The experimental results with clean tungsten differ considerably from Coomes' data. His yields were lower than any previously obtained. It is possible that his results could have been affected by the measurement of secondaries produced within the gun itself which would have the effect of apparently lowering the yield.

The results obtained for sodium on tungsten also disagree with Coomes' results which showed a lowering of the yield when the work function was reduced by the application of thorium on tungsten. However, they are in substantial agreement with work by Treloar,[1] deBoer and Bruining,[2] and others.[3,16] The careful distillation of the sodium, the outgassing procedure employed, and the reproducibility of the results appear to preclude the influence of oxygen on these results. Since the thickness of the sodium film was always less than a monomolecular layer, it is unlikely that absorption of electrons within the layer was appreciable. Thus we see that the secondary emission yield is not independent of the work function of the emitting surface but increases as the work function is decreased. This is in accord with Wooldridge's theory.[12]

I sincerely appreciate the many helpful suggestions given, and the continued interest shown, by Professor W. B. Nottingham, under whose direction this research was carried out. I am grateful to Dr. W. Painter of the RCA Manufacturing Company for his suggestions regarding electron gun design, to the electronics group at M.I.T. with whom many enlightening discussions were held, and to Mr. Lawrence W. Ryan who ably performed the difficult glass-working operations demanded in this research.

[16] K. Sixtus, Ann. d. Physik 3, 1017 (1929).

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Matter Waves and Electricity

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Classical four-dimensional relativity gives a most natural and harmonious interpretation of the three basic phenomena of nature: gravity, electricity, and the wave structure of matter, provided that the basic assumptions of the Einsteinian theory are modified in two respects: (1) the fundamental invariant of the action principle is chosen as a quadratic instead of a linear function of the curvature components; (2) the static equilibrium of the world is replaced by a dynamic equilibrium. Electricity comes out as a second-order resonance effect of the matter waves. The matter waves are gravitational waves but superposed not on an empty Euclidean space but on a space of high average curvature.

I. INTRODUCTION

THE belief in the fundamental unity of all nature is so deeply rooted that attempts to describe all natural phenomena from one unified basis occurred again and again throughout the history of physics. The great unification of geometry and physics by Einstein gave new impetus to such speculations. Various courses were open to generalize the original frame of relativity. The abandonment of the Riemannian line element in favor of a more general metrical structure by Weyl[1] and Eddington,[2] the introduction of "distance parallelism" by Einstein,[3] the change from a metrical to a projective platform by Veblen and Hoffmann,[4] the enlargement by a fifth dimension by Kaluza,[5] and the further development of that theory by Einstein and Bergmann[6] are some of the major landmarks in these efforts. The author's own attempt,[7] inaugurated about ten years ago, follows a somewhat different line. The frame of Riemannian geome-

[1] H. Weyl, Math. Zeits. 2, 384 (1918).
[2] A. S. Eddington, Proc. Roy. Soc. 99, 104 (1921).
[3] A. Einstein, Berl. Ber., pp. 217 and 224 (1928).
[4] O. Veblen and B. Hoffmann, Phys. Rev. 36, 810 (1930).
[5] Th. Kaluza, Berl. Ber., p. 966 (1921); A. Einstein, Berl. Ber., p. 23 (1927).
[6] A. Einstein and P. Bergmann, Ann. of Math. 39, 683 (1938).
[7] C. Lanczos, Phys. Rev. 39, 716 (1932).

try is not sacrificed nor is the four-dimensional structure abandoned. The generalization consists in the choice of a new action principle which leads to field equations more general than the Einsteinian gravitational equations. The new field equations are of fourth order in the g_{ik}. However, applying the methods of Hamiltonian dynamics to the variation of the action principle, one can vary metrical tensor and matter tensor as independent variables and consider the resulting system of field equations as a coupled system of differential equations of second order for the twenty variables g_{ik} and R_{ik}.[8] These equations are analogous to the basic equations of elasticity and correspond to an elastic theory of the ether.

