Theoretical Half-Lives of Forbidden β-Transitions

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Fermi's theory of β-decay is extended to the "nth forbidden" approximation. Precise formulas for the distribution in energy of the emitted β-rays are derived for arbitrarily charged nuclei, according to the five possible invariant forms of interaction, the so-called scalar, polar vector, tensor, axial vector, and pseudo-scalar interactions, respectively. The nuclear matrix elements of the transitions, made up of the components of certain irreducible tensors, are constructed. The selection rules appropriate to these matrix elements are given in Table II. The magnitudes of the nuclear matrix elements are estimated by a simple averaging process depending only on the directional properties of the tensors from which they are constructed and the order of magnitude of the tensor components. Theoretical half-lives of the "forbidden" β-decays of RaE, P^{32}, K^{40}, and Rb^{87} are calculated by numerical integration of the energy dependent electron emission probabilities. Upon comparing with the experimental determinations of the half-lives, the most satisfactory agreement seems to be obtained with the tensor form of interaction. The evidence in favor of Gamow-Teller selection rules is somewhat inconclusive for the case of the K^{40} decay because of the uncertainty of the experimental determination of the maximum electron energy.

I. INTRODUCTION

In order to explain the energy distribution of the β-particles from nuclei such as RaE and P^{32}, Konopinski and Uhlenbeck[1] extended the Fermi theory to the "forbidden" transitions. Upon comparing the theoretical distribution with the observed spectra, these authors were able to eliminate three of the five forms of interaction[2] which they considered. The invariant forms characterized as the scalar, axial vector, and pseudo-scalar interactions were shown to be inadequate to give the observed energy distributions of RaE and P^{32}. By assuming the nuclear spin changes, $J_i=2\rightarrow J_f=0$, for both RaE and P^{32} they found that the polar vector and tensor interactions were able to account satisfactorily for the observed distributions.

The above deductions will not be conclusive until not only the energy distributions of the β-particles emitted by radioactive nuclei are shown to agree with experimental determinations, but also the lifetimes characteristic of the decays are explained. A calculation of the half-lives may serve to verify the conclusions drawn from the theoretical treatment of the energy distributions, and furthermore may facilitate additional discrimination in the choice of interaction. Information concerning the nuclear spins of the initial and final nuclei is pertinent to these calculations. The "forbidden" transitions,

$$K^{40}(J_i=4)\rightarrow Ca^{40}(J_f=0)+\beta^-$$

and

$$Rb^{87}(J_i=3/2)\rightarrow Sr^{87}(J_f=9/2)+\beta^-,$$

having known spin changes,[3,4] are treated here in addition to RaE and P^{32}.

The theoretical half-lives of the four above-mentioned decays are calculated in detail according to the tensor and polar vector theories. For K^{40} the "allowed," "first," and "second forbidden" formulas derived in II are inadequate, in view of the fact that a spin change, $\Delta J=4$, requires at least the "third forbidden" approximation, and may entail the "fourth forbidden" formulas. The general formulas for "nth forbidden" transitions, calculated to the same degree of precision as those of II, are presented along with good approximations which are applicable to both heavy and light elements. The β-spectra of K^{40} and Rb^{87} are not of interest because their exact measurements are difficult; however, the more easily observed average electron energies are calculated.

The RaE and P^{32} lifetime problems are attacked for the first time in this paper. Good

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[1] E. J. Konopinski and G. E. Uhlenbeck, Phys. Rev. **60**, 308 (1941)—to be referred to hereinafter as paper II.

[2] H. A. Bethe and R. F. Bacher, Rev. Mod. Phys. **8**, 82 (1936). The five invariant interactions are also listed in II.

[3] J. R. Zacharias, Phys. Rev. **60**, 168 (1941).

[4] M. Heyden and H. Kopfermann, Zeits. f. Physik **108** 3-4, 232 (1938).

TABLE I. Data on half-lives.

