these 71 mesons, it cannot be ascertained whether or not they penetrated the tungsten plate. Of the remaining 530 pictures, 131 were taken with 2.54 cm of Pb above the chamber and 399 were taken with 7.62 cm of Pb above the chamber. Six and 27 of these, respectively, were stopped in the 3.81 cm of tungsten, equivalent to 6.8 cm of Pb. The use of the momentum *vs.* range curves of Rossi and Greisen[26] then gives the following results for sea level mesons: $33/530 = 6.2$ percent of the penetrating component with momentum between 1 and 3×10^8 ev/c. This is in good agreement with the meson spectrum obtained at sea level by Hughes.[27]

The author deeply appreciates the assistance of Dean A. H. Compton, who has made possible this work. Professor Auger has been of great value in interpreting the experimental results. Messrs. O'Donnel and Kahlberg of the machine shops were of great help in building the apparatus.

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[26] See page 249 of reference 11.

[27] D. J. Hughes, Phys. Rev. **57**, 592 (1940).

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A Precision Determination of h/e by Means of the Short Wave-Length Limit of the Continuous X-Ray Spectrum at 20 kv

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A redetermination of h/e by the method of the short wave-length limit of the continuous x-ray spectrum was undertaken in order to obtain further evidence as to the discrepancy among the determinations of atomic constants. The accuracy of the experiment is improved by (1) large primary x-ray intensity, (2) the use of a high resolving power double crystal spectrometer, (3) reduction in background and improvement in the sharpness of the limit by the use of balanced filters, (4) improved accuracy in voltage measurements, and (5) cleaning the target in vacuum. The sources of error both in the measurements and the interpretation of the data are discussed. Our result is $h/e = (1.3786\pm0.0002)\times10^{-17}$ erg sec./e.s.u. This value is found to be in fair, though not complete, agreement with measurements of other atomic constants.

I. THE PRESENT STATE OF OUR KNOWLEDGE OF THE VALUE OF h/e

EVER since absolute x-ray wave-length measurements and the revision of the value of the viscosity of air caused the accepted value of the electronic charge to be raised from the original Millikan value to the neighborhood of 4.8×10^{-10} e.s.u., there appeared a new discrepancy among the values of the natural constants.[1] This discrepancy arises from the fact that values of h/e derived from methods permitting a direct measurement of this quantity failed to agree satisfactorily with the "indirect" value of h/e, i.e., the value derived from other measurements of functions of the atomic constants e, m, and h. The present state of this discrepancy has been thoroughly discussed by Birge,[2,3] Dunnington,[4] Kirchner,[5] and DuMond.[6,7]

The methods for a direct measurement of h/e available at the present time, are: (1) the determination of the short wave-length limit of the continuous x-ray spectrum; (2) the photoelectric effect; (3) the determination of excitation and ionization potentials; (4) the determination of the excitation voltages of x-ray series as compared to the corresponding absorption edges; (5) measurement of the radiation constant c_2. Of these methods at present the first stands out

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[1] R. T. Birge, Phys. Rev. **48**, 918 (1935).

[2,3] R. T. Birge, *Reports on Progress in Physics*, London Physical Society (in print); R. T. Birge, Rev. Mod. Phys. **13**, 233 (1941).

[4] F. G. Dunnington, Rev. Mod. Phys. **11**, 65 (1939).

[5] Kirchner, "Die Atomaren Konstanten *e*, *m*, und *h*." Ergeb. d. exakten Naturwiss. (1939).

[6,7] J. W. M. DuMond, Phys. Rev. **56**, 153 (1939); J. W. M. DuMond Phys. Rev. **58**, 457 (1940).

in accuracy above the others and therefore it is the deviation of these measurements from the indirect value which causes most concern. For convenience the values of h/e as obtained by previous investigators are tabulated here:

1.3800—Ohlin[8] (corrected for cathode work function by Birge[9])
1.3775—Schwarz and Bearden[10] (mean value of Cu and W values)
1.3775—Schaitberger (re-computed by DuMond[7])
1.3765—DuMond and Bollman[11] (re-computed by DuMond[7])
1.3759—Feder[12] (re-computed by DuMond[7])
1.3754—Kirkpatrick and Ross[13] (re-computed by DuMond[7])
1.3749—Duane, Palmer, and Yeh[14] (re-computed by DuMond[7]).

As compared to these results Birge[3] computes as the most probable *indirect* value

$$h/e=(1.3793_3\pm 0.0002_3)\times 10^{-17} \text{ erg sec./e.s.u.}$$

It is seen from the above table that the only measurement in substantial agreement with the indirect value is the measurement of Ohlin. All the other measurements lie from 0.15 percent to 0.3 percent lower than the indirect value.

II. THE ISOCHROMAT METHOD

In the present experiment, as in the past ones listed above, the method of isochromats was used in locating the short wave-length limit. This method consists ideally in monochromatizing an x-ray beam and measuring the intensity in the monochromatic beam as a function of the voltage applied to the x-ray tube. The resulting curve is called an isochromat. In practice these ideal conditions can of course not be realized owing to the fact that (a) the "monochromatization" will actually consist of the production of an x-ray beam whose wave-length distribution has a sharp peak at a particular wave-length with the intensity decaying asymptotically on either side, and (b) since an electron beam perfectly homogeneous as to energy cannot be produced. The effect of these two deviations from an ideal isochromat condition is the same: both causes make an actually observed isochromat appear as a weighted superposition of ideal isochromats with voltage coordinates slightly shifted; if the wave-length of an ideal isochromat is changed by an amount $\Delta\lambda$, this change will cause the isochromat to shift along the voltage axis by an amount given by $\Delta V=-hce^{-1}\lambda^{-2}\Delta\lambda$. Let us analyze mathematically the effect of such a deviation from ideal conditions.

Let $y=f(z)$ be the equation of an ideal isochromat where z stands for the variable proportional to the x-ray tube voltage and y for the variable proportional to the x-ray intensity in the monochromatic beam. Let $F(z)$ be the experimentally observed function. Let us choose $z=0$ to be the short wave-length limit of the ideal isochromat; i.e., let $f(z)=0$ for $z\leqslant 0$. $F(z)$ is then formed by a weighted superposition of functions $f(z)$ whose independent variable differs by an amount x from the independent variable of the observed isochromat. The weights given to these various ideal components will then be a function of x only. Let this function be $g(x)$. We then have

$$F(z)=\int_{x=-\infty}^{x=z} g(x)f(z-x)dx, \tag{1}$$

the upper limit being $x=z$ since $f(z-x)=0$ for $x\geqslant z$. This equation theoretically permits solution for the function $f(z)$, provided the functions $F(z)$ and $g(x)$ are known for all values of their argument. Practically such a reversion is, however, not possible since the functions $F(z)$ and $g(x)$ are not known for large values of the variable. We must, however, remember that a solution for the complete ideal isochromat $f(z)$ is not necessary, but that only a location of the point at which the argument of $f(z)$ is zero is necessary. The location of this point can be effected as follows: let us examine the second derivative of

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[8] Per Ohlin, Dissertation, Uppsala (1941); Per Ohlin, "Meddelande Fran Fysika Institutionen," Uppsala 12–12–1939.

[9] Private communication.

[10] Bearden and Schwarz, Bull. Am. Phys. Soc., May 1–3 (1941).

[11] J. DuMond and V. L. Bollman, Phys. Rev. **51**, 400 (1937).

[12] H. Feder, Ann. d. Physik [5] **1**, 497 (1929).

[13] P. Kirkpatrick and P. A. Ross, Phys. Rev. **45**, 454 (1934).

