this method whether there is a 27-day periodicity displayed by the cosmic-ray data herein used, the entire interval of 556 days is divided into consecutive groups of 27 days each, beginning with May 25, 1938 as the first number-one day. In Bartels' [12] harmonic-dial method of analysis, the cosmic-ray data for each of these periods would be analyzed to two or more terms by Fourier analysis, and the amplitudes and phase constants corresponding to the assumed fundamental (or any particular harmonic) designated for successive intervals by the

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[10] H. T. Graziadei, Sitz. Akad. Wiss. Wien [2a] **145**, 495 (1936).

[11] W. Kolhörster, Physik. Zeits. **40**, 107 (1939). Kolhörster observed 27-day fluctuations of ±0.5 percent, out of phase with the sunspot relative numbers, flocculi, etc. He also indicated he was able, by observing changes in cosmic-ray intensity, to predict the beginning or at least the disposition toward great magnetic disturbances of several days' duration which may seriously interfere with radio communication. The writer has not yet been able to confirm this.

[12] J. Bartels, Terr. Mag. **40**, 1 (1935).

harmonic dial. In the more simple analysis being considered here, however, the simple average cosmic-ray intensity for all days of the same number in the several intervals is determined. These averages are then plotted against the day numbers to yield a sort of average or representative curve which, by its shape, is supposed to enable one to decide concerning the "reality" of the suspected 27-day periodicity of the cosmic-ray intensity.

The results of this treatment are shown in Fig. 14, which contains curves for the magnetic character-figure, the cosmic-ray intensity, and the sunspot area, obtained upon the assumption of a 27-day periodicity in each of these variables. Each curve is drawn twice, as if the representative curve were repeated for two cycles, in order to facilitate comparison of phase relations.

The curves of Fig. 14 appear to indicate a 27-day periodicity in all three variables. The cosmic-ray curve indicates average fluctuations of the order of 0.2 percent from the average, comparable to or larger than the cosmic-ray secondaries brought out by Chree's method. The deviations from the mean displayed by the sunspot-area curve are of about the same order as the secondary pulses yielded by the Chree method, while those displayed in the magnetic curve are considerably larger. A striking feature of the diagram is the similarity of the cosmic-ray and the sunspot-area curves. These indicate 27-day variations of these two variables almost exactly in phase. Not only is this true of the large variations, but in several instances smaller irregularities of the same type appear in phase in the two curves. Not only was this entirely unexpected in view of the generally inverse relation and the phase difference displayed in Figs. 6 and 7, and the inverse relation between cosmic-ray intensity and solar activity found by Kolhörster[11] by the same method, but comparison of the top and bottom curves of the figure shows an inverse relation between sunspot areas and terrestrial magnetic activity, in direct opposition to long accepted relations between these variables. Although the magnetic-character curve is quite irregular (particularly the positive loop) the main variations are generally out of phase with those in the cosmic-ray curve, as was expected. In fact, if the sunspot-area curve could be *inverted*, all the relations among the three variables would be less surprising.

An unsatisfactory characteristic of this method of analysis is that it requires the postulation of a particular period *a priori*. If one is not satisfied with the results obtained with an assumed period, he may start all over with another. Upon