Emitter	Experimental half-life/sec.	W_0/mc^2	Theoretical half-life/sec.	$\overline{W}/mc^2$	$(f_{nX}t_{\text{exp}})$/sec.
RaE	(4.3)10⁵ → $(4.3)10^5$	3.3			
$\Delta J=(2\rightarrow 0)$ "Yes"			$t_{1T}=(8)10^4$		$(3.4)10^4$
$\Delta J=(2\rightarrow 0)$ "No"			$t_{2T}=(5-13)10^5$		$(5-2.0)10^3$
			$t_{2V}=(1.9-3.4)10^5$		$(7-3.8)10^3$
P^{32}	$(1.2)10^6$	4.37			
$\Delta J=(2\rightarrow 0)$ "Yes"			$t_{1T}=(5.5)10^5$	2.4	$(13)10^3$
$\Delta J=(2\rightarrow 0)$ "No"			$t_{2T}=(0.54-11)10^7$	(2.4−2.2)	$(13-0.6)10^2$
			$t_{2V}=(0.27-4.2)10^7$	(2.4−2.2)	$(14-0.9)10^2$
K^{40}	$(4.5)10^{16}$				
$\Delta J=(4\rightarrow 0)$ "Yes"		2.4	$t_{3T}=(2.3)10^{17}$	1.6	$(1.2)10^3$
		3.6	$t_{3T}=(1.0)10^{15}$	2.2	$(2.7)10^5$
$\Delta J=(4\rightarrow 0)$ "No"		2.4	$t_{4T}=(0.23-13)10^{19}$	(1.6−1.5)	(119−2.0)
			$t_{4V}=(0.11-4.3)10^{19}$	(1.6−1.5)	(120−3.1)
		3.6	$t_{4T}=(1.1-32)10^{15}$	(2.2−2.0)	$(24-0.8)10^4$
			$t_{4V}=(0.55-11)10^{15}$	(2.2−2.0)	$(25-1.2)10^4$
Rb^{87}	$(2-6)10^{18}$	1.26			
$\Delta J=(3/2\rightarrow 9/2)$ "No"			$t_{2T}=(3.0)10^{15}$	1.13	$(4-12)10^6$
$\Delta J=(3/2\rightarrow 9/2)$ "Yes"			$t_{3T}=(0.3-13)10^{17}$	(1.13−1.09)	$(120-0.9)10^4$
			$t_{3V}=(0.14-3.3)10^{17}$	(1.13−1.09)	$(120-1.8)10^4$

agreement with the observed half-life of RaE[5] is obtained. Concurrently, the theoretical energy distribution predicted is in agreement with the well-defined experimental determinations of the shape of the spectra.[6–8] On the other hand, the calculated half-life of P^{32} differs considerably from its observed value. The shape of the P^{32} energy distribution calculated in II also varies somewhat from experimental determinations.

The half-life of K^{40} has been discussed by Bethe,[2] and recently was calculated by Marshak.[9] Both authors have shown that the "third forbidden" approximation is sufficient to explain its very long lifetime. Use of the K-U modification of the original Fermi theory (polar vector interaction) made it necessary for Bethe to assume $\Delta J=3$ which is in contradiction to the recent experimental data of Zacharias.[3] Marshak concludes that the K^{40} transition can be "third forbidden" if Gamow-Teller selection rules (tensor or axial vector interaction) are valid. The maximum β-ray energy, $W_0=3.6\ mc^2$, reported by Henderson,[10] as well as $W_0=2.4\ mc^2$, was used by Marshak. The former end-point energy yields a much smaller half-life than the latter. The results listed in Table I indicate that Marshak's conclusion is correct if the maximum energy is $2.4\ mc^2$, but if it amounts to as much as $3.6\ mc^2$, the half-life can be accounted for equally well by the "fourth forbidden" tensor or polar vector interactions. The uncertainty as to the end-point energy weakens the argument in favor of the Gamow-Teller rules based on K^{40}.