[14] Duane, Palmer, and Yeh, J. Opt. Soc. Am. **5**, 376 (1921).

the observed isochromat. From Eq. (1):

$$F''(z)=g(z)f'(0)+\int_{-\infty}^{z}g(x)f''(z-x)dx$$
$$=g(z)f'(0)+\int_{0}^{\infty}g(z-u)f''(u)du. \qquad (2)$$

Let us now assume that the ideal isochromat can be composed of straight line sections, i.e., let

$$f'(x)=a_i, \quad x_{i-1}<x<x_i, \quad i=1\cdots\infty. \qquad (3)^*$$

Equation (2) then reduces to

$$F''(z)=g(z)f'(0)+\sum_{i=1}^{\infty}g(z-x_i)(a_{i+1}-a_i). \qquad (4)$$

This equation shows that $F''(z)$ is composed of two terms, namely (a) the term $g(z)f'(0)$ representing the influence of the intercept at the origin of the ideal isochromat on the curvature of the real isochromat, and (b) the term

$$\sum_{i=1}^{\infty}g(z-x_i)(a_{i+1}-a_i)$$

which represents the influence of the breaks in slope at the points x_i on the curvature of the observed isochromat. In the case in which the term (b) is negligible we see therefore that the second derivative of the observed isochromat will reach a maximum at the same point at which $g(x)$ reaches a maximum; if therefore our voltage and wave-lengths are measured to the peak of their distribution curves, then *the second derivative* or "bending" (and hence approximately the curvature) of the observed isochromat will reach a maximum at the short wave-length limit of the ideal isochromat corresponding to these voltages. In discussing our actual results we shall approximately compute the term (b) and we shall show that it is negligible. For purposes of locating the threshold of the continuous spectrum it is therefore sufficient to determine the point of maximum bending of a physical isochromat.

It follows from the above analysis that a location of the threshold by extending the straight line portion of the isochromat onto the voltage axis is not permissible; in fact it can be shown that the choice of the point whence to project the tangent onto the axis can arbitrarily modify the value of the intercept. The use of the point of maximum bending for the location of the "true" intercept was first suggested by DuMond and Bollman.[11]

III. EXPERIMENTAL ARRANGEMENTS

(1) The X-Ray Tube

The precision attainable in this experiment is principally limited by the intensity available near the short wave-length limit; it was lack of sufficient primary intensity which forced former investigators[11] to decrease the resolving power of the monochromator to a serious extent. In this experiment our primary x-ray intensity was derived from the Watters Memorial x-ray tube and power supply, whose construction is described elsewhere.[15] In these experiments the tube current was maintained at 100 ma; this corresponds to an intensity five times larger than that in former experiments at this voltage.

(a) Discussion of Target Deposits

A serious problem in the performance of this experiment is the question of target deposits. A very thin layer of low atomic number material on the x-ray tube target will produce an appreciable energy loss of the electrons impinging on the target; owing to the low x-ray production efficiency in low atomic number materials the x-rays produced in such layers might easily escape observation. Low atomic number materials deposited on the target would therefore tend to cause an apparent shift in the observed isochromat toward higher voltages and therefore an apparent higher value of h/e. Preliminary experiments definitely established both the production of deposits on the target probably partly derived from the filament and partly derived from the

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* The assumption involved in Eq. (3) does not involve any significant restriction. It has been found by all earlier observers that an isochromat is generally composed of portions of practically constant slope, joined by short portions of the curve of sharp curvature, the so-called "knees." The assumption made in Eq. (3) is therefore in agreement with experiment. Part of the purpose of this analysis is to investigate the influence of these knees on the location of the threshold. Note that the spectra discussed here are "thick" target spectra. The x-radiation from a "thick" target has components originating at varying depths within the target. A thick target spectrum is therefore essentially a superposition of thin target spectra corresponding to varying electron energies.

[15] J. DuMond and J. P. Youtz, Rev. Sci. Inst. 8, 291 (1937).

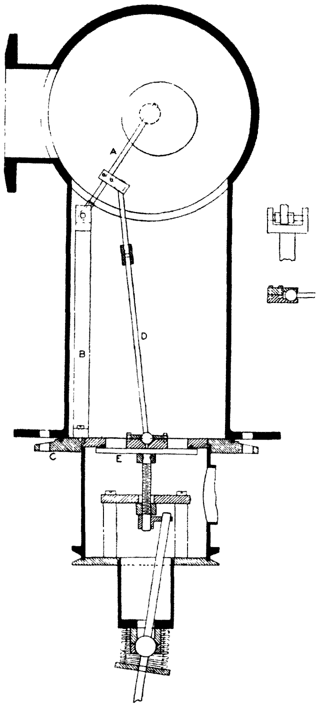

FIG. 1. Transverse cross section of x-ray tube looking at face of target from cathode end showing mechanism by which the target can be cleaned in vacuum. A steel wire brush is mounted on the arm A and is driven through the sylphon valve at the bottom. The target is outlined in one particular position (solid circle) and the path of the center of the target when gyrating is indicated (dotted circle).

electron beam; the latter deposits probably contain carbon as their principal component. In order to eliminate or at least retard the formation of these deposits the pumping system of the x-ray tube was redesigned to avoid diffusion of oil vapor into the x-ray tube and also to permit higher pumping speed.

(*b*) *Vacuum Target Cleaning Device*

Despite these measures the formation of deposits still proved a serious obstacle to the reliability of our measurements. As a solution to this problem a device was therefore installed in the x-ray tube to permit cleaning the target in vacuum (Fig. 1). An arm A is mounted on an upright post attached to the stationary plate C of a sylphon valve at the bottom of the x-ray tube. This arm is driven through an adjustable rod D from the moveable plate E of the valve. This rod has two $\frac{1}{2}''$ balls silver-soldered into its ends which engage cups in the moveable plate E and the arm A, respectively. Figure 1 shows both the outline of the target and the path (dotted line) described by its center as it is gyrated by means of the mechanism designed by DuMond and Youtz. The arm carries a brush facing the target; this brush is made of piano wire sections soldered into a brass ring. Cleaning of the target is effected by closing the valve as shown; the levers are so adjusted that the brush is now located between the cathode and anode just in front of the filament. The target is then gyrated; the brush will then clean exactly that area which the electron beam strikes during regular operation of the tube. The target is made of copper with a thin layer of gold deposited on its surface by an amalgamation process. A small area in the center of the target is silverplated for the production of the silver K lines for calibration purposes.

In order to obtain the Ag $K\alpha$ calibration lines the (ordinarily eccentric) target can be centered by external means to cause the electron beam to strike the silverplated center.

(*c*) *Effect of Internal Target Cleaning*

The effect of this device on the readings is shown in Fig. 2. Curve A on this figure is an isochromat obtained after operating the tube for a long time; curve B is an isochromat observed under identical conditions as curve A excepting for the fact that the "scratching" device was operated every two or three points. It is seen that in this extreme case a shift of nearly 1 percent is observed in the abscissa of each point, corresponding to an equal fractional change in the value of h/e. It is also seen that at least with the detection sensitivity used here, the radiation from the carbon is not observed.

The bearing of the existence of this effect on the former work on this subject is naturally difficult to infer. In particular the rate of formation of deposits of this nature is of course widely different in various investigations. In particular in regard to the work of Ohlin[8] the question of target deposits deserves serious consideration. At the voltages used in Ohlin's experiment a few molecular layers of low atomic number materials deposited on the target are sufficient to cause an error of several parts in ten thousand; Ohlin's apparatus is evacuated by means of an oil pump without the use of a refrigerating trap; the presence of organic vapors in the tube is therefore not unlikely.

(*d*) *Discussion of the Effect of the Pressure in the X-Ray Tube*

Another question which we might discuss here is that of the error introduced into an h/e determination by the presence of secondary electrons in the electron beam which have been released from the cathode by positive ion impact. Ohlin[8] describes an effect of pressure in the x-ray tube on the shape and position of his observed isochromats. Ohlin shows that in his experiment the sharpness of the short wave-length limit intercept is materially decreased as the pressure in the x-ray tube is increased; in his experimental curves this "blurring" is also accompanied by a very large rise in intensity. Ohlin tentatively ascribed this effect to the x-rays produced by the secondary electrons in the electron beam; these electrons will, of course, have larger energies than the thermionic electrons from the filament. Ohlin tried to measure the positive ion current in the tube directly and found it much too small to account for the observed effect. It seems therefore unlikely to ascribe the observed effect to the cause mentioned. In particular owing to the small energy of such secondaries[16] their influence on the value of h/e as measured at higher voltages ought to be entirely negligible. A possible explanation of the phenomena observed by Ohlin might be the fact that Ohlin used a vacuum curved crystal spectrometer in his investigation; the vacuum in the spectrometer was common to the x-ray tube. The x-ray beam is monochromatized by reflecting it off a curved crystal and then passing it through a narrow slit. Any scattering in front of this slit (e.g., by residual air in the poor vacuum case) would materially decrease the resolving power of this method of monochromatization; this decreased resolving power would both cause a blurring of the limit and also the greatly increased intensity, which Ohlin actually observed.

