this cannot be depended upon. However, the ratio of α_m for carbon and for lead should be correct within 10 percent at least. On the basis of this result it seems doubtful that any large systematic deviation from the Williams theory can occur with increasing atomic number. Recent measurements by others also support this conclusion.[7, 8] However, in order to examine the seeming discrepancy between the cloud-chamber and counter results it should be possible to perform an experiment in which measurements by the two methods could be made simultaneously.

The author wishes to express his appreciation to Mr. Sylvan Rubin, Mr. Eldred Hough, and Mr. R. E. Logan for assistance with the experimental work, and to Professor W. A. Fowler and Professor C. C. Lauritsen for advice and assistance.

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[7] Oleson, Chao, and Crane, Phys. Rev. **60**, 378 (1941).

[8] L. A. Kulchitzky and G. D. Latyshev, Phys. Rev. **61**, 254 (1942).

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Inefficiency and Other Sources of Error in Cosmic-Ray Measurements with Self-Quenching Counters

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The errors in coincidence measurements with self-quenching counters, due to inefficiency inherent in the counters and due to apparent inefficiency arising from showers and scattering, have been measured. The inefficiency inherent in the counters has been found to be almost entirely due to the dead time. The dead time, for the counters investigated and with the recording circuit employed, was 4×10^{-4} second. This gives rise to an inefficiency of 0.2 percent in counters with a normal counting rate of 300 per minute. The apparent inefficiency of the counters and the error in coincidence measurements, for most of the experimental arrangements used were found to be almost entirely due to showers. From the data presented below, we estimate that near sea level, with the counter telescope in the vertical direction and with 35 cm between the extreme counters, the coincidences due to side showers were about 15 percent of the normal coincidence rate when two counters were in coincidence, 7 percent when three counters were in coincidence, and 3 or 4 percent when the telescope contained five counters in coincidence. These percentages increased by a factor of 2 when the apparatus was taken from 259- to 4300-m elevation.

INTRODUCTION

As sources of error in coincidence measurements with a conventional cosmic-ray telescope we wish to consider the following effects: (1) *chance coincidences*, (2) *inefficiency* of the counters, (3) *scattering* in the absorber or in the counter walls, and (4) cosmic-ray *showers*. All four of these effects contribute to the "apparent inefficiency" of a counter; i.e., to the decrease in the counting rate when an additional counter is put into a cosmic-ray telescope.

The existence of these sources of error has been recognized for many years. The inefficiency of a counter was determined with an anticoincidence method by Rossi in 1930,[1] and a counter measurement of cosmic-ray showers was made by the same author in 1932.[2] In the years since then, there have been very numerous reports of measurements of these effects.[3] However, in the determinations of the influence of cosmic-ray showers on telescope measurements, some effects have been included which have made the determination of the error inexact; and the measurements of the inefficiency of counters have been influenced by the recording of showers so

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[1] B. Rossi, Rend. Acc. dei Lincei [6] **11**, 831 (1930).

[2] B. Rossi, Physik. Zeits. **33**, 304 (1932).

[3] See, for example, T. H. Johnson, Rev. Mod. Phys. **10**, 193 (1938); Phys. Rev. **40**, 638 (1932); D. K. Froman and J. C. Stearns, Can. J. Research **A16**, 29 (1938); M. E. Rose and W. E. Ramsey, Phys. Rev. **59**, 616 (1941); M. Cosyns, Bull. Tech. de'l Assoc. Ing. sortis del Ecole Polytech. Brux. (1936); W. E. Danforth and W. E. Ramsey, Phys. Rev. **49**, 854 (1936).

that the results were only accurate for counters of rather low efficiency.

The usual method of determining the inefficiency of a counter has been to place three counters in line and to compare the threefold coincidence rate with the coincidence rate between the two extreme counters. However, part of the difference in the rates is due to the fact that side showers discharge the two extreme counters much more often than they discharge all three counters. If the efficiency is low, the effect of the showers is comparatively unimportant. But for the highly efficient self-quenching counters, it is shown below that more than 90 percent of the "apparent inefficiency" determined in this way is due to showers.

The customary method of correcting for side showers has been to subtract the coincidence rate obtained with one counter placed just out of line from the rate obtained with all the counters in line. However, this correction includes not only the effect of side showers, but also the coincidences that occur when one particle traverses all the counters in line and an accompanying particle discharges the counter out of line. These cases should not be included in the correction, and give rise to a large error especially when electrons are being recorded.

In the present experiments the method of measuring the inefficiency has been slightly refined so as to eliminate the effect of showers and allow a precise determination of the real inefficiency of the self-quenching counters. In addition, the apparent inefficiency has been measured by the usual methods under a wide variety of experimental conditions. By subtracting the real inefficiency and the effect of chance coincidences from the apparent inefficiency, the effect of side showers under these conditions has been found.

This method of analysis gives accurately the apparent inefficiency due to side showers, but allows only an estimate of the number of side showers which produce coincidences in the counter telescope. The estimate is sufficiently accurate, however, to show that when the self-quenching counters are used, the coincidences due to side showers are a much more serious error under most experimental conditions than the error due to inefficiency. The precise determination of the inefficiency makes it possible to remove a large part of this error by using a large number of counters in coincidence. Then the number of side showers recorded becomes extremely small and the correction may usually be neglected. The error due to inefficiency is thereby increased but is still very small and may be accurately known.

The counters used in this investigation were made of brass cylinders 1 mm thick and 4.24 cm in diameter. The central wire was 4-mil tungsten, and contact was made to the wire by a thick Kovar rod (75-mils diameter) which emerged at one end of the counter through a glass seal. The length of the outer cylinder was 10 cm greater than the length of the central wire, and the length of the latter was 20 cm (except in a few cases specified below where the length was 60 cm). The counters were filled with a mixture of 91 percent argon and 9 percent alcohol, to a total pressure of 11 cm of Hg. Each counter was operated at 90 volts above its starting potential, as determined by the recording circuit. The starting potentials were approximately 1000 volts.

I. INEFFICIENCY INHERENT IN THE SELF-QUENCHING COUNTERS

A. Inefficiency Due to the Dead Time

Part of the inefficiency of a counter is due to the fact that following each discharge there is a certain period of insensitivity, during which the space charge which has accumulated around the central wire is removed from the immediate vicinity of the wire. Since this period depends on the motion of heavy ions, the order of magnitude is 10^{-3} or 10^{-4} second, which is much greater than the time required for an electron to traverse the counter; in fact, it is almost 100 times as large as the time required for the discharge to be initiated, to rise to the maximum, and to be quenched. The dead-time effect does not depend on how far from the wire the first ions are created.

During the course of this investigation, Stever[4] has published an analysis of the dead-time phenomenon in self-quenching counters, obtained by an oscillographic method. Stever's

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[4] G. H. Stever, Phys. Rev. **61**, 38 (1942).