The lifetime of Rb^{87} (half-life, $t=(2-6)10^{18}$ sec.)[11,12] was discussed by Konopinski and Bethe.[13] They obtained a theoretical estimate of the mean-life, $\tau\approx 10^{10}$ yr., according to the "third forbidden" polar vector interaction, with $\Delta J=3$. The half-life of Rb^{87} listed in Table I is predicted within the experimental error by the "third forbidden" tensor or polar vector interaction. This is in substantial agreement with the calculations of Marshak.

II. THEORETICAL CALCULATIONS

The reciprocal half-life of a radioactive nucleus according to Fermi's theory of β-decay is given by

$$1/t_{nX}=(G^2/2\pi^3\ln 2)\times\int_1^{W_0} C_{nX}F_0(W,Z)pWq^2dW, \quad (1)$$

$$=(G^2/2\pi^3\ln 2)f_{nX}(W_0,Z). \quad (2)$$

[5] A. Pompei, J. de phys. et rad. **6**, 471 (1935).
[6] A. Flammersfeld, Zeits. f. Physik **112**, 727 (1939).
[7] E. M. Lyman, Phys. Rev. **51**, 1 (1937).
[8] G. J. Neary, Proc. Roy. Soc. **175**, 71 (1940).
[9] R. E. Marshak, Phys. Rev. **61**, 431 (1942).
[10] W. J. Henderson, Phys. Rev. **55**, 238 (1939).
[11] O. Hahn, F. Strassmann, and E. Walling, Naturwiss. **25**, 189 (1937).
[12] A. Hemmendinger and W. K. Smythe, Phys. Rev. **51**, 1052 (1937).
[13] E. J. Konopinski and H. A. Bethe, Phys. Rev. **53**, 679 (1938).

The subscript, $n=0, 1, 2, \cdots n$, designates an "allowed," "first," "second," or "nth forbidden" transition; $X=S, V, T, A, P$ denotes the form of interaction, scalar, polar vector, tensor, axial vector, or pseudo-scalar, respectively. G is the Fermi constant; to obtain the half-life t_{nX} in seconds the magnitude of $(2\pi^3 \ln 2/G^2)$ is taken to be about $(6)10^3$ sec. for the interactions T or A. The value $\approx(3)10^3$ sec. is more appropriate for the others.[14] The function, $F_0(W, Z)$, precisely defined by Eq. (15), represents the effect of the nuclear charge in the "allowed" energy distribution, $F_0 pWq^2dW$. W is the electron's total energy in units[15] of mc^2; $p=(W^2-1)^{\frac{1}{2}}$ is the momentum of the electron, and $q=W_0-W$ is the energy taken by the neutrino. W_0 is the maximum energy available to the electron in the transition. The correction to the "allowed" formula which the "nth forbidden" transition requires is given by the factor,

$$C_{nX}=(2p^2F_0)^{-1}\int d\omega \sum_{jlm}\left|\int d\tau H_{nX}\right|^2, \quad (3)$$

in which H_{nX} has the invariant forms:

$$H_{nV}=[(V^*QU)(\psi^*\phi) - (V^*\boldsymbol{\alpha}^HQU)\cdot(\psi^*\boldsymbol{\alpha}\phi)]_n, \quad (4V)$$

$$H_{nT}=[(V^*\beta^H\boldsymbol{\sigma}^HQU)\cdot(\psi^*\beta\boldsymbol{\sigma}\phi) + (V^*\beta^H\boldsymbol{\alpha}^HQU)\cdot(\psi^*\beta\boldsymbol{\alpha}\phi)]_n. \quad (4T)$$

