Calculations on Classical Field Theory

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The equations of the classical field theory of elementary particles proposed by one of the authors were integrated numerically for the static, spherically-symmetric case. A solution was obtained corresponding essentially to minimum energy, and hence describing a classical electron according to the theory. It was found that the frequency associated with the solution was zero, to within the accuracy of the calculations, for the case of minimum energy. Hence the theory, in its present form, is not capable of accounting for the Sommerfeld fine-structure constant, as had seemed possible.

1. CALCULATIONS

SOME time ago one of the authors[1] proposed a classical field theory of elementary particles. This theory was based on a Lagrangian involving the electromagnetic field components $F_{\mu\nu}$, the electromagnetic potentials φ_u, and also a scalar "matter function" ψ, which bore a formal resemblance to the relativistic Schrödinger function. The field equations obtained were applied to the case of a static, spherically-symmetric charge distribution which was to be interpreted as a classical electron. It was found that, in spite of the fact that the charge density was static, the function ψ could have a periodic dependence on the time. The frequency associated with ψ was determined by the solution of the field equations. As there was a whole family of solutions satisfying the equations and the boundary conditions, that solution was to be chosen which made the total energy of the system a minimum. In this way one would obtain a classical particle which had associated with it both an energy and a frequency. If the latter turned out to be different from zero, there would exist the possibility of accounting for the Sommerfeld fine-structure constant on a purely classical basis.

The present paper has for its purpose to report on the results of calculations carried out to obtain solutions of these equations.

The field equations were taken in the form of Eqs. (44) and (45) of reference 1,[2] together with the boundary conditions of Eq. (46a). The solutions of the equations depended on the choice of the parameters a and b of Eq. (51). For each value of a taken, a number of values of b were tried until the one had been found which gave a solution satisfying the boundary conditions. The total energy (49) was calculated from this solution, and that value of a was sought which made this energy a minimum.

The equations were integrated numerically, by starting with a power series solution near the origin. Preliminary calculations showed that the desired solution corresponded to a value of a not far from 1.63. Hence first a solution was obtained for $a=1.63$, in which intervals of 0.05 were used for the independent variable x, in the range 0 to 2, and intervals of 0.1 for $x>2$. The correct value of b (2.21493) was determined by trial. The solution was determined to 7 figures, of which 6 were reliable in the important range of $x(0<x<2)$. Then solutions were obtained for values of a near 1.63 by working, not with the unknown functions themselves, but with the differences from their values for $a=1.63$. These differences were small and varied slowly with x, so that intervals of 0.1

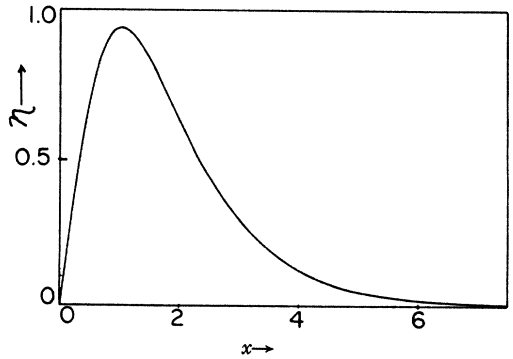

FIG. 1. Plot of η as a function of x.

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[1] N. Rosen, Phys. Rev. 55, 94 (1939).
[2] All equations referred to are in the paper of reference (1).

in the latter gave sufficient accuracy. From the solutions obtained in this way, the energy was calculated for $a=1.61$, 1.62, 1.63, 1.64. A cubic curve was passed through the calculated points, and the value of a at the energy minimum determined; it was found to be 1.629.

By interpolation, the constant β of Eq. (46b) was calculated for this value of a. This constant, according to the paper referred to, should be related to the Sommerfeld fine-structure constant by the equation

$$\alpha' = |2\alpha^2\beta/(\alpha\beta+\gamma)|,$$

where α' is the fine-structure constant, and α, β, and γ are quantities given in terms of the solution by Eqs. (48), (46b), and (50b), respectively. If one considers the values of the other quantities involved, in order to get the correct value for the fine-structure constant, the value β should have come out to be about ± 0.00283. The calculations gave $\beta = -0.00012$, too small in absolute value by a factor of about 24. One can say that, to within the accuracy of the calculations, β and therefore

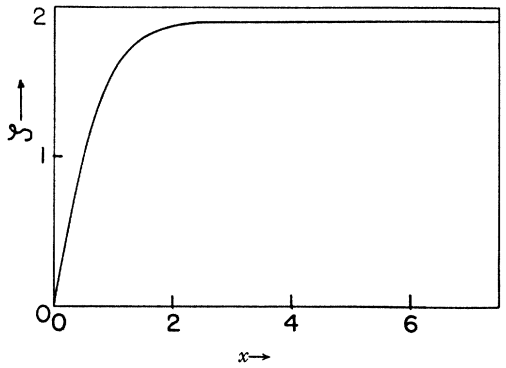

FIG. 2. Plot of ζ as a function of x.

the fine-structure constant are zero. Hence for the case of minimum energy, not only the charge density, but also the function ψ turns out to be static.

Figures 1 and 2 give graphs of the functions $\eta(x)$ and $\zeta(x)$ as defined by Eqs. (41a) for the case $a=1.63$. Figure 3 gives the corresponding charge distribution for the electron according to the field equations. These graphs can be taken to

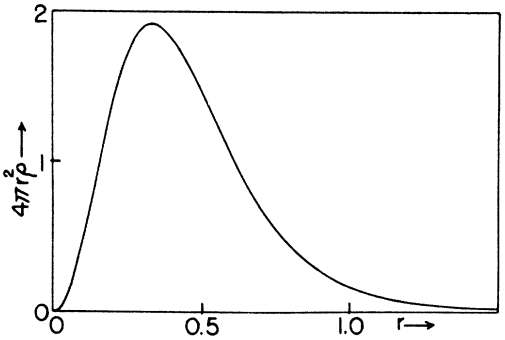

FIG. 3. Plot of $4\pi r^2\rho$ as a function of r, where ρ is the charge density, e being the unit of charge and e^2/mc^2 the unit of length.

represent the functions for $a=1.629$, since the small change in a will not alter them to any noticeable degree.

2. DISCUSSION

The results of the calculation indicate that the present classical electron theory cannot account for the Sommerfeld fine-structure constant (or for Planck's constant). This means that, in the quantization of the field equations, h must be introduced from the outside. Naturally this is less satisfying than if the theory had turned out to be self-contained.

As was pointed out in the earlier paper, there exist Lagrangians, other than the one adopted, which satisfy the same conditions as this one and which could be chosen instead. It is possible that some one of these Lagrangians might lead to a satisfactory explanation of the fine-structure constant.

On the other hand, the Lagrangian used is outstanding in its simplicity. If one takes it seriously, then one is led to say that a classical theory is incapable of providing a basis for an explanation of quantum phenomena. From this point of view the success of the present classical theory could only be judged by the extent to which, after the usual quantization, it could remove the difficulties in the quantum theory believed to be associated with the point electron. This is a question for investigation.