The author could show in his previous publication[7] that the generalized action principle is in perfect harmony with the equations of electricity. The integration of the field equations for infinitesimal fields gives rise to a free vectorial function ϕ_i which has all the properties of the vector potential. This vectorial function can be introduced as an undetermined Lagrangean multiplier, caused by the conservation laws of the matter tensor. The field equation for ϕ_i follows from the conservation laws and comes out as the nabla equation, in complete analogy to the field equation of the ordinary electromagnetic vector potential. The conservation law of the electric charge, expressed by the vanishing of the divergence of ϕ_i, can also be established. Finally, the dynamical interaction of charged particles can be deduced by an application of the action principle to the problem of motion. The resulting force is the electromotive force of Lorentz.

Hence, in spite of the fact that the deductions are based on the symmetric gradient

$$F'_{ik} = (\partial\phi_i/\partial x_k) + (\partial\phi_k/\partial x_i) \tag{1.1}$$

rather than on the customary anti-symmetric gradient

$$F_{ik} = (\partial\phi_i/\partial x_k) - (\partial\phi_k/\partial x_i), \tag{1.2}$$

the results are in agreement with the observable physical facts. The change of sign from plus to minus occurs in the ponderomotive force and that suffices to define the field strength according to (1.2) and deduce the Maxwellian equations.

While the theory is thus able to yield the basic theoretical aspects of electricity, it contains one fundamental difficulty which hampered all further progress up to recent times. As it was indicated in the closing chapter of the quoted paper,[9] the theory cannot explain the existence of free electric charges as long as static conditions prevail. Indeed, let us consider an electric field with the electrostatic potential ϕ_4. According to the theory the gradient of ϕ_4 represents the three components T_{i4} of the matter tensor and has to be interpreted as an energy flux. Such an energy flux must have a source. It requires the creation or destruction of matter which is in contradiction to the empirically well-established stability of the mass of electrically charged particles.

Recent researches concerning the dynamical problem of a particle[10] started the author on an entirely new track, furnishing him eventually with that new viewpoint which has to be added to the quadratic action principle before any further progress can be made. It is the *dynamical* viewpoint which replaces the essentially static considerations of the previous researches. The new departure reveals that the problem of electricity cannot be separated from the problem of matter waves since electricity is an accompanying second-order effect of the matter waves. The consistent pursuance of the dynamical viewpoint leads to such interesting results that a short non-technical draft of the new theory, without extended mathematical computations, seems to be justified.

2. THE QUADRATIC ACTION PRINCIPLE

If the Lagrangean function of the universal action principle shall be quadratic in the curvature components, there seems to be an abundance of five fundamental invariants. However, two of these invariants are inactive as far as variation goes while the remaining three invariants are reducible to but two on account of an algebraic identity which exists between them. These relations between the basic invariants were established by the author,[11] not knowing that they were deduced before by Weitzenboeck,[12] Bach,[13] and Juettner.[14]

[8] C. Lanczos, Zeits. f. Physik **96**, 76 (1935).

[9] Cf. reference 7, p. 735.

[10] C. Lanczos, Phys. Rev. **59**, 813 (1941).

[11] C. Lanczos, Ann. of Math. **39**, 842 (1938).

[12] R. Weitzenboeck, Wien. Ber. **129**, 683 (1920).

[13] R. Bach, Math. Zeits. **9**, 110 (1921).

[14] F. Juettner, Math. Ann. **87**, 270 (1922); cf. reference 17, footnote 1.

The two remaining invariants can be combined by an *a priori* undetermined numerical factor c and thus the Lagrangean function of the fundamental action integral becomes:

$$L=R_{\alpha\beta}R^{\alpha\beta}+cR^2. \tag{2.1}$$

The previous investigation left c an undetermined number since there seemed to be no logical reason why one particular value of c should be preferred. This distressing uncertainty can now be removed and the action principle of the world uniquely determined. There is actually one special value of c which is distinguished by particularly desirable properties.