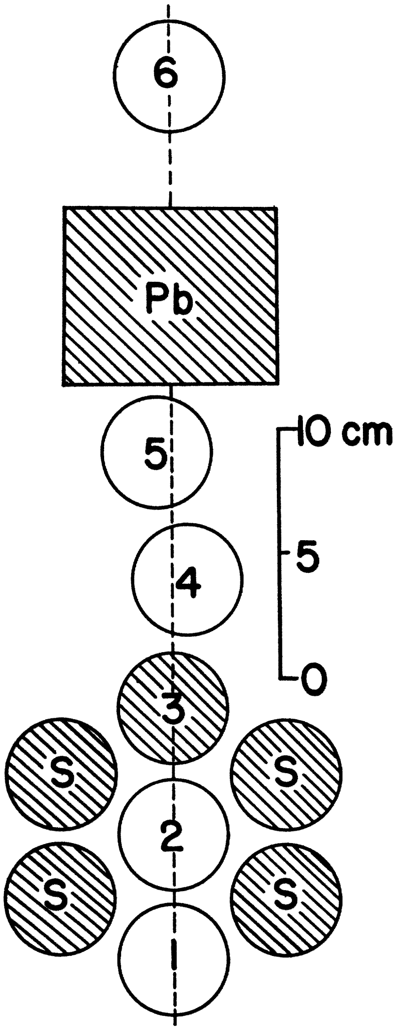

FIG. 1. Experimental arrangement used in the measurement of the dead time. Counters 1,2,4,5,6 are in coincidence, counters 3 and *S* in anticoincidence. All of the anticoincidences are due to inefficiency of counter 3. Results are in Table I. Results with counters 4 and 5 moved into line are given in Table II. When counters *S* are disconnected, some of the anticoincidences are due to showers: see results in Table IX.

photographs show that following a discharge there is a period of complete insensitivity (during which the counter acts as a proportional amplifier, and the pulses are too small to be seen on the oscilloscope), which is followed by a period of about the same length during which the pulse size gradually recovers to its full normal value.

Obviously, regardless of the method used to detect the pulses, the period during which the counter is apparently "dead" will depend on the sensitivity of the recording apparatus, just as the exact value of the "starting potential" of a counter also depends on the sensitivity of the detecting device. For our purposes, we wish to define the "dead time," t_0, as the time interval following a discharge until the counter has recovered to the point where another pulse may be recorded by our circuit. This definition is made fairly precise by the fact that there is a small region of field strength during which the pulse size increases very rapidly with the field strength in a counter. Thus in the present experiments the counters were operated at 90 volts above the recording circuit, and a decrease of 70 volts in the working voltage was found to produce no change in the measured dead time, within a statistical error of about 20 percent.

In order to suppress any apparent inefficiency except that due to the dead time, the usual method of measuring inefficiency was modified as shown in Fig. 1. The counter for which the inefficiency was determined is in position 3, and was connected in anticoincidence. The four counters *S* were connected in parallel with counter 3 in order to eliminate anticoincidences due to side showers. Counters 4 and 5 were placed slightly out of line in opposite directions with displacements of $\frac{1}{2}$ inch in the direction of the counter axes, and $\frac{1}{4}$ inch at right angles to the axes. This displacement prevented the recording of anticoincidences due to any inefficiency which occurs only near the edges of the counter, because a particle traveling very near the edge of counter 3 could not produce a coincidence. The lead was inserted between the counters so that practically all the coincidences recorded would be caused by mesotrons, which usually appear singly (at least at sea level), and which do not often undergo large-angle scattering, such as occurs frequently in radiation processes of electrons.

The criterion by which one can determine whether or not all the anticoincidences recorded with this arrangement are due to the dead time is the dependence of the apparent inefficiency on the discharge rate. If N is the discharge rate and t_0 is the dead time, the inefficiency due to the dead time is $(1-\exp(-Nt_0))$, or if $Nt_0 \ll 1$, it is simply Nt_0. Since the number of chance coincidences is completely negligible with five counters in coincidence, and the other causes of apparent inefficiency do not depend on the discharge rate, the apparent inefficiency will be proportional to the discharge rate if, and only if, the anticoincidences are all due to the dead-time effect. Accordingly, we have made the test with counters of different lengths in position 3, and during some of the measurements we have placed a radium source near the counters to stimulate the discharge rate.

The results are shown in Table I. For each measurement, we have computed the dead time t_0 with the formula

$$\text{Inefficiency}=\frac{\text{Anticoincidences}}{\text{Coincidences}}=Nt_0$$

and thus have assumed that there is no inefficiency except that which is due to the dead time. The consistency of the results obtained justifies the assumption.[5] The weighted average of all the results is

$$t_0=(3.8\pm0.2)\times10^{-4}\text{ sec.}$$

There is a small error in this determination of the dead time, because of the fact that occasionally when counter 3 fails to record the passage of a mesotron, a knock-on electron accompanying the mesotron discharges one of the side counters S, and thus an anticoincidence is not registered. The magnitude of this effect was determined by repeating the above experiment with counter 3 disconnected. With this arrangement, we should record an anticoincidence whenever we record a coincidence, except for the cases when a knock-on electron discharges one of the side counters, and for the cases when a large shower from the side discharges all of the coincidence counters as well as one of the counters S. The ratio of the difference $C-A$ between the numbers of coincidences and anticoincidences to the number C of coincidences obtained in this measurement is

$$(C-A)/C=(6.32\pm0.22)\times10^{-2}.$$

Since any side shower which discharges all five coincidence counters is almost sure to discharge one of the counters S, the above ratio sets an upper limit for the error in the coincidence rate which is due to side showers discharging all of the counters in coincidence. Likewise it sets an upper limit for the error in t_0 due to the knock-on electrons discharging the side counters. The major part of the above effect is probably due to these knock-on electrons. If we assume that the effect is *all* due to the knock-on electrons, then the corrected value of t_0 is

$$t_0=(4.0\pm0.2)\times10^{-4}\text{ second.}$$

B. Spread of the Discharge

One of the results in Table I is possibly in disagreement with the other results. The questionable measurement was made with a long counter in position 3, so that a large part of the counter protruded outside of the counter telescope. The low value of the dead time obtained in this measurement led us to suppose that the discharges which occurred in the "unused" part of the counter did not make the rest of the counter insensitive, or perhaps gave rise to a shorter dead time at the far end of the counter, i.e., that the discharge did not spread uniformly along the whole wire.

In order to test this hypothesis, we repeated the measurement of the dead time under two conditions: (1) with the outer or "unused" end of the counter stimulated by radium, and (2) with the inner or "useful" end of the counter stimulated by radium. This was accomplished simply by shielding the end of the counter which was not to be stimulated with about 40 cm of lead. The result obtained was entirely negative; the values found for the dead time were in

TABLE I. Determination of the dead time t_0. The experimental arrangement is shown in Fig. 1.