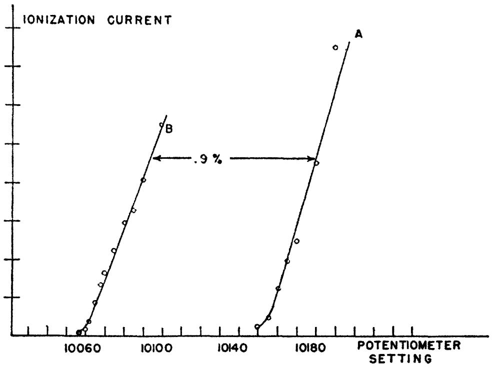

FIG. 2. Two isochromats showing the effect of the internal target cleaner. Curve *A* is observed with a highly contaminated target while curve *B* is observed with the target cleaner in operation.

(2) **The High Voltage D. C. Power Supply**

The power supply used in this experiment remains practically as originally described.[15] Since these experiments were performed at relatively low voltages only half of the voltage doubling circuit described in the earlier paper is used. The power supply is designed for maximum stability of x-ray tube voltage. Nevertheless, principally owing to instability in filament emission, considerable voltage fluctuations remain. In order to smooth out these remaining fluctuations a voltage regulator of the degenerative type was designed to operate in the high voltage line. This regulator was built in the summer of 1939 and has been in satisfactory operation since then. It is of a similar design to the circuit recently published by Parratt and Trischka.[17] The circuit of the regulator is shown in Fig. 3. Qualitatively it operates as follows: the voltage divider R_1-R_0 divides the voltage across the x-ray tube in the ratio 1 : 400; any changes in this voltage are amplified by a 3-stage d.c. coupled voltage amplifier of approximately 200,000 total gain. This amplifier feeds the amplified voltage fluctuation back into the line in reverse phase. It can be seen by a simple analysis that any incoming voltage fluctuations will be reduced in the ratio 1 : 500 approximately, provided that the magnitude of the incoming fluctuations does not exceed the allowable operating range of the power stage (3 6L6 tubes operated in parallel). The resistor R_8 protects the device against overload since at

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[16] L. Linford, Phys. Rev. **47,** 279 (1938); Hill, Buechner, Clark, and Fisk, Phys. Rev. **55,** 463 (1939).

[17] L. G. Parratt and J. W. Trischka, Rev. Sci. Inst. **13,** 17 (1942).

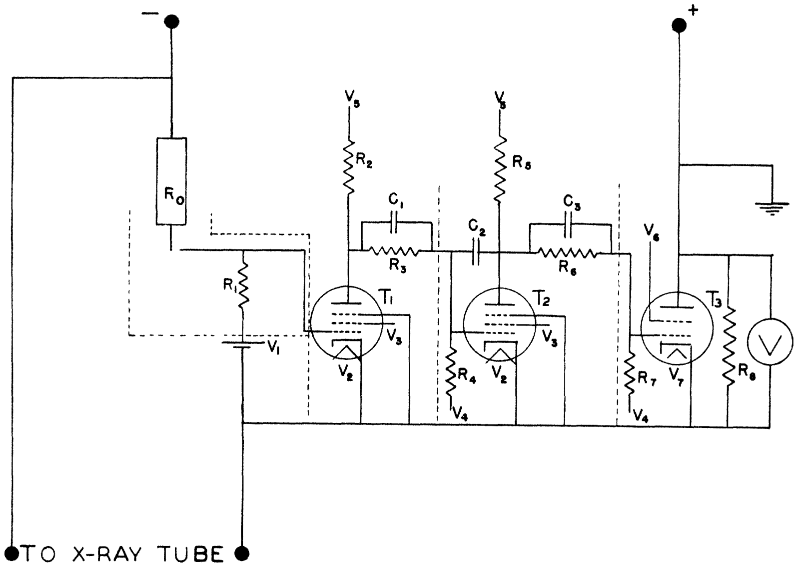

FIG. 3. Diagram of electronic voltage stabilizer: T_1, T_2—1852; T_3—3 6L6G in parallel; V_1—0–180 v variable bias voltage (battery); V_2—6 v (storage battery); V_3—45 v (battery); V_4—-90 v (battery); V_5—360 v (battery); V_6—250 v (regulated power supply); V_7—6.3 v (transformer); R_0—400×10^6 (Type MVR); R_1, R_6—1 meg; R_2, R_3, R_4, R_5, R_7—0.5 meg; R_8—7500, 100 watt; C_1, C_3—1000 $\mu\mu f$; C_2—adjust for best performance.

100-ma load current the drop across the power-stage cannot exceed 750 volts. The condenser C_2 was found to be necessary to suppress phase shift oscillations. Constant voltage is maintained by adjusting the variable bias battery V_1 to the required value; as long as the applied voltage is held (by external manual control of the generator field) so that the drop across the regulator does not vary more than 100 volts, the output voltage was found at an operating voltage of 20 kv to be constant to ±1 part in 50,000 for long periods of time.

(3) The Filament Supply

The 30-ampere, 5-volt filament of the x-ray tube is supplied by a set of large storage batteries constantly charged by a set of generators driven by a motor at ground potential through an insulating belt. Owing to the large internal impedance of the high voltage supply it is necessary to adjust the filament current within very close limits; also all sliding contacts in the filament circuit must be avoided in order to assure stability. The current was therefore controlled by means of a monel tubing mercury rheostat: A vertical tube of monel 0.008″ in wall thickness is jacketed with an oil bath and placed in series with the x-ray tube filament; the mercury can be raised and lowered inside this tube by means of a tubulated flask connected to the lower end of the tube through Koroseal tubing. This arrangement permits a total resistance variation of about 0.1 Ω.

(4) The Monochromator

(a) *The Spectrometer*

Monochromatization was effected by means of the two-crystal spectrometer described by DuMond and Marlow.[18] Two calcite crystals were cleaved out of a single block and were mounted and aligned according to the optical method of DuMond and Hoyt.[19] The crystals remained entirely untreated; i.e., they were neither ground nor etched. A rocking curve in the parallel position of the spectrometer showed a full width at half maximum of 10.5″ at 20 kv; this corresponds to a width of about 11 volts.

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[18] J. DuMond and D. Marlow, Rev. Sci. Inst. **8**, 112 (1937).

[19] J. DuMond and A. Hoyt, Phys. Rev. **36**, 1702 (1930).