We introduce a new tensor S_{ik} defined as follows

$$S_{ik}=R_{ik}-\tfrac{1}{4}\kappa Rg_{ik} \tag{2.2}$$

with the undetermined numerical factor κ. The Lagrangean function (2.1) can now be united into the single expression

$$L=S_{\alpha\beta}S^{\alpha\beta}, \tag{2.3}$$

provided that the following condition holds:

$$\kappa(2-\kappa)+4c=0. \tag{2.4}$$

To any given $c\geqq-\frac{1}{4}$ a corresponding real κ can be found.

Since the action integral is quadratic in the S_{ik}, the solution

$$S_{ik}=0 \tag{2.5}$$

is obviously a possible solution of the field equations. This solution is the most stable of all possible solutions since it minimizes the action integral to zero. It is reasonable to assume that this particularly stable solution of the field equations will correspond to the average metrical structure of the world on which the fields of the material particles are superposed as small perturbations. This metrical field is obviously of decisive importance for all phenomena of physics. Ordinarily we identify it with the Euclidean metrics but this would not serve our present purposes. It is desirable to have a specific name for the metrical background of the world as it exists if the central fields of the individual particles are stripped away; the name "metrical plateau" seems to be appropriate and will be adopted for the present paper. The metrical plateau contains the bulk of the geometry of nature; the fields of the individual particles are merely small perturbations of the basic plateau.

The condition (2.5) yields the following field equations:

$$R_{ik}-\tfrac{1}{4}\kappa Rg_{ik}=0. \tag{2.6}$$

Contraction gives

$$(1-\kappa)R=0. \tag{2.7}$$

Now if κ is different from 1, we get at once

$$R=0 \tag{2.8}$$

and (2.6) yields

$$R_{ik}=0. \tag{2.9}$$

Since the field equations $R_{ik}=0$, if singularities are excluded, do not go beyond the Euclidean metrics, the metrical plateau of the world would come out as the customary Euclidean space of special relativity. But then the theory would not contain anything that could possibly lead to a reasonable explanation of the atomistic structure of matter, in want of a universal constant of atomistic dimensions. Consider, however, the exceptional value

$$\kappa=1. \tag{2.10}$$

The vanishing of R is now avoided. With this value of κ the constant c becomes

$$c=-\tfrac{1}{4}. \tag{2.11}$$

The definition of S_{ik} is now:

$$S_{ik}=R_{ik}-\tfrac{1}{4}Rg_{ik}. \tag{2.12}$$

This S_{ik} was once suggested by Einstein[15] as a possible substitute for the matter tensor T_{ik}. It is an under-determined quantity, leaving the scalar curvature R arbitrary. The purpose of this under-determination was to provide gravitational forces which could counteract the electrostatic forces of an electron and thus keep the electron from exploding. However, this under-determination leads to unacceptable results.

The present theory does not lead to any under-determination, not even for the value (2.11) of the constant c. The field equations yield for the scalar R the exact equation

$$(1+3c)\Delta R=0, \tag{2.13}$$

where Δ is the invariant four-dimensional Laplacean operator. The critical value of c which

[15] A. Einstein, Berl. Ber., p. 349 (1919).

leads to the loss of one equation is thus $-\frac{1}{3}$ and not $-\frac{1}{4}$. The choice (2.11) preserves the field Eq. (2.13), and we can put

$$R=\text{const.}=4\lambda \qquad (2.14)$$

as the only reasonable solution of that equation. The field Eqs. (2.6) now become:

$$R_{ik}=\lambda g_{ik}, \qquad (2.15)$$

which are Einstein's cosmological equations. The constant λ is an arbitrary constant of integration. The λ term is of an entirely decisive importance because, as the following chapter will show, λ is a *microscopic* and not a macroscopic constant, establishing a universal gauge of atomic dimensions in the world.