Counter in position 3	Length (cm)	Discharge rate (min.$^{-1}$)	Anticoinc. $[(1,2,4,5,6)-(3+S)]$	$A/C\times10^3$	$t_0\times10^4$ (sec.)
a	20	299	46	1.84 ± 0.27	3.7
a	20	1090	64	8.0 ± 1.0	4.4
b	20	251	27	1.87 ± 0.36	4.5
c	60	769	45	2.97 ± 0.44	2.3
c	60	2330	90	13.7 ± 1.4	3.5
c	60	2406	92	16.0 ± 1.7	4.0
d	60	794	58	5.53 ± 0.73	4.2

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[5] One measurement in the seven gave a result for t_0 which differed from the average by almost four times the statistical error in the difference. If we assume the usual error curve for the probability of deviations, we find that the probability of obtaining a deviation this large or larger is 1/5000. It is therefore unlikely, but not impossible, that the result is purely a statistical fluctuation. See footnote 6.

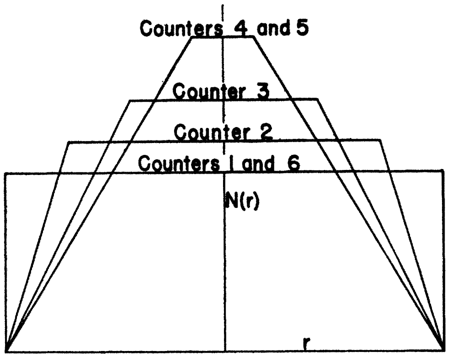


FIG. 2. "Usefulness" of different sections of the counters as a function of distance from the wire. $N(r)dr$ represents (in arbitrary units) the number of coincidence-producing rays which traverse a counter along paths with distances from the central wire lying between r and $r+dr$. The graphs are calculated for the counter arrangement shown in Fig. 1, with counters 4 and 5 moved into line. The assumption is made that over the small range of angle subtended by the diameters of the extreme counters the intensity of cosmic rays does not change.

agreement with each other and with all of the other measurements of the dead time. The data have therefore been included in Table I, and are the two sets of data taken on counter *c* with high discharge rates.

This result is also in agreement with Stever's findings; i.e., that the discharge does spread uniformly along the whole counter wire.

C. Inefficiency not Due to the Dead Time. Measurement of the Effective Diameter

Rose and Ramsey[3] have shown the existence of a type of inefficiency which is not due to the dead time, and have investigated this inefficiency for counters filled with oxygen or with an argon-oxygen mixture. The effect which they have observed is greatest when the primary particle passes through the counter near the edge, and seems to depend critically on the type of gas used, being much greater for an oxygen-filled counter than for a counter containing an argon-oxygen mixture of which only 6 percent is oxygen.

The causes for this phenomenon may be: (1) when the primary particle passes near the edge of a counter, the path length through the counter is short; therefore, it may happen occasionally that no ions are produced; or (2) when the particle passes near the edge, only a few ions are produced, and the field intensity is low; it is therefore likely that the electrons may be captured, and slowly-moving negative ions may be formed. This may either delay or prevent the discharge. The dependence of the inefficiency on the relative abundance of oxygen in the counters indicates that the formation of heavy ions is responsible for at least a part of this inefficiency. Because of the dependence on the number of ions produced, the probability for either of the processes (1) or (2) depends on the distance r from the wire to the path of the primary ray in the form $\exp[-(a^2-r^2)^{\frac{1}{2}}/k]$, where k is a constant and a is the radius of the counter. In addition, the probability for process (2) may have a term depending on the field strength, but this varies slowly with r except near the wire.

It has been shown above that the inefficiency of self-quenching counters, except possibly near the edge, is very small (0.18 percent for the short counters) and can be accounted for entirely by the dead time. However, a function of the form given above may be negligible for small r and yet be very large for $r \approx a$. The net result would be to make the effective diameter of the counter slightly smaller than the geometrical diameter.

The experiment described in Section A was repeated with counters 4 and 5 placed exactly in line. In this condition the ratio A/C of the anticoincidences to the coincidences gives a measure of the "over-all" inefficiency of counter 3, which should be larger than the inefficiency measured in the first experiment if there is any additional inefficiency near the edges of the

TABLE II. Evidence for inefficiency near the edge of self-quenching counters. The edge effect is taken as the difference between the "over-all" inefficiency (column 4) and the inefficiency due to the dead time (column 5). Weighted average of differences (discarding data on counter *c*):* Percent inefficiency due to edge effect $=0.093\pm0.034$ percent.

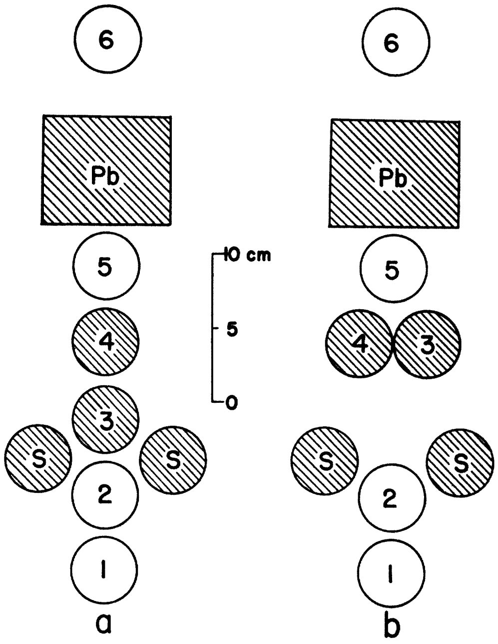
Counter in position 3	Length (cm)	Anticoincidences with counters 4 and 5 in line	$A/C\times10^3$ with counters 4 and 5 in line	$A/C\times10^3$ with counters 4 and 5 out of line	Differences ($A/C\times10^3$)
a	20	49	2.66 ± 0.38	1.84 ± 0.27	0.82 ± 0.47
b	20	52	3.07 ± 0.43	1.87 ± 0.36	1.20 ± 0.56
c	60	79	5.98 ± 0.67	2.97 ± 0.44	—
d	60	87	6.10 ± 0.65	5.53 ± 0.73	0.57 ± 0.98

* See reference 6.

counter. The results are given in Table II and compared with the results of the first experiment.[6]

The comparison in Table II indicates that the inefficiency of counter 3 is slightly greater when the coincidence-producing particles may traverse any part of the counter than when they cannot traverse the counter near the edge. The difference in efficiency, however, is very small, and is not quite definite, because the effect is only three times the statistical error. But this does not preclude the possibility that a small region near the edge of the counter may be very inefficient. In fact the effect of such an inefficiency on the "over-all" inefficiency of a counter depends on the position of the counter in the counter telescope, and is least for a counter near the center of the telescope. In Fig. 2 we have plotted the number of coincidence-producing rays traversing unit area of a counter, as a function of the distance r from the central wire to the path of the particle, for the counters in our telescope. It is seen that if the counter were completely insensitive within 1 mm of the outer walls, the resulting inefficiency would be only 0.28 percent for counter 3, but would be 4.75 percent for one of the extreme counters. Thus the data in Table II may indicate that the outer mm of the counter has a low efficiency, which may be due to the failure of some particles to produce ions in the short length of path through the counter near the edge. This effect, if real, might be of considerable importance for the extreme counters in the array.