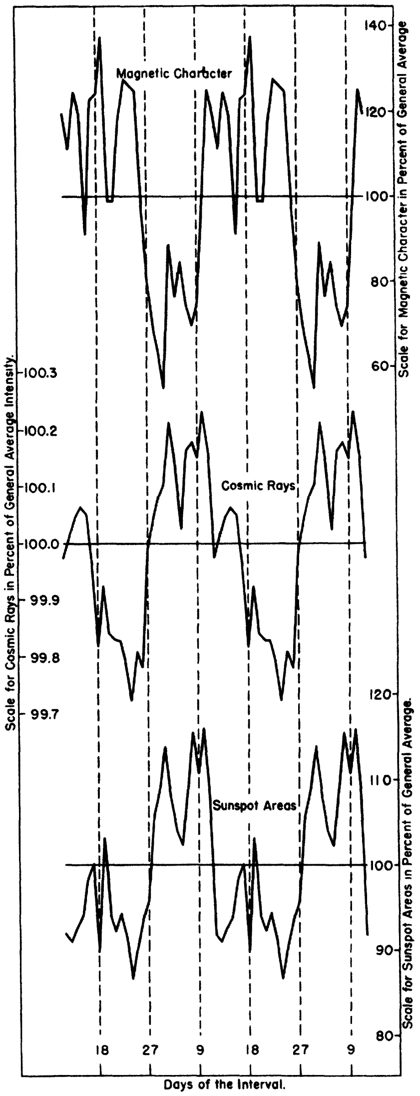

FIG. 14. For these curves, the entire period of observation was divided into successive intervals of twenty-seven consecutive days, each. The average values of the international magnetic-character figure, the cosmic-ray intensity, and the total sunspot area, were determined for all days of the same number in the several twenty-seven-day intervals. These averages are represented by the three curves, abscissae representing the order of days in an interval. Two complete cycles are included to facilitate comparison.

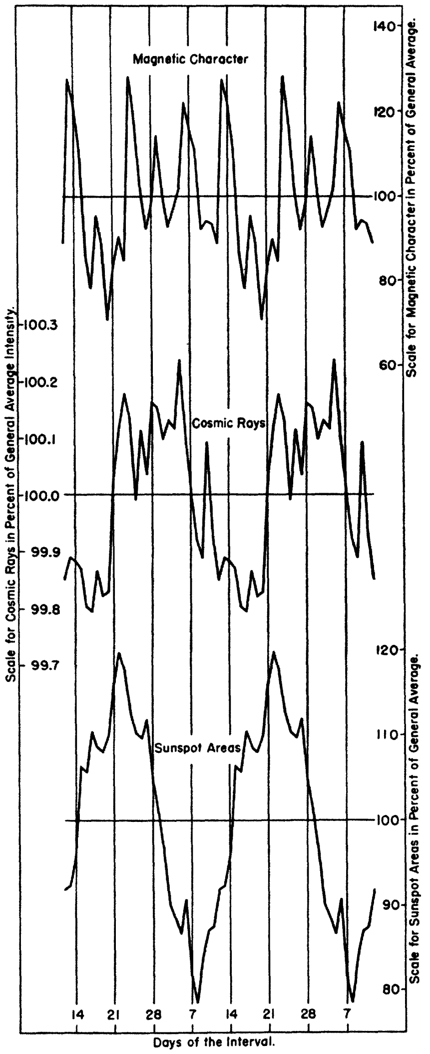

FIG. 15. These curves correspond to those of Fig. 14, but in this instance the successive intervals consist of twenty-eight days.

comparison of the shapes of the final curves and the amplitudes of the variations they indicate, he may decide one is preferable to another, but he can hardly be sure that an untried period would not be still more satisfactory. Because of the relation of the sunspot curve to the others when the 27-day period was assumed, it was decided to repeat the analysis with other periods. Since the Chree method had indicated a 28-day period for the cosmic-ray pulses and a 34-day period for the sunspot pulses, it was decided to repeat the procedure with each of these intervals.

Figure 15 shows the average curves obtained for the three variables upon arranging the data in groups corresponding to successive intervals of 28 days. In this the cosmic-ray curve is perhaps a little less regular, but regarding both regularity and amplitude, it seems about as satisfactory as the cosmic-ray curve of Fig. 14. Although the fluctuations are still quite large, the magnetic-character curve of Fig. 15 is so irregular that it can scarcely be considered to provide any evidence in favor of a 28-day period. The sunspot curve of Fig. 15, however, is far more regular than that of the preceding diagram, and the amplitude of the variations is greater. The phase relations between the cosmic-ray and sunspot-area variations in the two diagrams are quite different. In Fig. 15 the sunspot-area fluctuations appear to lead the cosmic-ray fluctuations by about 90° or some 6 or 7 days instead of being in phase as in Fig. 14.