In order to remove the degeneracy of the Eq. (2.12), we can put

$$\kappa=1-\epsilon \qquad (2.16)$$

and consider ϵ as an exceedingly small number. We now have

$$S_{ik}=R_{ik}-\lambda g_{ik}+\epsilon\lambda g_{ik} \qquad (2.17)$$

and the basic metrical plateau is no longer characterized by the equations $S_{ik}=0$, but by the "cosmological equations"

$$S_{ik}=\epsilon\lambda g_{ik}, \qquad (2.18)$$

where $\epsilon\lambda$ is of cosmological smallness.

3. FOURIER ANALYSIS OF THE RIPPLED METRICAL PLATEAU

We assume that the basic metrical plateau on which the fields of the individual particles are erected is not the smooth Euclidean space of four dimensions with which special relativity operates but a space rippled by a diffuse cosmic radiation which fills out the entire four-dimensional space-time manifold. Although we know that our space is probably of a closed spherical structure, we replace that sphere for the purpose of the mathematical analysis by a huge box of cubic shape. We assume that the g_{ik} of this world are nearly Euclidean so that we may put

$$g_{ik}=\delta_{ik}+\epsilon\gamma_{ik}, \qquad (3.1)$$

where ϵ is an infinitesimal expansion parameter. The quantities γ_{ik} are of an oscillatory nature on account of the radiation which fills the space. We apply a Fourier analysis to the γ_{ik}, expanding them into an infinite Fourier series with a basic wave-length which is extremely large. The terms of the Fourier analysis have constant amplitudes. The Fourier analysis is of the following form:

$$\gamma_{ik}=\sum_{\nu_1\cdots\nu_4 \atop -\infty}^{+\infty} A_{ik}{}^{(\nu_1\cdots\nu_4)} \times\exp\left[i(\nu_1 x+\nu_2 y+\nu_3 z+\nu_4 ct)\right]. \qquad (3.2)$$

We now examine the expression of the Einsteinian curvature tensor R_{ik}. We notice that the terms of this tensor can be split into two parts. The part R'_{ik} is formed of partial derivatives, while the part R''_{ik} cannot be transformed into partial derivatives. Assuming that our reference system is normalized by the customary condition:

$$\partial g^{\frac{1}{2}} g^{i\alpha}/g^{\frac{1}{2}}\partial x_\alpha=0, \qquad (3.3)$$

we obtain the following expression for R''_{ik}, neglecting quantities of higher than second order which are irrelevant for our present purpose:

$$R_{ik}''=\frac{1}{4}\left(\frac{\partial\gamma_{\alpha\beta}}{\partial x_i}\frac{\partial\gamma_{\alpha\beta}}{\partial x_k}-\frac{1}{2}\frac{\partial\gamma}{\partial x_i}\frac{\partial\gamma}{\partial x_k}\right) -\frac{1}{2}\left(\frac{\partial\gamma_{i\beta}}{\partial x_\alpha}\frac{\partial\gamma_{k\beta}}{\partial x_\alpha}-\frac{1}{2}\frac{\partial\gamma_{ik}}{\partial x_\alpha}\frac{\partial\gamma}{\partial x_\alpha}\right). \qquad (3.4)$$

These terms give rise to a phenomenon that Schroedinger called "resonance catastrophe." The terms of R'_{ik} are obtained by differentiation and thus can give nothing but strictly periodic contributions; a constant term could arise only if the function to be differentiated were a linear function of t or the other coordinates which contradicts the boundary conditions in infinity. The terms of R''_{ik}, however, lead to constant terms, in addition to the periodic terms. Each frequency of the Fourier spectrum (3.2) is in resonance with itself and produces a constant term. Although these constants are individually small, yet in view of the extended nature of the Fourier spectrum and the very high frequency of the peak of the radiation (the realm of cosmic-ray frequency) these constants sum up to excessively high values, in spite of the smallness of ϵ.