In order to determine the effective diameter of the counters more exactly, the following experiment was performed. The counters were first arranged as in Fig. 3a, and by means of the coincidence-anticoincidence circuit we recorded simultaneously the coincidences (1,2,5,6) and the anticoincidences $[(1,2,5,6)-(3+4+S)]$. Next, the counters were arranged as in Fig. 3b, and the same phenomena were recorded. In arrangement (b), the anticoincidences were caused by: (1) particles traversing any inefficient region between counters 3 and 4, (2) inefficiency of counter 3 or counter 4 due to the dead time, or (3) shower particles discharging counters 1, 2, 5, and 6 without discharging either of the counters 3, 4, or S. With arrangement (a), the effect of the inefficiency is negligible, so that the anticoincidences are due only to the last of the above-named causes. The shower effect should have been almost identical with arrangements (a) and (b), and was in any case small, because of the presence of the side counters S. The lead above counter 5 assured that we were recording mesotrons (which usually occur singly) rather than electron showers.



FIGS. 3a AND 3b. Experimental arrangement used in measuring the effective diameter of a counter. Counters 1,2,5,6 are in coincidence, counters 3,4, and S in anticoincidence.

The ratios of the anticoincidences to the coincidences obtained in this experiment were:

$$A/C=8.2\pm0.30 \text{ percent with arrangement (b)},$$

$$A/C=0.4\pm0.10 \text{ percent with arrangement (a)}.$$

The dead-time inefficiency of the counters accounts for an inefficiency of only 0.2 percent with arrangement (b), and zero with arrangement (a); therefore 7.6 percent of the coincidence-

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[6] Here again we have evidence that the inefficiency of counter c measured with counters 4 and 5 slightly out of line was subject either to large statistical fluctuation or to some other error, because the difference brought about by putting counters 4 and 5 in line is much larger for this counter than for the other three. If the inefficiency first measured were replaced by the expected result (calculated from the average dead time), the change brought about by putting counters 4 and 5 in line would agree with that found for the other counters.

producing rays traversed the inefficient area between counters 3 and 4 in arrangement (b). This figure is too low by about five percent (of itself) because of the mesotron secondaries generated above counters 3 and 4, which occasionally discharge these counters when a mesotron traverses the dead space;[7] hence the corrected result for the fraction of the particles which traverse the dead space is 8.0±0.4 percent.

If we assume that this number is accounted for by the existence of a sharply defined and perfectly inactive dead space of width d between counters 3 and 4 in arrangement (b), we find (by reference to Fig. 2 and the graph for counter 4) that $d=1.94\pm 0.10$ mm. The thickness of the counter walls between the interior of counter 3 and the interior of counter 4 was 2.0 mm. Therefore *within 1/10 of a millimeter, the effective diameter of the counter is equal to the inside diameter of the counter wall.*

If the interior of the counters were perfectly efficient, however, the dead space found should have been less than the wall thickness, because the mesotrons traversing the counter walls must often produce secondaries (which need not have very high energy) that discharge one of the counters. Therefore these results are in agreement with the small difference found above between the "over-all" efficiency and the efficiency near the center of the counter.

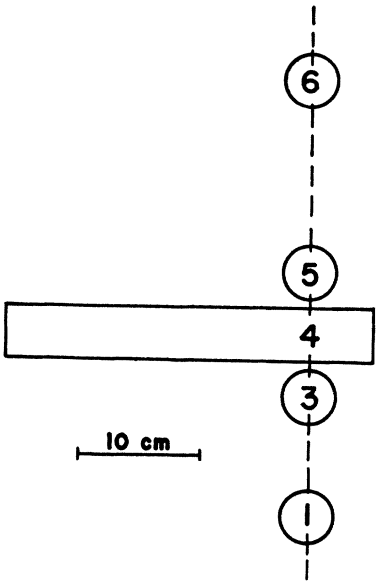

FIG. 4. Experimental arrangement used to determine the effective length of a counter. Results are plotted in Fig. 5.

Finally, we may state that for self-quenching counters of 4-cm diameter and 20-cm length, filled with an argon-alcohol mixture, the efficiency near the center is 99.8 percent;[8] but for rays traversing the counter within about $\frac{1}{2}$ mm of the walls, the efficiency may be much smaller. As a result, the over-all inefficiency of the central counter in an array is about 0.3 percent. The edge effect in the extreme counters, however, is exactly compensated by the occasional discharge of the counters when a ray goes through the walls; therefore the effective inefficiency of these counters depends only on the dead time and is about 0.2 percent.

D. Effective Length of the Counters

The effective length of a counter was measured by using the arrangement shown in Fig. 4. The coincidence rate between the five counters was recorded as a function of the position of counter 4, as the latter counter was moved parallel to its own axis from a position completely out of the beam on one side to a position completely out of the beam on the other side.

The data are indicated by the open dots in Fig. 5, and the statistical errors are shown by the vertical lines. The solid curve is the theoretical curve for a cylindrically-shaped counter of 4.2-cm diameter in our counter array, with an added background rate of 0.05 (min.$^{-1}$) which was obtained from the coincidence rate with the counter drawn completely out of the beam. The maximum height of the theoretical curve has also been taken from the experimental data, and the length has been chosen so as to give the best fit.

The effective length of the counter, as obtained by this comparison between the theoretical and experimental curves, is $l=18.8$ cm. The actual length of the fine collecting wire in this counter was 20 cm. Since the outer cylinder extended 5 cm beyond this wire at both ends, and since contact was made to the wire by means of a

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[7] See discussion and experimental results in Section IA.

[8] This value, of course, as explained above, is subject to variations depending upon the normal counting rate and upon the sensitivity of the recording circuit.