Only the polar vector and tensor interactions are presented here; however, the formulas for the correction factors, C_{nS}, C_{nA}, and C_{nP} are derived as well. $\int d\tau H_{nX}$ is the matrix element of the interaction between the nucleons of the parent nucleus and the electron-neutrino field. U and V are the wave functions of the initial and final nucleus. The operator Q by definition transforms a neutron into a proton. The square of the matrix element is integrated over the momentum and spin directions of the neutrino and summed over the spin and angular momentum quantum numbers, j, l, m, of the electron in the coulomb field. The Dirac electron wave functions ψ are to be normalized to one particle per sphere of unit radius.[16] ϕ is the four-component wave function of the neutrino. The operators $\boldsymbol{\sigma}$, $\boldsymbol{\alpha}$, and β, in (4) are the usual Dirac four-by-four matrices. The superscript H indicates that the operations are on the nuclear wave function.

The evaluation of the correction factors through "second forbidden" transitions was described in II. Briefly, the method is to expand the electron and neutrino wave functions in powers of their cartesian position coordinates x_i. The matrix elements will then contain, besides the vector operators $\boldsymbol{\sigma}$ and $\boldsymbol{\alpha}$, components x_i of the position vector $\mathbf{r}$. The final expressions must be invariant with respect to three-dimensional spacial rotations. Thus they must be expressible in terms of tensors made up of components of the vectors $\boldsymbol{\sigma}$ and $\boldsymbol{\alpha}$ in combination with the components x_i. For second and higher rank tensors formed in this way it becomes necessary to separate them into their irreducible representations. The irreducible tensors required through the "second forbidden" transitions, already constructed by Konopinski and Uhlenbeck (II), are designated in this paper as follows:

Scalars:

$$Q_0(s)=\int d\tau V^* s U, \quad (5)$$

where s may be any of the heavy particle scalar or pseudo-scalar operators,[17]

1, β, $(\boldsymbol{\sigma}\cdot\mathbf{r})$, $(\boldsymbol{\alpha}\cdot\mathbf{r})$, $\beta(\boldsymbol{\sigma}\cdot\mathbf{r})$, $\beta(\boldsymbol{\alpha}\cdot\mathbf{r})$, γ_5, or $\beta\gamma_5$.

Vectors:

$$Q_i(\mathbf{a})=\int d\tau V^* a_i U, \quad (6)$$

where $\mathbf{a}$ may be any of the vector operators, $\mathbf{r}$, $\boldsymbol{\sigma}$, $\boldsymbol{\alpha}$, $[\boldsymbol{\sigma}\times\mathbf{r}]$, or $[\boldsymbol{\alpha}\times\mathbf{r}]$, multiplied by any scalar operator.

Second-rank tensors:

$$Q_{ij}(\mathbf{a}, \mathbf{r})=\int d\tau V^*[a_{(i}x_{j)}-\tfrac{1}{3}\delta_{(ij)}(\mathbf{a}\cdot\mathbf{r})]U, \quad (7)$$

where the a_i's are components of the vector

[14] E. J. Konopinski, *Advances in Nuclear Physics* (Interscience Publishers, Inc.), in press.

[15] The units used throughout this paper are: mc^2=unit of energy, mc=unit of momentum, $\hbar/mc^2$=unit of time, and $\hbar/mc$=unit of length, where m is the rest mass of an electron.

[16] M. E. Rose, Phys. Rev. **51**, 484 (1937).

[17] The superscript H is dropped; all operators in the nuclear matrix elements are obviously heavy particle operators. The operator Q is also omitted from all the nuclear matrix elements.

TABLE II. Selection rules.