These constants in the Fourier sum of R_{ik} cannot be compensated. They remain as unsaturated terms and the "resonance catastrophe" would occur if the metrical plateau had to satisfy the field equations $R_{ik}=0$. We have to abandon these field equations in favor of a more general possibility which avoids the difficulty of resonance terms.

Since each component of the tensor R_{ik} yields a constant as the average value of that component, a constant tensor ρ_{ik} is generated, characterizing the average curvature of the basic metrical plateau. The principal axes of this tensor designate a distinguished reference system of the space-time manifold. If, however, the cosmic radiation is evenly distributed over all directions of the four-dimensional world, then the tensor ellipsoid of that constant tensor degenerates into a sphere and the principal axes become undetermined. In that case the tensor ρ_{ik} must get the form:

$$\rho_{ik}=\lambda\delta_{ik}. \tag{3.5}$$

This, however, is exactly the condition that the constant tensor ρ_{ik} has to fulfill if the field equations $R_{ik}=0$ are modified to the "cosmological equations"

$$R_{ik}=\lambda g_{ik}. \tag{3.6}$$

We thus return to our Eq. (2.15) obtained before by integrating the field equations.

In view of the dynamical equilibrium of the world the cosmological Eqs. (3.6) obtain an entirely new significance. Originally these equations were introduced by Einstein in order to describe the cosmological behavior of the world at large. The λ term is an excessively small correction term since λ is inversely proportional to the square of the average curvature radius of the world. The cosmological term can have no influence on the microscopic structure of matter. Its function is merely to regulate the metrical behavior of the world in astronomical dimensions. In the present theory the λ term is *excessively large.* The constant λ is inversely proportional to the square of a length of nuclear dimensions. We get the apparently paradoxical result that *a statistically Euclidean rippled world whose metric is nearly constant, may nevertheless generate an exceedingly strong average curvature.* The change from statical to dynamical relativity spans the tremendous gap between atomistic and cosmic dimensions. In the static consideration a high average curvature would either mean that the world shrinks to nuclear dimensions, or that the curvature radius of the world explodes with a terrific speed. The tremendous pressure created by the cosmological term must lead to an explosion of the world. In the dynamical consideration that tremendous pressure is evenly distributed over a great many vibrations, each one carrying a very small amount of the burden. The situation is comparable to the problem of the electron which should explode under the influence of the tremendous electrostatic repulsive forces. Here, too, if the force is considered as a dynamic and not a static phenomenon, the difficulty is at once removed, as the next paragraph will show.

It is obvious that the rippled structure of the metrical plateau must lead to observable physical phenomena. Dynamically these fields will not show up easily. They produce extremely high vibrations with extremely small amplitudes which cannot be observed directly, except if occasional phase relations produce a case of temporary resonance. And thus, the wavy, eternally oscillating metrical plateau must create statistical effects. It is responsible for the general "uncertainty" and practical unpredictability of motion phenomena. Heisenberg's uncertainty principle may become realistically interpretable in terms of the perpetual statistically distributed ripples of the metrical plateau which incessantly travel in every direction. Moreover, these vibrations are responsible for the metrical structure of the material particles. These particles are not the static solutions of an eigenvalue problem, growing out from a smooth background as small hills on a generally flat surface. The material particles are themselves formed of vibrations which are in dynamical equilibrium with the cosmic waves of the basic metrical plateau. The strange and apparently classically unexplainable wave mechanical phenomena are interaction effects between the metrical waves which emanate from the individual particles. The following mechanical analogy may illustrate the situation. If a car travels on a gravelled road, it cannot keep to a straight line exactly. It reaches its destination only because the dimensions of the car are large compared with the size of the gravel. If the size of the car shrinks to dimensions which are comparable to the dimensions of the gravel, the car will reach its destination with large statistical fluctuations. If, in addition to that, the wheels of a car were notched into a wavy profile and the rhythm of this wave were in resonance with the ripples on the road on which the car travels, the statistics which control the motion of this car would be necessarily of the nature of interference effects of products of wave functions, as it is the case in wave mechanical considerations. This crude picture is far from giving any explanation. Its purpose is not more than to show that the wave mechanical effects are not so puzzling any more if the metrical structure of the basic plateau on which the particles move has a dynamic instead of static character.