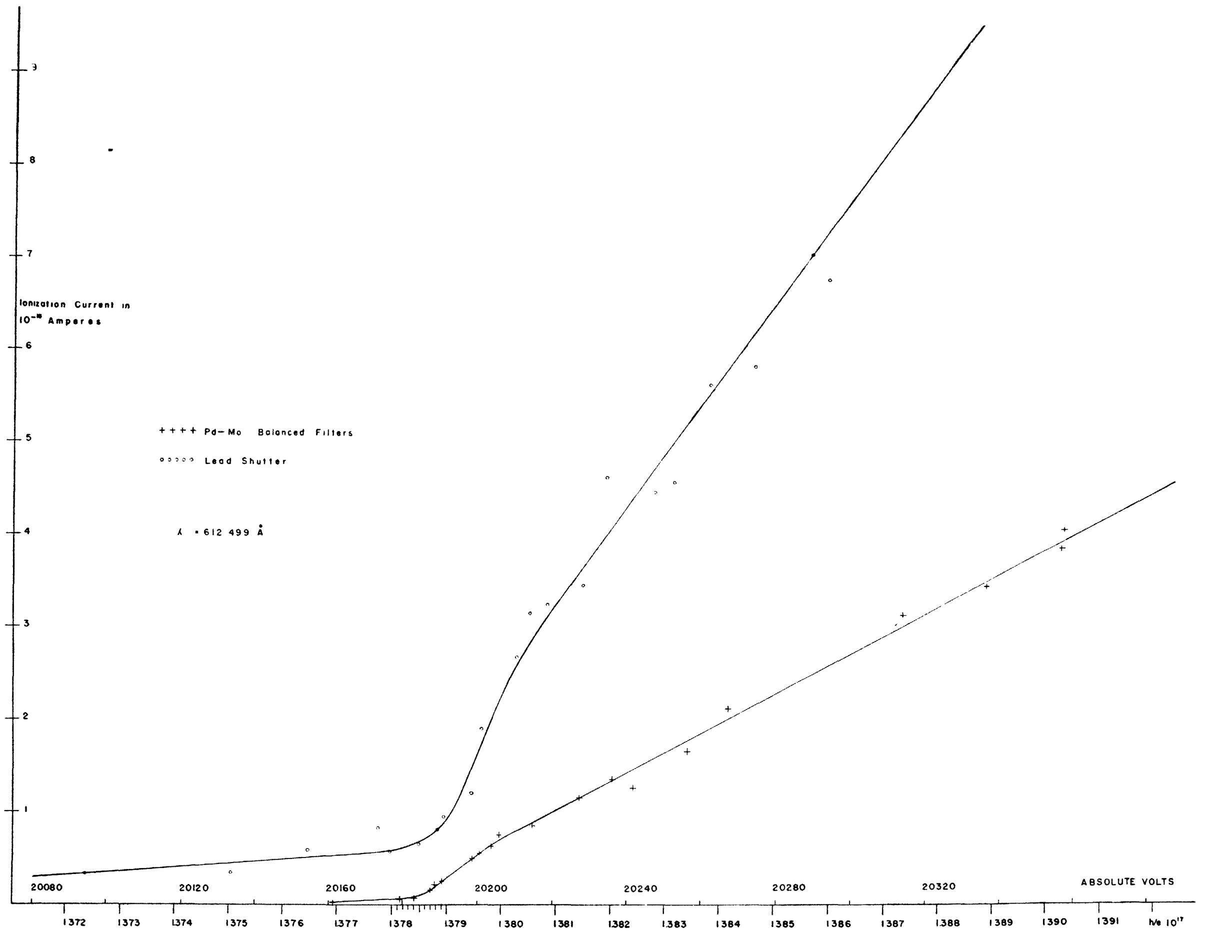

FIG. 4. Isochromats observed at $\lambda = 612.499$A. The upper curve is observed using a lead shutter for obtaining difference readings; the lower curve is obtained by the use of a set of Pd-Mo balanced filter. The corrected x-ray tube voltage is absolute volts and the computed h/e values are plotted on the abscissa.

(b) *Calibration of the Spectrometer*

For reasons described below, it was not feasible to choose a monochromatization wave-length at the peak of some well-known spectral line. It was therefore necessary to standardize the spectrometer at a well-known wave-length chosen near to the desired wave-length and then to compute the actual wave-length from the small angular shift (about 30′) of the crystals away from the standard position. The wave-length chosen for reference was the Ag $K\alpha_1$ line whose wave-length has been carefully investigated by numerous workers.[20] In our computation we adopted the value of Elg[21] (λ Ag $K\alpha_1 = 558.231$ X.U.);[*] the wave-length chosen for monochromatization was $\lambda = 612.499$A.

(c) *The Ion Chamber and Ion Current Amplifier*

The x-ray intensity was measured by means of a methyl bromide filled ionization chamber and a Western Electric electrometer tube No. D96475, operated in the Barth[22] circuit as described by Penick.[23] The circuit is operated with a 5×10^{11} ohm grid shunt; its over-all sensitivity is 4×10^{-17} amp./mm.

(d) *The Use of Balanced Filters*

A considerable number of lead stops and conduits were used in order to reduce scattering. These stops were located by means of a careful photographic survey of the position of the beam and the scattered radiation. In particular it was found necessary to place a stop directly after the second crystal in order to block tangential noncoherently scattered radiation from the crystal face.

As was mentioned above, an isochromat as actually observed will deviate to a considerable extent from the ideal shape partly owing to the effect of imperfect monochromatization. This imperfection is partly caused by the finite width of the crystal diffraction pattern and partly by incoherent scattering in the monochromator. The incoherent scattering causes a non-monochromatized x-ray leakage through the monochromator. Since the scattering is dependent on the x-ray wave-length, it will contribute to the isochromat a strongly voltage dependent background. Both these causes will contribute a "tail" to the isochromat of only slow convergence; it is the presence of this tail or "fillet" which caused difficulty in the interpretation of the results of former investigators. In this work it was attempted to effect a further reduction of this fillet by the use of a set of Ross[24] balanced filters. As is well known, a Ross filter consists of a set of two foils of different atomic number; the effective thicknesses of the foils (adjusted by inclination to the x-ray beam) is such that the absorption of the two foils is identical both at wave-lengths shorter than the K-absorption edge of the heavier material and at wave-lengths longer than at the K edge of the lighter material. In practice these two conditions cannot be precisely fulfilled simultaneously with two homogeneous foils.

Let us now assume that the pass band of the spectrometer is located just on the short wave-length side of the K edge of the lighter material constituting the balanced filter (see Appendix). Let us assume that the filters are perfectly balanced on the long wave-length side but not on the short wave-length side. Now consider the change in intensity as one filter is substituted for the other in the x-ray beam. Let us assume that the voltage is close to the short wave-length

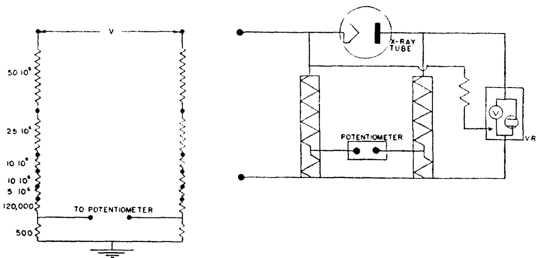

FIG. 5. Diagram of voltage dividers and connections.

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[20] Kellstrom, Zeits. f. Physik **41**, 516 (1927); C. D. Cooksey and D. Cooksey, Phys. Rev. **36**, 85 (1930); J. A. Bearden, Phys. Rev. **43**, 92 (1933); V. Zeipel, Arkiv f. Mat. Astr. och Fysik **25A**, No. 8 (1935); G. Berger, as quoted by Ingelstam; Ingelstam, Nova Acta Reg. Soc. Scien. Ups. [4] **10**, No. 5 (1936); S. Elg, Zeits. f. Physik **106**, 315 (1937).

[21] S. Elg, reference 20.

[*] The wave-length is expressed on the Siegbahn scale of wave-lengths in which the effective grating space of calcite in the first order of reflection is taken as 3029.04 X.U. at 18°C. Wave-lengths expressed in these units must be multiplied by $\lambda g/\lambda s = 1.00203_4$ in order to be reduced to absolute c.g.s. units.

[22] Barth, Zeits. f. Physik **87**, 399 (1934).

[23] D. B. Penick, Rev. Sci. Inst. **6**, 115 (1935).

[24] P. A. Ross, J. Opt. Soc. Am. and Rev. Sci. Inst. **16**, 433 (1928).

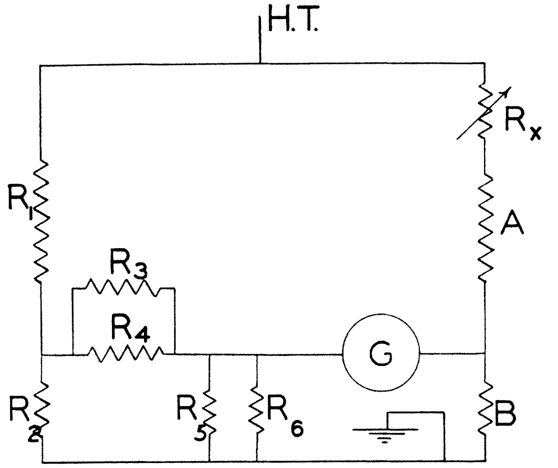

FIG. 6. Bridge circuit for a precision determination of the resistance ratio A/B. The nominal values of the resistances are:

R_1—10^6 ohms R_3—10^6 ohms R_5—5000 ohms
R_2—100 ohms R_4—50,000 ohms R_6—5000 ohms
R_x—0–10^6 ohms (variable).

The ratio R_1/R_2 is known to a high degree of precision.

limit. Since there is no primary intensity at wave-lengths shorter than the limit, the slight unbalance of the filters on the short wave-length side will not contribute any change in intensity. The entire intensity shift must therefore derive from the wave-length band contained between the short wave-length limit and the K edge of the lighter metal of the filter pair. The action of the balanced filter set is therefore to "clip off" the pass band of the spectrometer a small wave-length interval (the length of the interval will depend on the angular setting of the crystals) beyond the center of the pass band on the long wave-length side. The "fillet" from imperfect monochromatization will therefore be materially reduced. The background from incoherent scattering will also be largely eliminated.† The overall intensity is decreased by a factor of about 2.5 if the thickness of the filters is correctly chosen. Figure 4 shows two isochromats observed in one case using a set of balanced filters (Mo–Pd) and in the other case simply a lead shutter; the spectrometer is set at a wave-length differing by about 6 X.U. from the Mo K edge. The effective suppression of the "tail" and background is evident from the figure.