In Fig. 16 are shown the results obtained with a postulated period of 34 days. Although the fluctuations in the cosmic-ray curve are still quite large, both this and the magnetic-character curve are so irregular that it seems one must conclude there is no evidence of a 34-day periodicity in these variables. However, the sunspot-area curve is reasonably regular, and the extreme variations are somewhat greater than in either of the two preceding diagrams. If the magnitude of extreme variations from the mean were the only criterion, we should conclude that the method provides better evidence for the 34-day, than for either the 27-day, or the 28-day period in the sunspot-area variations. However, the 28-day curve is somewhat more regular than either of the others.*

When the three diagrams are considered together, they indicate a 27-day periodicity in the magnetic character, a 27-day or a 28-day periodicity in the cosmic-ray intensity, and while there is some indication of all three periodicities in the sunspot area, the 28-day and 34-day periods are more strongly indicated in this case. Different phase relations between the cosmic-ray

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* *Note added in proof:* Sunspot-area curves for data arranged in groups corresponding to successive intervals of 29, 30, 31, 32, and 33 days, respectively, have been constructed by an assistant, Mr. S. W. Rasmussen. Of these, the 30-day, and the 33-day curve actually display somewhat greater extreme variations than does the 34-day curve of Fig. 16, but are quite as irregular. In fact, none is as nearly symmetrical as the 28-day curve of Fig. 15.

and the sunspot fluctuations are indicated in the different diagrams. When we consider the two statistical treatments together, there appears to be fair agreement in the indication of a 28-day (or possibly a 27-day) period for the cosmic radiation, a 27-day period for the magnetic activity, and a 34-day period for the sunspots.

DISCUSSION OF RESULTS

To explain the results obtained by the two methods of investigation employed here, and further, to explain the differences between these and the results of other investigators, does not appear to be a simple matter. This may be due primarily to the fact that the interrelations among the three variables (or four, including the time) are not simple. Perhaps some of the difficulties are due to the brevity of the time interval. In this connection, it should be noted that both methods have been applied to data for even shorter intervals by some of the investigators mentioned. That there are relations among the larger fluctuations of the three variables is indicated by Figs. 1, 2, 6, and 7, obtained by the first method. If the three variables actually fluctuate with different periods (particularly if one or more has only a temporary or quasi periodicity) then the relations among the secondary pulses obtained by this method may be fairly comprehensible. The different results obtained by the second method upon assumption of different periodicities, illustrate the fact that considerable caution should be used in employing this method, particularly for rather short intervals of time.

Other investigators have announced periodicities in the several variables. Not only did Chree[4, 6] arrive at the 27-day period in magnetic activity by his method, but his findings have been (approximately) confirmed by Bartels,[13] T. and B. Düll,[8] and Benkova,[14] among others. Benkova states that the 27-day recurrence tendency of magnetic storms was detected as early as 1858. He found the probability of such recurrence to be greatest during years of decreasing activity, which presumably would apply to the interval of the present investigation. He recorded intervals varying from 26.63 to 28.5 days. The 27-day (or 28-day) variations in the cosmic-ray intensity have been announced by Hess,[15] Graziadei,[10] Gill,[16] Monk and Compton,[17] Kolhörster,[11] and Forbush.[18] A sunspot (sunspot relative numbers rather than areas are commonly used) period in the neighborhood of 27 days appears to be generally accepted, although the