4. ELECTRICITY AS A RESONANCE EFFECT OF MATTER WAVES

We consider our action integral

$$A=\int S_{\alpha\beta}S^{\alpha\beta}d\tau, \tag{4.1}$$

where $d\tau$ is the four-dimensional volume element. The principle of least action requires that the variation of this integral shall vanish for arbitrary variations of the g_{ik}. We are not interested at present in the exact derivation of the field equations. We want to investigate the infinitely weak fields only. For that purpose we temporarily separate the variation of the g_{ik} and the variation of the S_{ik}. We know that the variation of the S_{ik} is reducible to the variation of the g_{ik}, by differentiation. But it is equally true that the variation of the g_{ik} is reducible to the variation of the S_{ik}, by integration. Hence it is permissible to consider the variation of the S_{ik} as the independent and the variation of the g_{ik} as the dependent variables. But the terms which contain the δg_{ik} are of second order and are thus negligible for our present problem.

The variation of the S_{ik} is not a free variation because the tensor S_{ik}, defined by (2.17), is subject to the conservation law of momentum and energy. This law takes the form:

$$\frac{\partial g^{\frac{1}{2}}S^{i\alpha}}{\partial x_\alpha}=-g^{\frac{1}{2}}\Gamma_{\alpha\beta}{}^{i}S^{\alpha\beta}. \tag{4.2}$$

If we consider this equation as an auxiliary condition of the variation, the application of the Lagrangean multiplier method gives a vectorial function ϕ_i as Lagrangean multiplier and we obtain for infinitesimal S_{ik} the field equation:

$$S_{ik}=(\partial\phi_i/\partial x_k)+(\partial\phi_k/\partial x_i)-2\Gamma_{ik}{}^{\alpha}\phi_\alpha. \tag{4.3}$$

This field equation is not restricted to infinitesimal *metrical* fields g_{ik}. Its only restriction is that the *matter tensor* S_{ik} shall be infinitesimal. It cannot be used in the central nuclear region of a particle but it gives the field of a particle in the peripherical regions where the tensor S_{ik} is sufficiently small.

To show that the vectorial function ϕ_i has actually the properties of the ordinary vector potential, we at first notice that the definition (2.12) of the tensor S_{ik} requires the vanishing of the scalar S which gives:

$$\frac{\partial g^{\frac{1}{2}}\phi^\alpha}{\partial x_\alpha}=0. \tag{4.4}$$

This equation expresses the conservation law of the electric charge.

The conditioning differential equation for ϕ_i follows if the solution (4.3) is put back into the divergence Eq. (4.2). We obtain with sufficient accuracy:

$$\frac{\partial^2\phi_i}{\partial x_\alpha{}^2}=-\Gamma_{\alpha\beta}{}^{i}S^{\alpha\beta}. \tag{4.5}$$

This is the classical determining equation of the vector potential if the right side is interpreted as the four-dimensional "current vector."

We notice that ϕ_i comes out as an infinitesimal quantity of *second* order because both factors of the right side are infinitesimal.