IV. VOLTAGE MEASUREMENT

(1) Description of the Devices used for Voltage Measurement

The voltage applied to the x-ray tube was measured by means of a voltage divider and potentiometer arrangement as shown in Fig. 5. The divider consists of two $100\times10^6 : 500\Omega$ resistance ratios immersed in oil. Their construction is described in an earlier paper.[15] It will be noted that the two resistance ratios measure very unequal voltages; hence in order to have the potentiometer actually measure a voltage proportional to the total x-ray tube voltage, it is essential that the two ratios be equal, to a sufficiently high precision. It can be shown that for the ratios used here the error due to this cause cannot exceed 1.6×10^{-5}. It can also be shown that to an accuracy of 1 in 10^5 the dividing ratio of the divider which has the larger part of the x-ray tube voltage across it can be used in the computation. The problem of voltage measurement therefore reduces essentially to establishing an accurate ratio of order 1 : 200,000. By the method of calibration shown below, it became evident that the stability of the high resistance unit was not too satisfactory; the ratio would change by several parts in 10,000 during a day's run. In order to avoid introducing error into the final result due to this cause it was decided to adopt a method of calibration which was sufficiently rapid so the voltage divider ratio could be checked in a few minutes during a run.

(2) Voltmeter Calibration

Through the kind cooperation of Dr. Frank Wenner of the Bureau of Standards we had available two high quality manganin resistance boxes, made by Otto Wolff, Berlin. One of these contains 10 sections of $100{,}000\Omega$ each and the other one contains 10 sections of 1000Ω each. None of the sections differs by more than 3×10^{-4} from their mean value. Dr. Wenner suggested a method by which these resistance standards could be used to build up an accurately known resistance ratio of nominal value 1 : 10^4; we shall briefly describe the method here. Let

R_s = resistance of the 10^6 box with its units series connected,

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† See Appendix.

R_p = resistance of the 10^6 box with its units parallel connected,

r_s = resistance of the 10^4 box with its units series connected,

r_p = resistance of the 10^4 box with its units parallel connected.

It can then be shown[25] that

$$R_s/r_p=(10^4R_p/r_s)\{1+0(\delta^2)\},$$

where δ is the deviation of the resistance sections from their mean. Hence to an accuracy of 10^{-7} we have that

$$R_s/r_p=10^4R_p/r_s.$$

Since $R_p/r_s \sim 1$ we can measure this ratio to very high precision in a Wheatstone bridge by using a method of interchanges of R_p and r_s. It was thus possible to measure R_p/r_s to a precision of better than one part in 10^6. Great care was taken to eliminate errors due to contact resistances and crossleakages. After having thus established an accurate 10^4 ratio, these resistances ($R_s=R_1$; $r_p=R_2$) were connected with other manganin standards as shown in Fig. 6. The nominal value of the resistances R_3, R_4, R_5, and R_6 are $10^6\Omega$, $50{,}000\Omega$, 5000Ω, and 5000Ω, respectively. These values were determined to an accuracy of 1 in 10^6 in terms of sections of the 10^6 or $10^4\Omega$ boxes, by using in all cases an interchange method on an equal arm bridge. The bridge circuit shown in Fig. 6 was then balanced by adjusting the resistance R_x. The equation of balance is

$$\frac{A+B+R_x}{B}=\frac{R_1+\rho_1}{\rho_1}\cdot\frac{\rho_3+\rho_2}{\rho_3},$$

where

$$\rho_2=R_3R_4/(R_3+R_4),$$

$$\rho_3=R_5R_6/(R_5+R_6),$$

$$\rho_1=R_2(\rho_2+\rho_3)/(R_2+\rho_2+\rho_3).$$

Owing to the fact that the power dissipation of the 10^6 ohm box is limited to 100 watts (the $10^6\Omega$ box consists of 40 coils through each of which oil is circulated by Bakelite tubes extending into the coils), this measurement could not be made at a voltage in excess of 10 kv. It was shown by direct measurement of the temperature coefficient of the $10^6\Omega$ unit that heating errors would amount to less than 2 in 10^5. Since the voltage divider ratio $(A+B)/B$ is needed at 20 kv, an independent measurement of the load coefficient of the voltage divider was made. This was done by determining each half of the 100×10^6 : 500 ratio (i.e., two 50×10^6 : 500 ratios) at voltages from 3 kv to 10 kv. A bridge similar to the one shown in Fig. 6 was used. Since a 20-kv load across the entire divider is equivalent to a 10-kv load across each half, the load coefficient can safely be inferred from such a measurement, in particular since it is small. The load coefficient was determined to cause a correction of -1.0×10^{-4} to the voltage ratio as observed directly at 10 kv.

A complete calibration of the divider was made between every 2 or 3 points on the isochromats shown in Fig. 4; the effect of the drift in divider ratio on the precision of the results can therefore be considered as eliminated.

(3) Ripple Measurement

The ripple was measured by placing an oscilloscope in series with a 0.2-μf condenser directly across the x-ray tube. The oscilloscope was calibrated by placing the terminals of a 150~ oscillator across the x-ray tube terminals and observing the oscilloscope deflection. The measured ripple voltage is 6 volts peak to peak. This ripple volt-

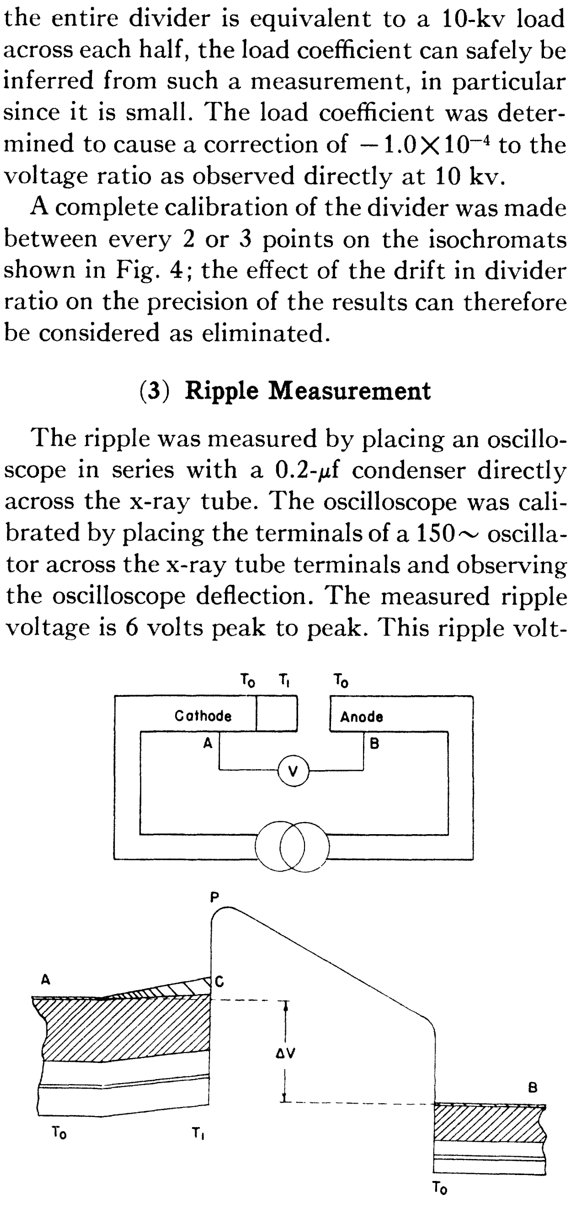

FIG. 7. Schematic diagram of the x-ray tube circuit and the corresponding energy bands of the electrons. T_0 represents room temperature while T_1 is the cathode temperature. ΔV is the voltage measured by a voltmeter attached to A and B.

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[25] Wenner, J. Res. Nat. Bur. Stand. **25**, 229 (1940) RP 1323.

age will constitute an additional component of the "smearing" function $g(x)$; it will not, however, give rise to a correction to the voltage scale.