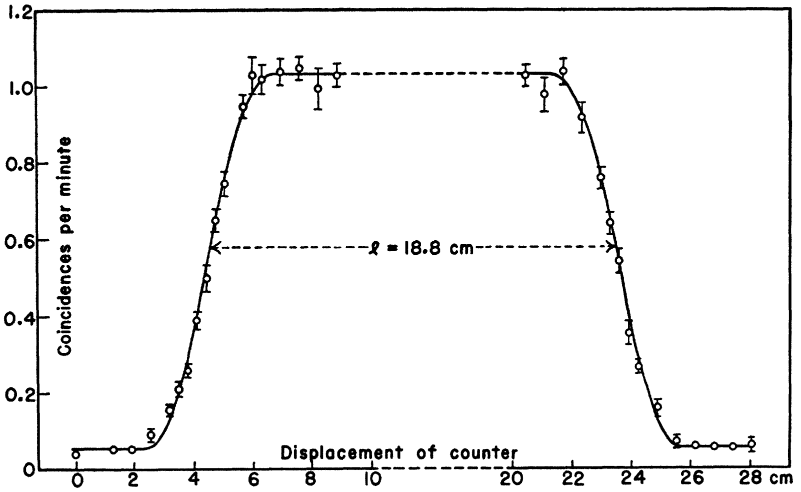

FIG. 5. Determination of the effective length. The open dots represent the coincidence rates obtained with the arrangement shown in Fig. 4 for various positions of the middle counter. The solid curve is calculated for a cylindrical counter, with background rate added.

thick rod, it had been expected that the effective length would be equal to the length of the fine central wire; however, it appears that the effective length is less than this by 1.2 cm. As a result of this measurement, we find that the absolute intensities of cosmic rays reported recently by one of us[9] are all too small by 10.3 percent.

II. APPARENT INEFFICIENCY DUE TO SHOWERS AND SCATTERING

A. Apparent Inefficiency Due to Scattering

With the apparatus shown in Fig. 1, the ratio of anticoincidences to coincidences recorded was increased by a factor of 5 when counters 4 and 5 (those placed slightly out of line) were disconnected from the coincidence group. That is, the anticoincidences $[(1,2,6)-(3+S)]$ were almost 1 percent of the coincidences (1,2,6) whereas the anticoincidences $[1,2,4,5,6-(3+S)]$ were only 0.2 percent of the coincidences (1,2,4,5,6). This result contrasts strongly with the small increase in anticoincidence rate, due to the edge effect, obtained when counters 4 and 5 were not disconnected but placed exactly in line with the others. The result might easily have been explained had the scattering of mesotrons in the lead absorber been appreciable, since a fraction of the mesotrons traversing counter 6 might have been scattered by the lead so as to traverse counters 1 and 2 but miss counter 3.

In order to test whether or not scattering was really responsible for the anticoincidences $[(1,2,6)-(3+S)]$ observed, the latter were recorded both as described above and also with the width of lead absorber reduced to exactly the width of the counters. In the latter condition, scattering could not possibly cause counters 1, 2, and 6 to be discharged without counter 3 also being discharged. No difference was observed between the two measurements (within a statistical error of 12 percent of the effect). Later measurements, described below, showed that the entire increase in the apparent inefficiency of counter 3 upon disconnecting counters 4 and 5 was due to *showers*, in spite of the presence of the side counters S.

The unimportance of the scattering effect in measurements on mesotrons at sea level is also predicted theoretically. Using the formula given by Rossi and Greisen,[10] one finds that the root mean square angle of scattering for mesotrons of momentum 2×10^9 ev/c in 7 cm of lead is only 1.5 degrees.

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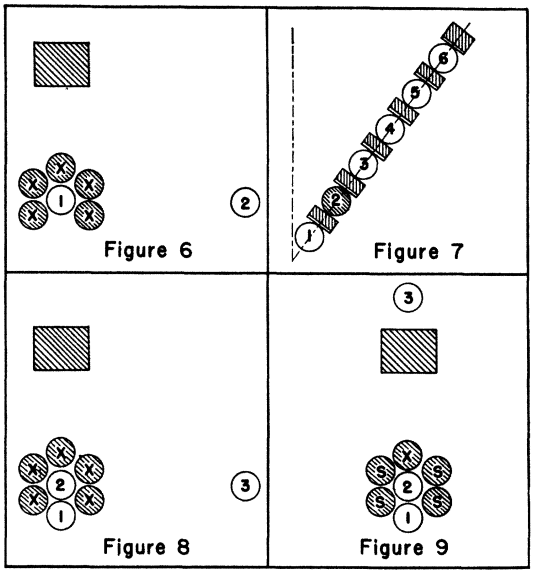
[9] K. Greisen, Phys. Rev. **61**, 212 (1942).

[10] B. Rossi and K. Greisen, Rev. Mod. Phys. **13**, 240 (1941); formula given on page 265.



FIGS. 6–9. Experimental arrangements used to study the effect of side showers on coincidence measurements. Results corresponding to Fig. 6 are in Table III; results corresponding to Fig. 7, with varying numbers of counters in coincidence and counter 2 in anticoincidence, are in Tables IV, V, VIII, X, XI; results corresponding to Fig. 8 are in Table VI; and results corresponding to Fig. 9 are in Table VII.

B. Apparent Inefficiency Due to Showers, and the Effect of Side Showers on Coincidence Measurements

The effect of showers on an array of counters is strongly dependent on the number of counters in the array, their geometrical arrangement, the physical surroundings of the apparatus, and the altitude at which measurements are made, as well as on the type of absorber used and on its shape and position. In the experiments described below these factors have been varied in an attempt to determine the effect of showers on coincidence measurements made under various conditions. The experiments are grouped below according to the number of counters connected in coincidence.

1. Experiments with Two Counters in Coincidence

(*a*) *Results with experimental arrangement indicated in Fig. 6.*—The results are given in Table III. In order to distinguish the effect of showers, the number of chance coincidences had to be determined. Therefore measurements were made both with no artificial stimulation and with the counters stimulated by radium. The difference between the coincidence rates observed was assumed to be due to chance coincidences. The resolving time thus determined is $\tau = 7.7 \pm 0.6$ microseconds. With this value of τ, we have calculated the chance coincidence rates given in Table III, and by subtraction of the chance coincidences from the total we have obtained the shower rate. From these figures we see that when the counters are not stimulated with radium, 96 percent of the coincidences are due to showers and only 4 percent are chance coincidences.

From the data in Table III we may also discover how many of the showers which discharge counters 1 and 2 also discharge one or more of the five anticoincidence counters surrounding counter 2. An anticoincidence $(1,2-X)$ may occur when a chance coincidence $(1,2)$ occurs, provided none of the counters X is also discharged. However, the anticoincidence rate with no artificial stimulation was 0.072 (min.^{-1}), while the chance coincidence rate was only 0.018. Therefore at least three-fourths of the anticoincidences are not due to chance coincidences. These anticoincidences are accounted for by showers discharging counters 1 and 2 without discharging any of the counters X. Thus the anticoincidence rate due to showers is at least 0.054 (min.^{-1}); i.e., at least 14 percent of the showers (0.40 per minute) which discharge counters 1 and 2 fail to discharge any of the five surrounding counters X.