It is shown in analysis that a potential function cannot be determined uniquely if the conditioning differential equation is not known everywhere. The differential Eq. (4.5) does not hold in the central region of the particle and thus the Eq. (4.5), although it gives the right mathematical law for ϕ_i, does not suffice to determine it. Fortunately, this uncertainty afflicts only the possible dipoles and multipoles of ϕ_i. *The "charge"-part of the potential,* which decreases with $1/r$ and which is the dominating and physically decisive term, *can be uniquely determined, in spite of the uncertainty of the central region.*

In order to understand the true significance of the vector potential and its main mathematical properties, let us examine the nature of the conservation law (4.2) more closely. We encounter here once more a "resonance catastrophe" of a definite type. This time the difficulty does not arise from the fact that the left side of the equation is a sum of derivatives while the right side is not. As it was shown by Einstein,[16] the right side of the equation can be transformed into a sum of derivatives. However, let us apply the Gaussian integral transformation to both sides of the divergence equation. The surface integral obtained on the left side vanishes if S_{ik} is zero everywhere on the boundary. The right side, however, does not vanish, even if S_{ik} is zero everywhere outside the particle. This shows that the field equation

$$S_{ik}=0 \tag{4.6}$$

for infinitesimal fields cannot be established without imposing a definite vectorial condition on the central part of the field. The physical interpretation of this situation is that although the matter tensor vanishes outside the particle, the momentum and energy *flux* through a closed surface surrounding the particle need not vanish. But then that flux has to be maintained and that is only possible if matter destroys itself or is created incessantly. We have here a similar resonance phenomenon as in the case of the plateau vibrations. Just as there a safety valve was provided by the cosmological term λg_{ik}, so in the new situation a similar safety valve is provided by the Lagrangean multiplier ϕ_i. This shows that the vector potential ϕ_i plays the same role for the matter waves that the cosmological constant λ plays for the plateau waves. In both cases a resonance catastrophe has to be avoided. Since the resonance is a second-order effect, we understand why electricity appears as a second-order surplus effect of the matter waves. *The flux of the electrostatic field balances the gravitational flux of the matter waves.* If this "pre-established harmony" between matter waves and electricity seems rather astonishing, one can explain it as a logical consequence of the nature of the vector potential as a Lagrangean multiplier. It is the avowed *purpose* of this multiplier to make the auxiliary condition of the matter tensor—and that is the conservation law of momentum and energy —possible and thus it is only natural that it is the function of the electric charge to provide an energy flux which compensates the permanent irradiation of the matter waves.

Of particular interest is the electrostatic charge e which enters in the "scalar potential" ϕ_4. We obtain:

$$-4\pi e=\int\frac{\partial\phi_4}{\partial x_\alpha}\nu^\alpha d\sigma=\int S_{4\alpha}\nu^\alpha d\sigma=\frac{1}{2}\int\frac{\partial g_{\alpha\beta}}{\partial x_4}S^{\alpha\beta}dv, \tag{4.7}$$

where dv is the three dimensional volume element. This volume integral can be transformed into a surface integral according to the fundamental theorem of Einstein[16] that the gravitational momentum and energy flux can be rigorously transformed into a surface integral. This shows that the electric charge can be computed by making use of a closed surface surrounding the particle *at any arbitrary distance* so that the behavior of the unknown central field does not influence the electric charge.

The expression (4.7) shows directly that it is the oscillation of the metrical tensor g_{ik} which generates the electric charge. We notice, too, that a change of the sign of the time axis changes the charge from plus to minus, without changing its absolute value. Such a change in the direction of $x_4=ict$ means for the matter waves that *outgoing waves are changed into incoming waves.* The duplicity of positively and negatively charged particles finds thus its explanation by the fact that there is an equal chance for outgoing as for incoming waves. Notice that this possibility is in no contradiction to the phenomena of the retarded potential which are always based on outgoing and not on incoming waves. The matter waves are eternal oscillations which have no beginning and no end and thus "outgoing" and "incoming" refers merely to a phase relation and not to something which comes in collision with our regular experiences as to the sequence of time events.