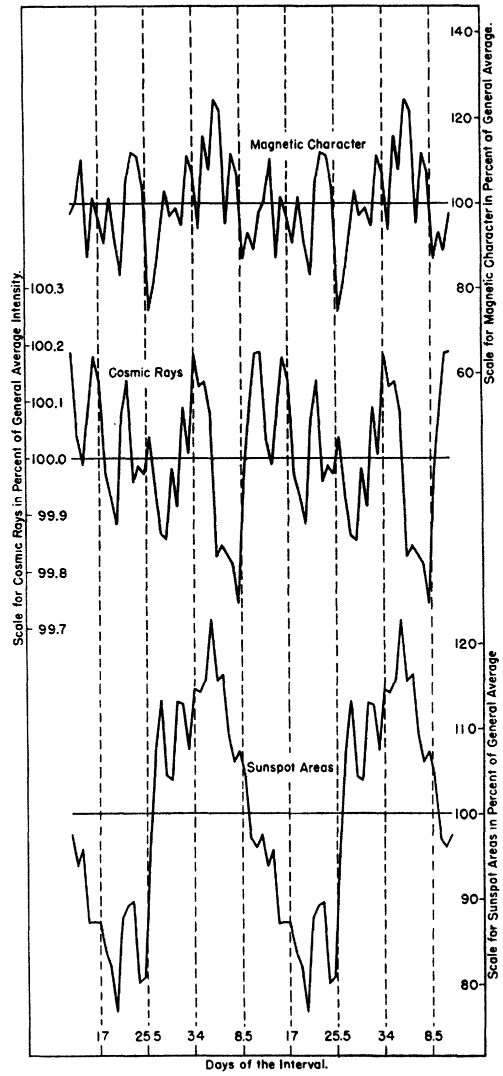

FIG. 16. These curves correspond to those of Fig. 14, but in this instance the successive intervals consist of thirty-four days.

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[13] J. Bartels, Terr. Mag. **37**, 1 (1932).

[14] N. P. Benkova, Terr. Mag. **47**, 147 (1942).

[15] V. F. Hess, Terr. Mag. **41**, 345 (1936).

[16] P. S. Gill, Phys. Rev. **55**, 429 (1939).

[17] A. T. Monk and A. H. Compton, Rev. Mod. Phys. **11**, 173 (1939).

[18] S. E. Forbush, Bull. Int. Union Geod. and Geophys. No. 11, 438 (1940).

period varies with time and for individual spots. Among those on the subject are papers by Sanford,[9] Alter,[19] and Clayton.[20] Alter observed a sunspot periodicity of 28 days in late 1936 and early 1937. While Clayton found an average period of 27.26 days, she observed a period of 29 days in late 1935 and early 1936. Sanford concluded the sunspot period was 27.25 days. So far as the writer is aware, there is no confirming evidence for a 34-day sunspot period. Apparently, this could correspond to solar rotation only in the neighborhood of the poles, well out of the sunspot zone.

The lag of 3 (or 4) days of the cosmic-ray, primary pulses relative to the inverse, sunspot pulses of Figs. 6 and 7 is not without its counterparts. Chree[4] found a 4-day lag of the magnetic (H-range) pulse corresponding to his primary, sunspot-area pulse for the years, 1890–1900, inclusive. T. and B. Düll[8] observed a 5-day lag of the magnetic character relative to the sunspot relative number.[21] According to Chapman and Bartels,[22] Maurain found for the forty-one-year period, 1883–1923, that days of maximum sunspottedness preceded days of great magnetic disturbance (selected by him) by some 2 to 4 days, the average being about 2.5 days for the 41 years. Also, according to them, Stagg found the time of maximum (projected) sunspot area preceded selected days of high magnetic disturbance by 1.67 to 2.25 days during the years, 1901–10. Graziadei,[10] employing a period of 27.2 days and extrapolating from Sanford's[9] work, concluded that his cosmic-ray maximum must have lagged the sunspot-relative-number maximum by about 4.5 to 7.5 days. It has been suggested that electrical particles may be emitted from sunspots or flocculi or perhaps other active regions[13] only loosely associated with sunspots, that these alter the ring current about the earth, that this alters the terrestrial magnetic field, and that this finally alters the cosmic-ray intensity at the earth's surface. If the train of events occurs in this order, then some of the phase lags might be explained in terms of the time required for the transit of the electrical particles, or perhaps by the supposition that the particles may be able to reach the earth or its ring current only in case they start from solar regions with certain locations relative to the earth which might cause their chief effect to be established some time after their culmination, as suggested by Graziadei.[10]

The sun may be able to affect the cosmic radiation through agencies other than the ring current. Vallarta,[23] Jánossy,[24] and Epstein[25] have shown that the terrestrial intensity of the cosmic radiation may be influenced directly by the solar magnetic field. Vallarta and Godart,[26] assuming a solar magnetic dipole of 10^{34} gauss cm^3 with its axis inclined to the solar axis of rotation, and assuming a 27-day period of rotation, arrived theoretically at a 27-day variation of appropriate magnitude in the cosmic-ray intensity at the earth. They do not appear to have stated what they assumed to be the angle between the supposed solar dipole and the sun's axis of rotation. They did write: "Taking the angle between the earth's dipole and the solar dipole as 6°, we have for the amplitude of the 27-day period the values given in Table I, . . . It should be emphasized that the calculated amplitude is quite sensitive to the angle between the two dipoles." Although these statements are very definite, it does not seem they could have assigned this constant value to the angle between these two dipoles, since it would actually vary with three periods, and the magnitude does not seem appropriate. Therefore, the writer is inclined to suspect that they actually assumed the angle between the solar dipole and its axis of rotation to be 6°. This suspicion is strengthened by the fact that Abetti[27] has written: "From observations carried out at Mount Wilson from 1912 to 1914 at the epoch of minimum solar activity in order to profit by relatively tranquil conditions of the photosphere,

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[19] D. Alter, Pub. Astronom. Soc. Pac. **49**, 242 (1937).