(*b*) *Results with experimental arrangement indicated in Fig. 7.*—This experiment was performed outdoors, under a thin wooden roof at elevations

TABLE III. Measurements taken with the experimental arrangement shown in Fig. 6, in the basement of the physics building in Ithaca (elevation 259 m).

Quantity measured	No artificial stimulation	Counters stimulated by radium
Coinc. (1,2) per min.	0.417 ± 0.015	0.908 ± 0.033
Anticoinc. $(1,2-X)$ per min.	0.072 ± 0.006	0.470 ± 0.024
Counts per min. in counter 1, N_1	276	1586
Counts per min. in counter 2, N_2	257	1254
Chance coinc. per min.	0.018 ± 0.0013	0.51 ± 0.038
Shower rate (min.^{-1})	0.40 ± 0.015	0.40 ± 0.050

TABLE IV. Measurements taken in the vertical direction in Ithaca with the apparatus shown in Fig. 7. Counters 3, 4, and 5 were not in use. N_1, N_2, and N_6 are the counting rates in the individual counters 1, 2, and 6.

	No artificial stimulation		Counters stimulated by radium	
Quantity measured	No lead between counters	13-cm lead between counters	No lead between counters	13-cm lead between counters
Coinc. (1,6) per min.	3.14 ±0.022	2.46 ±0.042	3.32 ±0.048	2.60 ±0.037
Anticoinc. (1,6–2) per min.	0.197±0.006	0.119±0.009	0.313±0.015	0.239±0.011
N_2 (min.$^{-1}$)	252	204	806	736
N_1 (min.$^{-1}$)	252	220	515	475
N_6 (min.$^{-1}$)	268	218	1127	1086
Anticoinc. rate due to inefficiency	0.005	0.003	0.018	0.013
Chance coinc. per min.	0.015±0.002	0.011±0.001	0.126±0.013	0.113±0.012
Anticoinc. per min. due to side showers	0.177±0.006	0.105±0.010	0.169±0.020	0.113±0.016

TABLE V. Measurements taken at various elevations and two zenith angles, with and without lead, with the apparatus shown in Fig. 7. Counters 3, 4, and 5 were not in use. C refers to the coincidence rate between counters 1 and 6; A is the anticoincidence rate [1,6–2]. The anticoincidence rates may be considered as proportional to the number of side showers which discharge counters 1 and 6.

		Zenith angle 0°		Zenith angle 46°	
Elevation (meters)	Absorber (cm of lead)	A (min.$^{-1}$)	A/C (percent)	A (min.$^{-1}$)	A/C (percent)
259	0	0.197±0.006	6.3±0.2	0.139±0.008	10.0±0.6
	13	0.119±0.009	4.9±0.4	0.110±0.009	9.5±0.8
1616	0	0.35 ±0.02	8.0±0.5	0.23 ±0.02	11.9±1.2
	13	0.20 ±0.02	6.1±0.5	0.23 ±0.02	13.6±1.5
3240	0	0.81 ±0.03	10.6±0.5	0.52 ±0.03	15.4±0.9
	13	0.39 ±0.02	8.4±0.5	0.48 ±0.03	19.0±1.1
4300	0	1.31 ±0.04	11.8±0.4	1.02 ±0.04	20.5±0.8
	13	0.78 ±0.03	12.5±0.6	0.77 ±0.03	22.2±1.1

TABLE VI. Measurements taken with the experimental arrangement shown in Fig. 8, in the basement of the physics building in Ithaca (elevation 259 m).

Quantity measured	No artificial stimulation	Counters stimulated by radium
Coinc. (1,2,3) per min.	0.125 ±0.006	0.139±0.008
Ratio of anticoinc. (1,2,3–X) to coinc. (1,2,3) in percent	9.9 ±1.6	13.3 ±2.2
Coinc. (1,2) per min., N_{12}	38	38
Counts per min. in counter 3, N_3	276	1586
Chance coinc. (1,2,3) per min.	0.0027±0.0002	0.015±0.0011
Coinc. (1,2,3) per min. due to showers	0.122 ±0.006	0.124±0.008

259, 1616, 3240, and 4300 meters, both with and without lead absorbers between the counters. The thickness of the absorber when in use was 13 cm of lead, which is sufficient to prevent electrons from traversing the counter telescope. The absorber had the same width as the counters, so that it could not possibly scatter particles in such a way as to produce a coincidence without discharging counter 2.

An analysis of anticoincidence rates (1,6–2) obtained in Ithaca with the counter telescope in the vertical direction is given in Table IV. The anticoincidences are due to (1) inefficiency of counter 2, (2) chance coincidences between counters 1 and 6, and (3) side showers which discharge counters 1 and 6 without discharging counter 2. The effect due to inefficiency has been calculated by using the value of the dead time determined above (4×10^{-4} second).

The increase in the anticoincidence rates which accompanied the stimulation with radium allows two independent determinations (from the measurements with and without lead) of the resolving time. The two values thus found are 6±1 and 7±1 microseconds. The average of these two values has been used to determine the total number of anticoincidences arising from chance coincidences.

We see from the values in Table IV that inefficiency normally accounts for less than 3 percent of the anticoincidences, chance coincidences account for about 8 percent, and side showers account for about 90 percent of the anticoincidences.

The measurements at other zenith angles and

TABLE VII. Measurements taken with the experimental arrangement shown in Fig. 9, in the basement of the physics building in Ithaca (elevation 259 m).

Quantity measured	Counter *a* (20-cm length) in position X, no artificial stimulation	Counter *a* in position X, stimulated with radium	Counter *d* in position X (60-cm length), no artificial stimulation
Coinc. (1,2,3) per min.	2.47 ±0.018	2.55 ±0.042	2.53 ±0.032
Anticoinc. $[(1,2,3)-(X+S)]$ per min.	0.026±0.0019	0.041±0.0054	0.032±0.0036
Counts per min. in counter X, N_x	299	1090	794
Chance coinc. (1,2,3) per min.	0.003	0.011	0.003
Anticoinc. per min. due to inefficiency	0.005	0.018	0.013
Anticoinc. per min. due to side showers	0.018±0.002	0.012±0.006	0.016±0.004

other altitudes are presented in Table V. Here we have not separated the anticoincidences into their various components. However, over the range of altitudes included in our measurements the chance coincidences between two counters vary with altitude in very nearly the same way as do the showers; therefore the showers account for about 90 percent of the anticoincidences at all of the altitudes and zenith angles.

Since an appreciable fraction of the side showers which discharge the two coincidence counters will also discharge the anticoincidence counter and hence fail to produce an anticoincidence, the number of side showers *which produce coincidences* must be even larger than these anticoincidence rates. Thus we see that the error due to side showers, which would be involved in a coincidence measurement with two counters, is a large error, completely overshadowing the errors due to inefficiency and chance coincidences. The error becomes more serious at greater zenith angles of the counter telescope, where the coincidence rates decrease faster than the shower rate. And the error becomes more serious at high altitudes, because the shower rate increases with altitude faster than does the mesotron intensity. In this experiment the shower rate was 7 times as large at Mount Evans (elevation 4300 m) as at Ithaca, and the ratio to the intensity of single particles was twice as great at Mount Evans as at Ithaca.