Interaction		"nth forbidden"	Completely "forbidden"
Scalar	Matrix ΔJ Parity Change	$Q_n(\beta\mathbf{r},\mathbf{r})$ $(n-1)', n$ n_{odd} — yes, n_{even} — no	$\Delta J=(0\leftrightarrow n_{\text{even}})$ — yes $\Delta J=(0\leftrightarrow n_{\text{odd}})$ — no
Polar V.	Matrix ΔJ Parity Change	$Q_n(\mathbf{r},\mathbf{r})$, $Q_n(\boldsymbol{\alpha},\mathbf{r})$, $Q_{n-1}([\boldsymbol{\alpha}\times\mathbf{r}],\mathbf{r})$ $(n-1)', n$, $(n-1)', n$, $(n-1)$ n_{odd} — yes, n_{even} — no	$\Delta J=(0\leftrightarrow 0)$ — yes
Tensor	Matrix ΔJ Parity Change	$Q_n(\beta[\boldsymbol{\sigma}\times\mathbf{r}],\mathbf{r})$, $Q_n(\beta\boldsymbol{\alpha},\mathbf{r})$, $Q_{n+1}(\beta\boldsymbol{\sigma},\mathbf{r})$ n, n, $n', (n+1)$ n_{odd} — yes, n_{even} — no	None
Axial V.	Matrix ΔJ Parity Change	$Q_n([\boldsymbol{\sigma}\times r],\mathbf{r})$, $Q_{n+1}(\boldsymbol{\sigma},\mathbf{r})$ n, $n', (n+1)$ n_{odd} — yes, n_{even} — no	$\Delta J=(0\leftrightarrow 0)$ — no
Pseudo-S.	Matrix ΔJ Parity Change	$Q_n(\beta\gamma_5\mathbf{r},\mathbf{r})$ $(n-1)', n$ n_{odd} — no, n_{even} — yes	$\Delta J=(0\leftrightarrow n_{\text{even}})$ — no $\Delta J=(0\leftrightarrow n_{\text{odd}})$ — yes

operators given in (6). Q_{ij} is the irreducible tensor with zero spur.[18] The parentheses around the subscripts indicate that there is a term for each permutation of the indices. For "third" and "fourth forbidden" transitions, third-, fourth-, and fifth-rank tensors are required. The third-rank irreducible tensors are defined by

$$Q_{ijk}(\mathbf{a},\mathbf{r})=\int d\tau V^*[a_{(i}x_jx_{k)} -1/5\delta_{(ij}a_{k)}r^2-2/5\delta_{(ij}x_{k)}(\mathbf{a}\cdot\mathbf{r})]U. \quad (8)$$

Similarly the fourth-rank tensors have the form

$$Q_{ijkl}(\mathbf{a},\mathbf{r})=\int d\tau V^*[a_{(i}x_jx_kx_{l)} -3/7\delta_{(ij}x_kx_{l)}(\mathbf{a}\cdot\mathbf{r})-3/7\delta_{(ij}a_kx_{l)}r^2 +3/35\delta_{(ij}\delta_{kl)}(\mathbf{a}\cdot\mathbf{r})r^2]U. \quad (9)$$

All rank tensors are completely symmetric with respect to any permutation of their indices, and all "spurs" are zero; i.e.,

$$\sum_{\mu=1}^{3} Q_{\mu\mu kl\cdots m}=0. \quad (10)$$

"Fourth forbidden" tensor and axial vector interactions would require the fifth-rank tensor, $Q_{ijklm}(\boldsymbol{\sigma},\mathbf{r})$. However, in the specific cases treated it will not be required.

The selection rules for which the various irreducible tensors are non-vanishing were given in II through the "second forbidden" approximation. Table II extends the spin and parity selection rules to the "nth forbidden" transitions.

The first column of Table II lists the five forms of interaction. For each form of interaction the second column contains the irreducible tensor matrix element appropriate to the "nth forbidden" approximation. The subscript n indicates that the tensor is of nth rank. There are tensors of lower rank in the "nth forbidden" transitions than those listed. The selection rules corresponding to these are already included in the "$(n-2)$ forbidden" degree. Therefore, it is seen that they constitute negligible corrections to the "$(n-2)$ forbidden" matrix elements. Beneath the matrix element is listed the spin changes for which the element is non-vanishing. The primes indicate that $|J_i|+|J_f|\geqslant J_{\max}$. As in II, "yes" means that the transition must be even$\leftrightarrow$odd; "no" indicates that the parities of initial and final nucleus must be the same. Particular combinations of spin and parity changes are completely "forbidden" in any approximation by four of the interactions. These cases are given in the last column. Table II is complete except for the "first" and "second forbidden" tensor and axial vector interactions. These omissions are certain scalars which are given in Konopinski and Uhlenbeck's Table I.[1]