V. INTERPRETATION OF RESULTS

(1) The Measurements

Our final isochromats are shown in Fig. 4. On the axis of abscissa are plotted values of absolute voltage and values of h/e based on the auxiliary constants: $c=2.9977_6\times 10^{10}$ cm/sec.; $d_1=3.02904\times 10^{-8}$ Siegbahn cm; $\lambda_g/\lambda_s=1.00203$; $pq=1.00034$. The points on the graph have been corrected for (1) the individual calibrations of the divider, (2) $\frac{1}{2}$ the P.D. across the x-ray tube filament, (3) the cathode work function. We estimate the point of maximum curvature of both graphs to be at

$$h/e=1.3786\times 10^{-17} \text{ erg sec./e.s.u.}$$

(2) The Work Function Correction

The corrections applied to the voltage scale have been discussed in a former paper.[11] It might be added, however, that Ohlin[8] claims that the work function correction does not rest upon a sound basis. This justifies discussing this correction again in some detail. Let Fig. 7 represent the schematic diagram of the x-ray tube circuit and the corresponding energy level diagram. A potentiometer measurement such as we have used for measuring the voltage will measure the difference in energy of the edge of the Fermi distribution of A and B, respectively. An electron in passing from A to P will have acquired the following energies: (1) Energy AC, owing to change in levels caused by Kelvin P.D., (2) energy CP, thermionic work function; (3) decrease in work function due to external field; (4) thermal energy of the electron corresponding to its electron temperature. It is these terms plus the energy $e\Delta V$ which are available for conversion into radiation at the anode. These four terms are the same energy terms which account for the so-called *calorimetric* work function; i.e., the work function measured by observing the extra heat input into a cathode, required to keep it at constant temperature when emission current is drawn. Such determinations are numerous;[26] all of them give results agreeing within a few tenths of a volt with the *thermionic* work function. Probably the most direct proof for the necessity of the work function correction is offered by the early work of Richardson and Cooke.[27] In their experiments Richardson and Cooke measured the heat input into the anode as a function of anode voltage and found that if this power be extrapolated to zero external voltage, a power input approximately corresponding to the work function remained. The precision of these experments is insufficient to distinguish between the calorimetric and thermionic work functions. We believe therefore that the cathode work function correction must be applied.

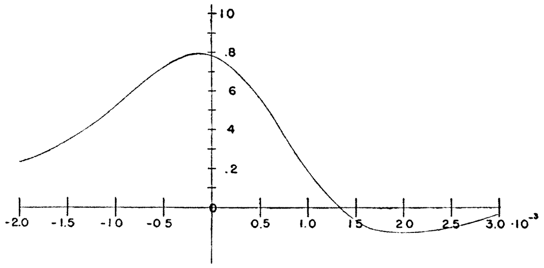

FIG. 8. Plot of the function

$$\frac{1}{1+(2/1.3\times 10^{-3})^2}-\frac{1}{2}\left\{\frac{1}{1+[(z-1.5\times 10^{-3})/1.3\times 10^{-3}]^2}\right\}$$

indicating the shift in the point of maximum bending introduced by a change in slope at $z=1.5\times 10^{-3}$ equal to one-half of the slope of the isochromat at the origin. The abscissa is expressed as fractional voltage.

(3) Discussion of Errors

Because of the lack of a generally accepted procedure for obtaining an error estimate we shall here adopt the following procedure:

(1) In cases where effective "limits" of error are available (such as in the case of N.B.S. certificates of standard cells, etc.) we shall divide these limits by 3.5‡ to obtain the "probable" error; (2) in cases were a "probable" error can be estimated (such as in the case of the interpretation of a set of scattered data), we shall combine its value directly with the values obtained from (a).

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[26] Richardson and Cooke, Phil. Mag. **20**, 173 (1910); **21**, 404 (1911); **25**, 624 (1913); **26**, 472 (1913); Lester, Phil. Mag. **31**, 197 (1916); Wilson, Proc. Nat. Acad. Sci. **3**, 426 (1917); Phys. Rev. **24**, 666 (1924); Davisson and Germer, Phys. Rev. **20**, 300 (1922); **24**, 666 (1924); Michel and Spanner, Zeits. f. Physik **35**, 395 (1928); Dushman, Rowe, Ewald, and Kidner, Phys. Rev. **25**, 338 (1925).

[27] Richardson and Cooke, Phil. Mag. **20**, 173 (1910); **21**, 404 (1911).

‡ On the basis of a Gaussian distribution the probability of an error greater than 3.5 times the probable error is 1 percent approximately.

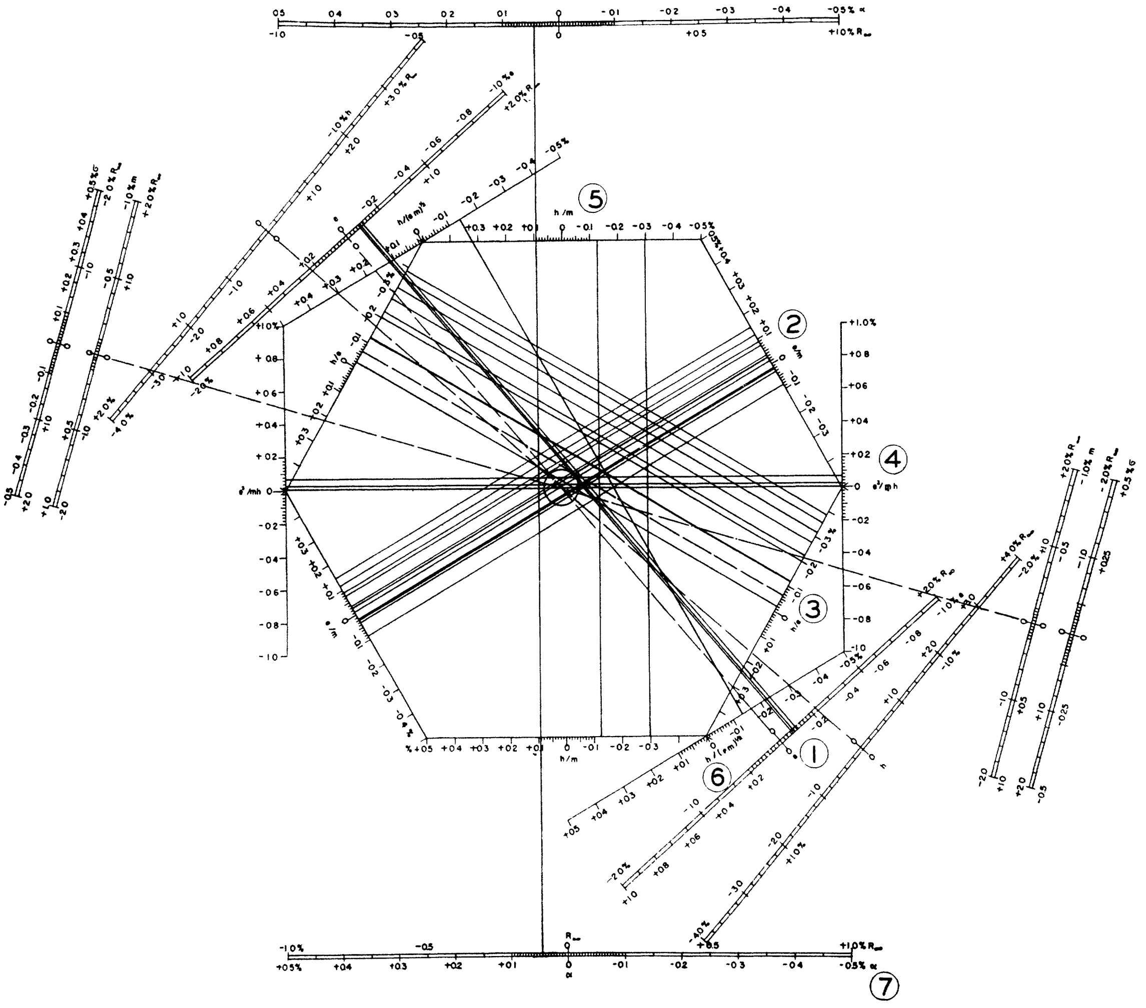

FIG. 9. Isometric consistency chart of the atomic constants. The values plotted are shown in the adjacent table (Table II).