2. *Experiments with Three Counters in Coincidence*

(*a*) *Result with experimental arrangement shown in Fig. 8.*—The results are given in Table VI. The chance coincidence rates have been calculated by using the value of the resolving time determined above; the remainder of the coincidences are assumed to be due to showers.

In order to have a check on the reliability of the separation of the anticoincidence rate into its components, the experiment was performed both with no artificial stimulation and with the counting rates stimulated by radium. The results are seen to be in good agreement, since the shower rate should have been the same in the two conditions.

The figures in Table VI indicate that when the counters are not stimulated by radium, 98 percent of the coincidences are due to showers. The anticoincidence rate shows that 10 percent of the showers fail to discharge any of the five surrounding counters.

(*b*) *Results with experimental arrangement shown in Fig. 9.*—This experiment differs from the one described just above only in that counter 3 is placed in line with counters 1 and 2. This is the usual arrangement of three counters for intensity measurements. For our present investigation we are primarily interested in the anticoincidences $[(1,2,3,)-(X+S)]$. These are due to (1) inefficiency of counter X, (2) chance coincidences between counter 3 and counters 1 and 2, and (3) side showers. The chance coincidences and the effect of inefficiency have been calculated by using the values of the resolving time and the dead time given above, and the remainder of the anticoincidences is due to the side showers *which fail to discharge any of the counters X or S.* That the effect of scattering is negligible has already been shown (see Section IIA).

The results are given in Table VII. In order to have a check on the determinations, some of the measurements were made with a long counter in position X (counter *d*, of length 60 cm) and some with the counters stimulated by radium; for these two cases the inefficiency is larger, but the shower rates should be the same as when counter *a* is used (20-cm length) with no artificial stimulation. The results verify this prediction.

If we compare these results with the coincidence rate itself, we see that for counter *a* in position *X*, the number of chance coincidences between the three counters is only 0.1 percent of the coincidence rate, and the anticoincidences due to inefficiency are only 0.2 percent of the coincidence rate, whereas the number of anticoincidences due to showers is 0.7 percent of the coincidence rate. But the first experiment with three counters (Fig. 8, Table VI) showed that 90 percent of the showers which produce a threefold coincidence also discharge one of the counters *X* or *S* and so do not produce anticoincidences. Hence the number of coincidences produced by side showers is 10 times the number of anticoincidences produced by the showers, or *7 percent* of the coincidence rate. This indicates that even in the basement of a building at low altitude, the error due to side showers in coincidence measurements with three counters is very considerable.

The experiment described just above (Fig. 9, Table VII) was repeated with the side counters disconnected and only counter *X* in anticoincidence. The ratio of the anticoincidences to the coincidences obtained in this condition was $A/C=(3.3\pm0.6)$ percent. This is larger than the ratio obtained from the data in Table VII (i.e., 1.06 percent, for counter *a* in position *X*), because when the side counters are disconnected, a larger fraction of the showers which produce coincidences also produce anticoincidences. Chance coincidences and inefficiency are responsible for only $\frac{1}{10}$ of the 3.3 percent. Since the shower rate, however, is 7 percent of the coincidence rate, we see that about *half* of the showers which discharge the three counters in coincidence also discharge the single counter *X* and thus fail to produce an anticoincidence. This fact is used in the analysis of the following experiments. It also indicates the way to eliminate the errors due to showers. If we had used four counters in coincidence instead of three, the fourth being placed in the position of the anticoincidence counter, the number of coincidences produced by showers would have been reduced by a factor of 2.

(*c*) *Results with experimental arrangement shown in Fig. 7.*—The results are given in Table VIII. We know from the experiments described above that the anticoincidences arising from inefficiency and from chance coincidences are only a small fraction of the total number of anticoincidences, especially when no side counters are used.[11] Therefore the anticoincidence rates given in Table VIII may be considered as proportional to the number of showers discharging the coincidence counters. Moreover, since about half of the showers which discharge the three coincidence counters also discharge the anticoincidence counter (as shown above), we may consider the total number of showers which discharge the three counters in coincidence as about twice the number of anticoincidences. It is seen that this implies a large error in coinci-

TABLE VIII. Measurements taken at various elevations, outdoors, with the apparatus shown in Fig. 7. Counters 3 and 4 were not in use and no absorber was used. *C* refers to the coincidence rate between counters 1, 5, and 6. *A* is the anticoincidence rate [1,5,6–2]. The anticoincidences are almost entirely due to showers, and are about half the number of showers which produce coincidences.

Elevation (meters)	Zenith angle 0°		Zenith angle 46°	
	A (min.$^{-1}$)	A/C (percent)	A (min.$^{-1}$)	A/C (percent)
259	0.105 ± 0.008	3.6 ± 0.29	0.042 ± 0.005	3.2 ± 0.39
1616	0.153 ± 0.014	3.8 ± 0.34	0.094 ± 0.017	5.4 ± 1.0
3240	0.45 ± 0.035	6.3 ± 0.51	0.161 ± 0.021	5.9 ± 0.80
4300	0.69 ± 0.044	6.9 ± 0.46	0.29 ± 0.032	7.6 ± 0.88

TABLE IX. Evidence for side showers discharging an array of five counters in line. The data were taken with the apparatus shown in Fig. 1, in the basement of the physics building in Ithaca. The figures in the first three rows of the table represent anticoincidence rates in counts per 100 minutes.

	Counters 4 and 5 displaced as shown in Fig. 1	Counters 4 and 5 placed in line
$A\times100$, with only counter 3 in anticoinc.	0.76 ± 0.10	1.13 ± 0.17
$A\times100$, with counter 3 plus side counters *S* in anticoinc.	0.40 ± 0.06	0.64 ± 0.09
Difference	0.36 ± 0.12	0.49 ± 0.19
Percent ratio of difference to coinc. rate	0.17 ± 0.06	0.20 ± 0.08

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[11] This has only been shown for the measurements in Ithaca, but would be even more true at high altitudes, because the shower rate is known to increase more rapidly with altitude than the total counting rates.

TABLES X and XI. The following data were taken with the apparatus shown in Fig. 7, outdoors, under a thin wooden roof. The anticoincidences are mainly due to the side showers which discharge the coincidence array but miss the anticoincidence counter.

TABLE X. Anticoincidences (1,3,4,5,6 −2) per 100 min.