[18] The tensors (7) are identified with those of paper II as follows:

$$Q_{ij}(\boldsymbol{\sigma},\mathbf{r})\equiv B_{ij},\quad Q_{ij}(\boldsymbol{\alpha},\mathbf{r})\equiv A_{ij},\quad Q_{ij}(\mathbf{r},\mathbf{r})\equiv 2R_{ij},$$
$$Q_{ij}([\boldsymbol{\sigma}\times\mathbf{r}],\mathbf{r})\equiv T_{ij},\quad Q_{ij}([\boldsymbol{\alpha}\times\mathbf{r}],\mathbf{r})\equiv \int[\boldsymbol{\alpha}\times\mathbf{r}]_{(i}x_{j)}.$$

The notation used here is convenient for expression of the general nth-rank tensors, $Q_n(\mathbf{a},\mathbf{r})$, introduced.

III. GENERAL FORMULAS FOR C_{nX}

The correction factors through "fourth forbidden" were derived for all the forms of interaction by the method outlined in the previous section. Upon inspection it was found that all the correction factors could be written in a generalized "nth forbidden" form.

$$C_{nS}=|Q_n(\beta\mathbf{r},\mathbf{r})/n!|^2\sum_{\nu=0}^{n}(A_{n\nu}q^{2(n-\nu)-2}M_\nu+2C_{n\nu}q^{2(n-\nu)-1}N_\nu+D_{n\nu}q^{2(n-\nu)}L_\nu),$$

$$\begin{aligned}C_{nV}=&|Q_n(\mathbf{r},\mathbf{r})/n!|^2\sum_{\nu=0}^{n}(A_{n\nu}q^{2(n-\nu)-2}M_\nu-2C_{n\nu}q^{2(n-\nu)-1}N_\nu+D_{n\nu}q^{2(n-\nu)}L_\nu)\\&+|Q_n(\boldsymbol{\alpha},\mathbf{r})/n!|^2\sum_{\nu=0}^{n}(A_{n\nu}q^{2(n-\nu)-2}L_\nu)+i[Q_n(\boldsymbol{\alpha},\mathbf{r})/n!\cdot Q_n^*(\mathbf{r},\mathbf{r})/n!-\text{c.c.}]\\&\times\sum_{\nu=0}^{n}(-A_{n\nu}q^{2(n-\nu)-2}N_\nu+C_{n\nu}q^{2(n-\nu)-1}L_\nu)+|Q_{n-1}([\boldsymbol{\alpha}\times\mathbf{r}],\mathbf{r})/(n-1)!|^2\\&\times\sum_{\nu=0}^{n-1}(A_{(n-1)\nu}q^{2(n-1-\nu)-2}M_\nu+2C_{(n-1)\nu}q^{2(n-1-\nu)-1}N_\nu+[D_{(n-1)\nu}-B_{(n-1)\nu}/n]q^{2(n-1-\nu)}L_\nu),\end{aligned}$$