(a). *Interpretation of Isochromat*

The probable error in locating the point of maximum curvature is estimated as 10^{-4}. The question of the validity of the maximum curvature method was checked quantitatively as follows: The term

$$\sum_{i=1}^{\infty} g(z-x_i)(a_{i+1}-a_i)$$

in Eq. (4) was computed directly. We used for $a_{i+1}-a_i$ the break in slope observed in the isochromats of Fig. 4, and assumed $g(z)$ to have the form

$$g(z)=\frac{C}{1+(z^2/a^2)}. \qquad (5)$$

The width a at half maximum was assumed to be composed of the terms: (1) The half-width of the 2-crystal spectrometer "window curve," $a_1=5\times10^{-4}$ (expressed as fractional width); (2) the ripple voltage spread, $a_2=1.5\times10^{-4}$; (3) the spread caused by "vertical divergence" of the x-ray beam in the spectrometer, $a_3=5\times10^{-4}$; (4) the spread caused by the potential drop across the filament, $a_4=1.5\times10^{-4}$. If we make the conservative assumption that these widths be additive, we obtain

$$a=13\times10^{-4}.$$

If, with the use of these parameters, $F''(z)$ is plotted (Fig. 8), a curve is obtained whose maximum is displaced by 1×10^{-4} from $z=0$. Considering the highly conservative estimate of the a parameter in Eq. (5) we can put 10^{-4} as a *limit* of error due to this cause.

Table II. Isometric consistency chart of the atomic constants.

The consistency of determinations of seven functions of the atomic constants e, h, and m by various methods is compared in Fig. 9 graphically by the method of DuMond. The origin of the chart is arbitrarily chosen as $e_0=4.80650\times10^{-10}$ e.s.u. $m_0=9.11780\times10^{-28}$ gram. $h_0=6.63428\times10^{-27}$ erg-sec. $e_0/m_0=1.75850\times10^{7}$ e.m.u./gram. $h_0/e_0=1.38028\times10^{-17}$ e.s.u. $h_0/m_0=7.27621$ c.g.s.u. $e_0^2/h_0m_0=3.81921\times10^{24}$ e.s.u. $h_0/(e_0m_0)^{\frac{1}{2}}=1.00216\times10^{-8}$ e.s.u. $R_\infty=109737$ cm^{-1}. $\alpha_0=7.29870\times10^{-3}$.

The measurements are based on the following set of auxiliary constants: $c=2.99776\times10^{10}$ cm/sec. $F=96514$ absolute coulomb/physical gram-equivalent. $p=1.00048$. $q=0.99986$. $pq=1.00034$. $\lambda g/\lambda s=1.00203$.

The values plotted are, reading in all cases from the highest to the lowest values:

(1) e	4.8030×10^{-10}	Söderman, Nature **135**, 67 (1935);
	4.8026	Bearden, Phys. Rev. **37**, 1210 (1931); **47**, 883 (1935); **48**, 385 (1935);
	4.8021	Bäcklin, Zeits. f. Physik **93**, 450 (1935).
(2) e/m	1.76110×10^{7}	Perry and Chaffee, Phys. Rev. **36**, 904 (1930);
	1.76048	Houston and Chu, Phys. Rev. **51**, 446 (1937); **55**, 423 (1939); **55**, 175 (1939);
	1.76006	Bearden, Phys. Rev. **54**, 698 (1938);
	1.75982	Dunnington, Phys. Rev. **52**, 475 (1937);
	1.75914	C. Robinson, Phys. Rev. **55**, 423A (1939);
	1.75913	Drinkwater, Richardson, and Williams, Proc. Roy. Soc. **A174**, 104 (1940);
	1.75900	Kirchner, Ann. d. Physik [5] **8**, 975 (1931); [5] **12**, 503 (1932);
	1.75870	Goedicke, Ann. d. Physik [5] **36**, 47 (1939);
	1.75820	Shaw, Phys. Rev. **54**, 193 (1938);
	1.75815	Shane and Spedding, Phys. Rev. **47**, 33 (1935);
	1.75797	R. C. Williams, Phys. Rev. **54**, 558 (1938);
	1.75700	Kinsler and Houston, Phys. Rev. **45**, 104 (1934).
(3) h/e	1.3800×10^{-17}	Ohlin, Dissertation, Uppsala (1941); Arkiv For Mat. Astr. och Fysik **27B**, No. 10 (1940).
	1.3793	(Dotted line) indirect value, Birge;
	1.3786	This research;
	1.3775	Schwarz and Bearden, Bull. Am. Phys. Soc. May 1–3, (1941);
	1.3775	Schaitberger, Ann. d. Physik [5] **24**, 84 (1935);
	1.3765	DuMond and Bollman, Phys. Rev. **51**, 400 (1937);
	1.3759	Feder, Ann. d. Physik **51**, 497 (1929);
	1.3754	Kirkpatrick and Ross, Phys. Rev. **45**, 454 (1934);
	1.3749	Duane, Palmer, and Yeh, J. Opt. Soc. Am. **5**, 376 (1921).
(4) $e^2(mh)$	3.8219×10^{34}	Robinson, Phil. Mag. **22**, 1129 (1936);
	3.8203	Robinson and Clews, Proc. Roy. Soc. **A176**, 28 (1940);
	3.8194	Kretschmar, Phys. Rev. **43**, 417 (1933).
(5) h/m	7.255	Ross and Kirkpatrick, Phys. Rev. **45**, 223 (1934);
	7.267	Gnan, Ann. d. Physik [5] **20**, 361 (1934).
(6) $h/(em)^{\frac{1}{2}}$	1.00079	von Friesen, Proc. Roy. Soc. **A160**, 424 (1937); Dissertation Uppsala (1936).
(7) α	7.3019×10^{-4}	Christy and Keller, Phys. Rev. **58**, 658 (1940).

The values of e are those calculated by Birge,[3] but corrected to correspond to the value of the Faraday given above.

The values of e/m are those calculated by Birge (Phys. Rev. **60**, 766 (1941)).

The values of h/e are those re-calculated by DuMond,[7] excepting for the value of Ohlin, which is corrected for the cathode work function by Birge (private communication).

The values of h/m are as corrected by Kirchner.[5]

The values of e^2/mh of Robinson were corrected here for the changes in auxiliary constants.

The value of e^2/mh of Kretschmar was corrected by Kretschmar.

(*b*). *Wave-Length Measurement*

The error in wave-length measurement is composed of the following terms: (1) Uncertainty in the Ag $K\alpha_1$ line wave-length 2×10^{-5} (P.E.); (2) uncertainty in $\lambda g/\lambda s$ 6×10^{-5} (P.E.); (3) uncertainty in location of Ag $K\alpha_1$ line 3×10^{-5} (P.E.); (4) accuracy of angular setting of spectrometer 7×10^{-5} (limit); (5) stability of crystal position during runs 8×10^{-5} (limit) (by direct measurement). This gives 8×10^{-5} for the probable error of wave-length measurement.**

(*c*). *Voltage Measurement*

The uncertainty in voltage measurement is composed of the following terms: (1) uncertainty in standard cell voltage 1×10^{-4} (limit); (2) uncertainty in potentiometer calibration 2×10^{-4} (limit); (3) uncertainty in calibration of auxiliary standards 1×10^{-4} (limit); (4) uncertainty in calibration of voltage divider 1×10^{-4} (limit); (5) uncertainty[3] in pq 3×10^{-5} (P.E.). This gives 8×10^{-5} as the probable error of voltage measurement.

(*d*). *Combination of Errors*

If we take the error in c^2 to be 3×10^{-5} (P.E.) (Birge[3]) and combine these estimates we obtain 1.6×10^{-4} for our total probable error. Our result therefore becomes $h/e=(1.3786\pm.0002)\times10^{-17}$ erg sec./e.s.u.

(4) Comparison of the Result with other Determinations

This result is shown in relation to other determinations of the atomic constants on the DuMond consistency chart,[6] Fig. 9. Our measurement is shown by the heavy lines in the h/e group of measurements while Birge's "indirect" value is shown as the dotted line.

Our value lies 0.07 percent lower than the indirect value but considerably higher than other "direct" determinations other than the work of Ohlin.[8] It is of course difficult to attribute causes to these differences which are considerably beyond the probable error in all cases. We might mention however that it has been found in the case of the experiment of DuMond and Bollman,[11] considerable error might have been introduced by failing to consider the temperature coefficient of the voltage divider used by them. A

** The probable errors are combined by geometrical addition.

recalibration of the 14,000 : 1 resistance ratio used by them by an improved bridge method revealed that a change of several tenths of a percent toward higher ratios (higher h/e) would occur if the dividers were loaded up to 20 kv. We can of course see no particular reason for a disagreement with other experiments excepting for the fact that these experiments described here were executed under conditions of higher resolving power of monochromatization of the x-ray beam and the careful elimination of other sources of error already described above, many of which precautions were not mentioned as taken by the other observers.