[20] H. H. Clayton, Terr. Mag. **46**, 71 (1941).

[21] If the magnetic-character and cosmic-ray curves of Fig. 14 are accepted, then it might be inferred that the times of maximum death rate designated by the curves of T. and B. Düll would be times of decreased cosmic-ray intensity.

[22] S. Chapman and J. Bartels, *Geomagnetism* (Oxford University Press, 1940), Vol. 1, pp. 382–3.

[23] M. S. Vallarta, Nature **139**, 839 (1937).

[24] L. Jánossy, Zeits. f. Physik **104**, 430 (1937).

[25] P. S. Epstein, Phys. Rev. **53**, 862 (1938).

[26] M. S. Vallarta and O. Godart, Rev. Mod. Phys. **11**, 180 (1939).

[27] G. Abetti, *The Sun* (translated by Zimmerman and Borghouts), (D. Van Nostrand Company, 1938), p. 222.

Seares deduced the following results:

$$i = 6.0° \pm 0.4°,$$
$$P = 31.52 \pm 0.28 \text{ days.''}$$

i was defined to be the inclination of the solar magnetic axis with respect to the axis of the rotation, and P, the period of rotation of the magnetic axis around the axis of rotation.

Forbush[18] has offered two objections to the conclusions of Vallarta and Godart: first, that the effect does not vary with latitude as their theory demands; and second, that the observed variations are not truly or permanently periodic. The second of these objections might be met if, instead of supposing it to be due to an inclination of the sun's dipole axis to its axis of rotation, we attribute the effect to the fields of the rather evanescent sunspots, superimposed upon the field of the permanent solar dipole with its axis supposed coincident with the solar axis of rotation. According to our supposition regarding the assumptions of Vallarta and Godart, the 27-day periodicity in the cosmic radiation could be regarded as due to a solar dipole-moment component of $10^{34} \sin 6° = 10^{33}$ e.m.u. rotating with a period of 27 days in the sun's equatorial plane. If, then, a sunspot or a combination of them could produce the equivalent of a dipole moment of 10^{33} e.m.u. parallel to the equatorial plane, we could have the 27-day periodicity derived by Vallarta and Godart, but vanishing and reappearing with the sunspots. Or we might suppose the permanent solar-dipole axis to be inclined at 3° to the axis of rotation so that the component in the equatorial plane would be only half as great as with the 6° angle. Then the corresponding sunspot moment would need only to be 5×10^{32} e.m.u. in order that the 27-day effect might be just as computed, at certain times, and vanish at others, depending upon the phase relations between the sunspot moment and the equatorial component of the permanent moment.

Whether such a notion is worth considering would appear to depend upon whether the sunspots could ever be regarded as magnetically equivalent to a dipole of such a magnitude. To investigate this possibility we must first decide upon a method of estimating the equivalent dipole moment of a sunspot. In doing this we must keep in mind the known characteristics of the magnetic field of a sunspot. Spectroscopic observations show that this field is most intense at the center of the spot and is there directed along the radius of the sun or the axis of the spot. At the edge of the spot the field becomes vanishingly small and is directed along the radius of the spot, tangent to the sun's surface. In the region between the center and the edge of the spot, the total magnetic field intensity is proportional to $(1 - r^2/a^2)$, where r is the distance from the center of the spot to the point and, a is the radius of the spot. If we assume these observed values of the spot's magnetic field actually represent conditions in a plane (the plane of the spot) normal to the solar radius through the center of the spot, then we may imagine the observed magnetic field of a unipolar spot to be represented fairly approximately by the field of a dipole with its axis coincident with the solar radius through the center and located below (nearer the sun's center) the plane of the spot at a distance $a/\sqrt{2}$ from its center. A dipole so located would yield fields in the correct directions at the center and the edge of the spot. The total intensity in the plane of the spot would not vary with the distance from its center as specified above, but would be more nearly proportional to $(1 - r/a)$, and at the edge would still have a magnitude nearly 14 percent as great as at the center. However, the dipole moment computed in this manner might be regarded as providing an adequate approximation for computation of the order of magnitude of a sunspot dipole moment for present purposes. To provide a field intensity H at the center of a spot of radius a, the dipole moment would need to be $\sqrt{2}a^3H/8$.