$$\begin{aligned}C_{nT}=&|Q_n(\beta[\boldsymbol{\sigma}\times\mathbf{r}],\mathbf{r})/n!|^2\sum_{\nu=0}^{n}(A_{n\nu}q^{2(n-\nu)-2}M_\nu-2C_{n\nu}q^{2(n-\nu)-1}N_\nu\\&+[D_{n\nu}-B_{n\nu}/(n+1)]q^{2(n-\nu)}L_\nu)+|Q_n(\beta\boldsymbol{\alpha},\mathbf{r})/n!|^2\sum_{\nu=0}^{n}(A_{n\nu}q^{2(n-\nu)-2}L_\nu)\\&-[Q_n(\beta[\boldsymbol{\sigma}\times\mathbf{r}],\mathbf{r})/n!\cdot Q_n^*(\beta\boldsymbol{\alpha},\mathbf{r})/n!+\text{c.c.}]\sum_{\nu=0}^{n}(-A_{n\nu}q^{2(n-\nu)-2}N_\nu+C_{n\nu}q^{2(n-\nu)-1}L_\nu)\\&+|Q_{n+1}(\beta\boldsymbol{\sigma},\mathbf{r})/(n+1)!|^2\sum_{\nu=0}^{n}(B_{n\nu}q^{2(n-\nu)}L_\nu),\end{aligned}\tag{11}$$

$$\begin{aligned}C_{nA}=&|Q_n([\boldsymbol{\sigma}\times\mathbf{r}],\mathbf{r})/n!|^2\sum_{\nu=0}^{n}(A_{n\nu}q^{2(n-\nu)-2}M_\nu+2C_{n\nu}q^{2(n-\nu)-1}N_\nu\\&+[D_{n\nu}-B_{n\nu}/(n+1)]q^{2(n-\nu)}L_\nu)+|Q_{n+1}(\boldsymbol{\sigma},\mathbf{r})/(n+1)!|^2\sum_{\nu=0}^{n}(B_{n\nu}q^{2(n-\nu)}L_\nu).\end{aligned}$$

C_{nP} is the same as C_{nS} except that $Q_n(\beta\gamma_5\mathbf{r},\mathbf{r})$ replaces $Q_n(\beta\mathbf{r},\mathbf{r})$. $|Q_n/n!|^2$ is the sum of the squares of the absolute values of the components of the tensors, the summation sign over the n indices being omitted. The quantities, $A_{n\nu}$, $B_{n\nu}$, $C_{n\nu}$, and $D_{n\nu}$, are the numerical coefficients:

$$\begin{aligned}A_{n\nu}&=\frac{(n-\nu)2^{n-2\nu}(2\nu+1)!}{(2n-2\nu)!(\nu!)^2},\quad B_{n\nu}=\frac{2^{n-2\nu}(2\nu+1)!}{(2n-2\nu+1)!(\nu!)^2},\\C_{n\nu}&=\frac{(n-\nu)2^{n-2\nu}(2\nu+1)!}{(2n-2\nu+1)!(\nu!)^2},\quad D_{n\nu}=\frac{2^{n-2\nu}(\nu+1)(2\nu)!}{(2n-2\nu+1)!(\nu!)^2}.\end{aligned}\tag{12}$$

L_ν, M_ν, and N_ν are combinations of the radial part of the electron wave functions evaluated at $r=\rho$, the nuclear radius.

$$\begin{aligned}L_\nu&=(2p^2F_0)^{-1}(g_\nu^2+f_{-(\nu+2)}^2)\rho^{-2\nu},\\M_\nu&=(2p^2F_0)^{-1}(f_\nu^2+g_{-(\nu+2)}^2)\rho^{-2(\nu+1)},\\N_\nu&=(2p^2F_0)^{-1}(f_\nu g_\nu-f_{-(\nu+2)}g_{-(\nu+2)})\rho^{-2\nu-1}.\end{aligned}\tag{13}$$

Expanding the radial functions in ascending powers of ρ, one obtains the first approximation formulas for L, M, and N.