ACKNOWLEDGMENTS

In the performance of this experiment we have received generous help from many sources. In the first part of this work, notably in the construction and tests of the voltage divider a great deal of the work was performed by Dr. Howland H. Bailey. We are also greatly indebted to Dr. Frank Wenner for the kind loan of resistance standards and his advice concerning precision electrical measurements. The set of balanced filters used in this experiment was kindly loaned to us by Professor Paul Kirkpatrick of Stanford University.

The funds for this investigation were derived partially from the gift of Dr. Leon L. Watters of New York City for research work in x-rays and atomic structure known as the Watters Memorial Research Funds. We are grateful for this opportunity to express our sincere appreciation for this gift.

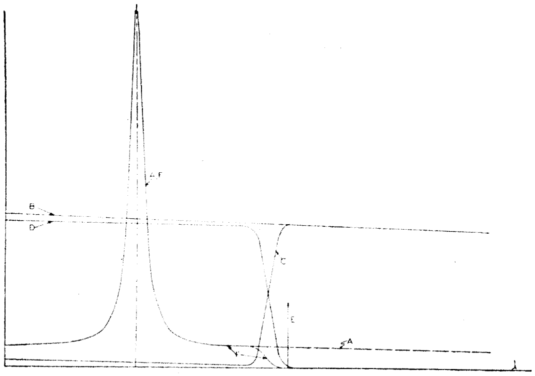

FIG. 10. Graphical representation of the action of balanced filters in conjunction with the 2-crystal spectrometer in "clipping" one wing of the monochromator pass band. Curve A represents the pass band of the 2-crystal monochromator alone. B is the transmission curve of Pd in this region; C is the transmission curve of Mo. D represents the differential transmission of the two filters; i.e., $D=B-C$. E is the wave-length at which the filters are balanced. F is the final pass band of the "compound monochromator." The width of the spectrometer window curve, the slope of the absorption edge, and the separation between them are drawn to the scale as used in this experiment. The curve A has been lifted bodily upward by the addition of a constant so as greatly to exaggerate the height of its wings in order to make visible the clipping effect.

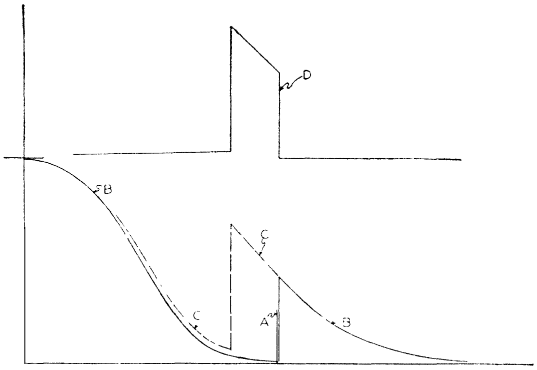

FIG. 11. The curves of Fig. 10 drawn on a small enough scale such as to show the Pd edge also. Note in particular the imperfect balance at short wave-length.

APPENDIX

Discussion of the Use of Balanced Filters in the Improvement of the Sharpness of the Isochromat Intercept

In the main body of this paper we discussed briefly the "clipping action" produced by the use of a set of balanced filters in conjunction with a two-crystal spectrometer. We shall now discuss this process in somewhat more detail, first as to the operation required in the actual use of the method, and second as to its theory.

(1) The pass band of the two-crystal spectrometer is adjusted so that its center falls a few X.U. on the *long* wave-length side of the K edge of the lighter of the two metals constituting the filter.

(2) The obliquity of the filters to the x-ray beam is adjusted until no difference in ionization is detectable when one filter is substituted for the other. During this process the x-ray tube is operated at a voltage about 10 kv above the voltage corresponding to the limit at the wave-length of the monochromator pass band. This assures that the filters have the same absorption on the long wave-length side of the region embraced by their K-absorption discontinuities.

(3) The pass band of the two-crystal spectrometer is moved until its center lies at a wave-length about 6 X.U. *shorter* than the K-edge wave-length of the lighter of the two metals. The peak of the pass band is now in that spectral region *between* the K edges of the two filters where

their absorptions are widely different. It is much closer to the softer of the two edges, however.

(4) Readings for plotting the isochromat are now obtained by taking differences between ionization readings observed through the heavier foil and readings observed through the lighter foil. The difference in ionization thus observed is between two to three times smaller than the total ionization observed without the use of filters.

Let curve A in Fig. 10 and also in Fig. 11 represent the transmission curve of the two-crystal spectrometer; the shape of this curve represents the combined effect of ideal crystal diffraction pattern width, crystal imperfection, incoherent scattering, and vertical divergence of the x-ray beam passing through the monochromator. As the accelerating voltage is varied in exploring an isochromat, this curve is "folded" with the ideal spectral distribution of the x-ray beam. The resultant curve will not exhibit a sharply defined break at the short wave-length limit but it will have a very wide region of changing curvature, owing to the wings or approaches of the monochromator pass band. The width of this region renders the exact location of the short wave-length limit difficult.

Let curve C represent the fractional transmission [approximately equal to exp $(-ad\lambda^3 \sec. \theta)$ save for absorption discontinuities; d = thickness of filter, θ = obliquity to x-ray beam] of the lighter metallic foil (Mo in this work) and curve B the transmission curve of the heavier foil (Pd). In steps (1) and (2) of the procedure outlined above the filters are adjusted so that the transmission difference curve $(D=B-C)$, averaged over the pass band located at E, will vanish. Owing to the slight difference in absorption jump ratio of Pd and Mo this adjustment will give rise to non-zero values of the difference curve on the short wave-length side of the K edge of the heavier filter (see curve D in Fig. 11). It is this difference curve D which, when multiplied point by point with the original pass band A will give the actual transmission curve (F; Fig. 10, 11) of the "compound monochromator." It will be observed that the long wave-length wing of the transmission curve is now almost completely suppressed. This transmission curve F, if folded into the ideal spectral distribution, will give rise to the actually observed isochromats, in which the quantum limit is now much more sharply defined. It will be noted that the imperfect balance of the two filters at short wave-lengths bears no influence on the resultant curve, since there is no spectral intensity there.

It might be noted that this system of monochromatization does not increase the number of readings necessary in obtaining an isochromat, since even without using balanced filters the ionization readings must be obtained relative to the background by means of lead shutter difference readings.

It is important to call attention to the fact that without this balanced filter refinement the steeply rising spectral intensity of the continuous spectrum as one passes from the limit towards longer wave-lengths, is available to "shine through" the rather slowly decaying foot or approach of the spectrometer pass band on its long wave-length side. This approach has a law of decay similar to the inverse square of the wave-length difference from the peak of the pass band. On the other hand, the law of increase of the continuous spectrum is more or less linear for a great distance relative to the width of the pass band. Thus the intensity transmitted at any particular wave-length will vary at large distance from the peak as the inverse first power of the distance from the peak; the integrated or total intensity would therefore be logarithmically divergent, provided that the continuous spectrum would continue to rise linearly indefinitely. Notwithstanding the fact that the latter condition is not fulfilled in practice, this divergence indicates that a dangerously strong emphasis falls on the shapes of the continuous spectrum and of the spectrometer pass band at *large distances* from the peak of the one and the limit of the other. It was to avoid this danger that we have here adopted the balanced filter technique for clipping off most of the long wave-length approach of the spectrometer pass band.

The use of balanced filters with the two-crystal monochromator has also another very important purifying effect already alluded to. The crystal monochromator alone permits a considerable amount of non-monochromatized radiation to enter the ionization chamber by the mechanism of incoherent scattering from the crystal faces as well as air scattering and scattering from lead stops and other obstacles. All wave-lengths present in the primary x-ray beam can contribute to this intensity. Compared to the weak monochromatized intensity when settings are close to the short wave-length limit this general scattered radiation can become very considerable in intensity. Furthermore as the voltage is varied to explore the isochromat, this scattering does not remain constant but varies rather rapidly with voltage and hence there is superposed upon the true isochromat a background which is neither level nor linear and which contributes still more to the difficulty of accurately locating the quantum limit. The balanced filter method, since it in effect "clips off" or suppresses the effect of all wave-lengths of the spectrum on the soft side of the region between the two K edges of the filters, therefore effectively eliminates the background of incoherent scattering.