Elevation (meters)	Absorber in cm of lead	Zenith angle			
		0°	29°	46°	56°
259	0	3.4±0.25	2.4 ±0.27	1.6 ±0.23	1.04±0.17
	13	2.2±0.20	0.94±0.18	0.79±0.17	0.62±0.15
1616	0	6.2±0.27	3.6 ±0.56	2.5 ±0.42	1.5 ±0.34
	13	2.8±0.51	2.1 ±0.43	0.98±0.25	1.1 ±0.26
3240	0	14.6±1.1	7.6 ±0.74	4.5 ±0.65	4.0 ±0.49
	13	5.6±0.72	4.4 ±0.59	2.9 ±0.52	1.9 ±0.36
4300	0	21.6±1.4	13.6 ±1.1	10.2 ±1.0	5.6 ±0.71
	13	8.5±0.87	6.2 ±0.74	5.2 ±0.70	4.6 ±0.65

TABLE XI. Percent ratio of anticoincidences (1,3,4,5,6 −2) to coincidences (1,3,4,5,6).

Elevation (meters)	Absorber in cm of lead	Zenith angle			
		0°	29°	46°	56°
259	0	1.19±0.09	1.14±0.13	1.28±0.18	1.40±0.24
	13	0.94±0.09	0.56±0.11	0.73±0.16	0.94±0.22
1616	0	1.56±0.07	1.27±0.20	1.44±0.25	1.45±0.33
	13	1.01±0.18	0.93±0.19	0.76±0.20	1.28±0.31
3240	0	2.19±0.17	1.65±0.16	1.73±0.25	2.66±0.34
	13	1.34±0.17	1.49±0.20	1.45±0.26	1.62±0.30
4300	0	2.30±0.15	2.01±0.17	2.62±0.25	2.53±0.33
	13	1.60±0.17	1.48±0.18	2.00±0.27	2.84±0.41

dence measurements with 3 counters, the magnitude of which becomes more serious at high altitudes.

3. *Experiments with Five Counters in Coincidence*

The experiments with three counters in coincidence have shown that when one of the counters was surrounded by five counters in anticoincidence, 90 percent of the side showers which discharged the three counters also discharged one of the anticoincidence counters, and hence failed to produce an anticoincidence. When the side counters were not used but only one counter in line was connected in anticoincidence, about half of the side showers recorded discharged the single anticoincidence counter. If five counters are used in coincidence instead of three, the side showers which discharge the array will be much fewer and, in general, will contain more particles. Such a shower would have a larger probability of discharging the anticoincidence counters. Therefore when five counters are used in coincidence, the set of five anticoincidence counters surrounding one coincidence counter should be discharged by practically all of the showers recorded, and we should obtain no anticoincidences due to showers. This conclusion is verified in the first experiment reported in this paper, where all of the anticoincidences could be accounted for by the dead-time inefficiency. Moreover, if the side counters are disconnected and only one counter (in line with the coincidence array) is used in anticoincidence, *more than half* of the side showers recorded will discharge the single anticoincidence counter. Thus the number of side showers which discharge the five coincidence counters is several times the number of anticoincidences recorded due to the showers.

(*a*) *Results with experimental arrangement shown in Fig. 1.*—The results are given in Table IX. The difference between the two anticoincidence rates should represent the number of showers which discharge the five counters in coincidence and fail to discharge counter 3.

It is seen that the number of anticoincidences due to showers is about as large as the number due to the dead-time inefficiency; the number of fivefold coincidences due to showers must be several times as large.

(*b*) *Results with experimental arrangement* *shown in Fig. 7.*—The results are given in Tables X and XI. The anticoincidences occurring at Ithaca because of the dead-time inefficiency should be about 0.2 percent of the coincidence rates. There should be no anticoincidences due to scattering because the lead absorber had the same width as the counters. The anticoincidences arising from chance coincidences between the coincidence counters are quite negligible. The effect of inefficiency near the edge of the anticoincidence counters might amount to 0.1 percent of the coincidence rate. The remainder of the anticoincidences should be due to showers discharging the anticoincidence counter. Thus we see that even at 259-m altitude the major part of the effect (about 70 percent) is due to the side showers.

At higher elevations the effect of inefficiency near the edge of the anticoincidence counter should remain the same fraction of the coincidence rates as at Ithaca. The inefficiency due to the dead time should go up in the same ratio as the counting rates. But the effect of showers increases with altitude much faster than does the counting rate. Therefore at the higher elevations the fraction of the anticoincidences which is due to showers is even larger than at Ithaca. The anticoincidence rates may thus be taken as a proportional measure of the number of showers which discharge the counter array. However, the number of side showers which discharge the five counters in coincidence is several times as large as the number which produce anticoincidences.

The variation of the anticoincidence rates with altitude, shown in Table X, should give a measure of the variation of the shower intensity with altitude, if the anticoincidences are caused mainly by showers. We may check this by computing the average ratio of the anticoincidence rates at the higher altitudes to the anticoincidence rates at Ithaca, and comparing these ratios with the corresponding ratios of the intensities of the soft component, measured recently at the same altitudes by one of us.[9] This is done in Table XII. We have also given in Table XII similar ratios for the total intensity of all particles which discharge a counter telescope of six counters (taken from the same reference). The comparison verifies the fact that the major part of the anticoincidences, both when five counters and when two counters were in coincidence, was due to some agent (showers) which increases in intensity with altitude in the same way as the electron intensity.

TABLE XII. Evidence that anticoincidences are mostly due to the soft component of cosmic rays. The second and third columns refer to data taken with the experimental arrangement shown in Fig. 7 (see Tables V, X, XI). The last two columns give the corresponding ratios for the intensity of the soft component and for the total intensity.

Elevation (meters)	Average ratio of anticoinc. rates, with 5 counters (1,3,4,5,6) in coinc.	Average ratio of anticoinc. rates, with 2 counters (1,6) in coinc.	Ratio of intensities of the soft component	Ratio of total intensities
259	1.0	1.0	1.0	1.0
1616	1.6	1.8	1.9	1.4
3240	3.5	3.9	4.4	2.3
4300	6.1	6.9	8.2	3.2

An interesting fact, noted in Table XI, is that not only the anticoincidence rates but also the ratio of anticoincidences to coincidences decreases when the lead absorber is put in place, by an average factor of 1.4. This may be attributed both to the absorption of shower particles and to the multiplication of the shower particles in the lead. That is, the lead prevents a shower of only a few low energy particles from producing a coincidence. Moreover, when a shower of high energy particles, or of a large number of particles, strikes the apparatus, the multiplication in the lead makes it very unlikely that the anticoincidence counter will not be discharged. Thus fewer showers produce coincidences, and a smaller fraction of these produce anticoincidences, when lead is used. The distribution of the 13 cm of lead in layers 2 cm thick between the counters assists to make this multiplication effect pronounced, because 2 cm of lead is nearly the optimum thickness for shower production.

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