One of the main riddles of electricity is the peculiar unsymmetry of the masses of electron and proton. Can the present theory give a satisfactory solution of this puzzle? The calculations are not advanced yet to the point where this question can be answered. However, we have good reason to believe that the results will come

[16] A. Einstein, Ann. d. Physik **49**, 769 (1916); cf. Eq. (50), p. 806; for the contravariant components, cf. reference 10, p. 817.

out satisfactorily. In the first place, the charge is a second-order effect, and so is the mass.[17] It seems possible that the charge shall have a linear influence on the mass. But apart from this, the present theory differs in one characteristic point from all previous attempts to apply relativity to the physical universe. We have called the tensor S_{ik} the "matter tensor" and we have interpreted the field equations

$$S_{ik}=0 \tag{4.8}$$

as "matter waves." These matter waves are gravitational waves but not of the ordinary type. If we consider the definition of the tensor S_{ik} and take in account that the field of a material particle is superposed on the cosmological Eqs. (2.15), we notice that the S_{ik} tensor of a material particle has to be defined as the *variation* of the quantity (2.17), taken for an infinitesimal change δg_{ik} of the metrical tensor g_{ik}:

$$S_{ik}=\delta R_{ik}-\lambda\delta g_{ik}. \tag{4.9}$$

This is not the ordinary curvature tensor with which we operate when the Einsteinian field equations are applied to infinitesimal fields. The curvature tensor (4.9) is superposed on a field of high average curvature and not on a field of zero curvature. The difference amounts to the same as the difference between Dirac's equation of the electron "with the mass term" and "without the mass term." If the mass term is omitted, the field quantities satisfy the ordinary wave equation and the waves propagate with light velocity. If the mass term is present, the wave equation is enlarged by a fifth term and the phase velocity of the matter waves is no longer the light velocity. Similarly, the ordinary gravitational waves of the Einsteinian theory, superposed on a Euclidean space, propagate with light velocity. The new matter waves, however, characterized by the vanishing of the tensor (4.9), propagate with an entirely different phase velocity. The introduction of a highly curved metrical plateau as the basic geometrical platform of the physical world removes the degeneracy of the Euclidean space and opens entirely new perspectives. The connection of the Eqs. (4.8) and (4.9) with the Dirac and possibly Schroedinger equations will be the subject of an independent investigation.

[17] C. Lanczos, Phys. Rev. **59**, 708 (1941).

5. CONCLUSIONS

The general viewpoints of a new theory have been developed which attempts to explain the basic physical phenomena of the world in a most harmonious and organic fashion. Although the fundamental action principle is chosen on strictly logical grounds, without augmenting it by phenomenological elements, yet the basic phenomena of electricity, matter waves, and gravity are derivable from it as necessary mathematical consequences. In addition to the quadratic action principle, an essentially new element enters the theory: the dynamical aspect. The metrical background of the world is not smooth but rippled by an extended spectrum of permanent vibrations. These vibrations, although observable only by their statistical effects and giving the source of the general uncertainty principle which prevails in nature, create a strong average curvature upon which the fields of the individual particles are erected. The matter waves associated with the particles are metrical waves which differ from the ordinary gravitational waves by the fact that they are metrical deformations of a strongly curved world and not of a flat world. This explains the abnormal phase velocity of these waves which differs widely from light velocity. Electricity is an accompanying resonance effect of the matter waves, necessary to maintain the undamped matter waves without energy losses. Although electricity is a second-order effect, yet for our world of observations it is by far more decisive than all other effects because it is *static* (in a proper reference system) while the waves of the basic metrical platform and also the matter waves of particles represent eternal oscillations of very high frequencies.

The theory presented here is far from being complete. It is not more than the first step toward a new land. But the author who has grappled with these ideas for ten years, groping in the dark and unable to solve the puzzle of the electric charge, until suddenly the dynamical aspect arrived, illuminating the entire scenery and melting away all difficulties—cannot doubt that here are the outlines of a theory which will be destined to bring the three basic phenomena of nature: gravity, electricity, and the wave theory of matter, into one inseparable unity.