As an example, let us consider one of the largest unipolar sunspot groups during the time of this investigation, Mount Wilson Number 6618, with a maximum area of 1939 (millionths of the visible hemisphere) on Sept. 23, 1939 and a maximum field intensity[28] of $H = 3600$ gauss. The spot radius corresponding to its area is $10^9(19)^{\frac{1}{2}}$ cm if the sun's radius is taken to be 7×10^{10} cm. Using these values, we find the corresponding dipole moment to be 5.2×10^{31} e.m.u. This is smaller than the required magnitude, by a factor of 10 or 20. If, then, the approximations

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[28] Pub. Astronom. Soc. Pac. **51**, No. 304, 14 (1939).

were adequate to yield the order of magnitude, and if the conclusions of Vallarta and Godart result from the assumption of an angle of 6° between the solar dipole axis and the axis of rotation, it would appear that the sunspots could not produce the observed 27-day periodicity in the cosmic-ray intensity directly by their own magnetic fields, for the area of this spot group was greater than the average daily total-sunspot area during the period of observation. Nevertheless, it would be interesting to know whether cosmic-ray intensities would display a better or worse correlation with sunspot magnetic moments[29] than with sunspot areas or relative numbers.

There are perhaps other circumstances which might give rise to periodic and perhaps related fluctuations of the variables under investigation. Chapman and Ferraro[30] have attributed magnetic storms to a geomagnetic ring current (or cylindrical current) which they show can be stable at a distance of a few earth radii. This they show to be naturally oscillatory in the radial direction, and upon assuming reasonable values for the variables, they compute natural periods varying from 2 sec. to 10 min. Thus any kind of impulse which would cause an alteration of the radius of the ring current might set up fluctuations of the terrestrial field with a period quite independent of any periodicity of the original agent. Evans[31] has recently provided an explanation for a relation between fluctuations in cosmic-ray intensity and in the earth's magnetic field, through the agency of currents of slow interstellar electrons flowing around the earth in spherical shells under the influence of the earth's electric and magnetic fields. The region of these shells is estimated to extend about as far as the distance to the sun. He suggests the possibility of an oscillatory fluctuation in terrestrial magnetic field and cosmic-ray intensity. Though he considers the effect of the emission of particles from the sun, he does not consider the effect of a solar magnetic field. It would seem that sunspots might, by means of their magnetic fields, produce pronounced effects upon such a system and, if it should prove to be naturally oscillatory, cause temporary fluctuations in the terrestrial field and the cosmic-ray intensity, with periods both equal to and differing from the periods of the spots.

Jánossy and Lockett[32] have mentioned objections to the theories based on the magnetic field of the sun. They write that recent spectroscopic measurements of Evershed failed to indicate the presence of a magnetic field at the surface of the sun, and that Chapman has shown that a magnetic dipole field proceeding from within the sun will generate electric currents in the sun's ionized atmosphere, which may limit the outward extension of the field.

The writer is indebted to Dr. Seth B. Nicholson of Mount Wilson Observatory for supplying information regarding sunspots, and acknowledges NYA assistance, chiefly in checking computations.

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[29] It may be worth noting that the distance from the earth to the dipole representing such a sunspot at low latitudes, would vary about one-fourth as much on account of the sun's rotation as it would on account of the eccentricity of the earth's orbit.

[30] S. Chapman and V. C. A. Ferraro, Terr. Mag. **46**, 1 (1941).

[31] F. Evans, Phys. Rev. **61**, 680 (1942).

[32] L. Jánossy and P. Lockett, Proc. Roy. Soc. **178**, 52 (1941).