$$L_\nu = (F_\nu/F_0)\left(\frac{2^\nu \nu!}{(2\nu+1)!}p^\nu\right)^2 \frac{\nu+1+S_\nu}{2\nu+2},$$

$$M_\nu = (F_\nu/F_0)\left(\frac{2^{\nu+1}\nu!}{(2\nu+2)!}p^\nu\right)^2 \left[\frac{2\nu+2}{\nu+1+S_\nu}\left(\frac{\alpha Z}{2\rho}\right)^2 + \left(\frac{S_\nu}{2S_\nu+1}p^2/W - \frac{(2\nu+1)(\alpha Z)^2 W}{(2S_\nu+1)(\nu+1+S_\nu)}\right)\left(\frac{\alpha Z}{\rho}\right)\right.$$
$$\left. + \frac{(\nu+1)(S_\nu-\nu)S_\nu}{(2S_\nu+1)^2}\left(1-\frac{4S_\nu+3}{S_\nu(S_\nu+1)}(\alpha Z)^2\right)p^2 + \left(1+\frac{\nu(4S_\nu+3)}{(S_\nu+1)(\nu+1+S_\nu)}(\alpha Z)^2\right)\left(\frac{\alpha Z}{2S_\nu+1}\right)^2\right], \tag{14}$$

$$N_\nu = -(F_\nu/F_0)\left(\frac{2^\nu \nu!}{(2\nu+1)!}p^\nu\right)^2 \frac{1}{\nu+1}\left[\left(\frac{\alpha Z}{2\rho}\right) + \frac{S_\nu}{2S_\nu+1}p^2/W - \frac{2(\alpha Z)^2}{2S_\nu+1}W\right],$$

where

$$F_\nu(W,Z) = \left(\frac{(2\nu+2)!}{\nu!}\right)^2 (2p\rho)^{2(S_\nu-\nu-1)} \exp(\pi y)\,|\Gamma(S_\nu+iy)|^2/\Gamma^2(1+2S_\nu) \tag{15}$$

and

$$S_\nu = [(\nu+1)^2-(\alpha Z)^2]^{\frac{1}{2}}, \quad y = \alpha Z W/p.$$

For positron emission $-Z$ replaces Z; $\alpha = 1/137$ is the fine structure constant. Formula (14) was used for RaE; however, for lighter elements, $A<100$, simplifications valid for $\alpha Z \ll 1$ are sufficient. In these cases $S_\nu \approx \nu+1$ and $F_\nu \approx F_0$ for any ν. L, M, and N then reduce to the simple expressions

$$L_\nu = a_\nu^2 p^{2\nu}, \quad M_\nu = a_{\nu+1}^2 p^{2\nu}\left[b_\nu^2\left(\frac{\alpha Z}{2\rho}\right)^2 + b_\nu\left(\frac{\alpha Z}{\rho}\right)p^2/W + p^2\right], \quad N_\nu = -a_\nu^2 p^{2\nu}\left[b_\nu\left(\frac{\alpha Z}{2\rho}\right) + p^2/W\right], \tag{16}$$

where a_ν, and b_ν are the numerical factors, $a_\nu = 2^\nu \nu!/(2\nu+1)!$, $b_\nu = (2\nu+3)/(\nu+1)$. The correction factors given above are complete except for C_{1T}, C_{1A}, and C_{2T}. The omitted terms apply only to $0 \leftrightarrow 0$ transitions and are given in II. The characteristic features of the correction factors (11), have been discussed by Konopinski and Uhlenbeck in their effect on the shape of the spectra.

The evaluation of the lifetimes requires some estimate of the size of the nuclear matrix elements, (5) to (9), inclusive. An exact evaluation will not be possible until more is known concerning nuclear states. The comparable problem was met in the case of the "allowed" transitions by assuming the order of magnitude unity for the square of the matrix elements, $|Q_0(1)|^2$ and $\sum_i |Q_i(\beta\sigma)|^2$, which is consistent with normalized nuclear wave functions. In the "forbidden" case the situation is complicated by the fact that the irreducible tensors Q_n were constructed without regard to normalization. In symmetrizing them the $n!$ permutations of their indices introduce many terms. The spurious effect of this repetition of terms may be removed by introducing the normalization factors, $1/n!$, as is done in the formulas (11).

In order to obtain a more conservative estimate of the order of magnitude of the nuclear matrix elements, a method of averaging the tensor directions over a sphere may be adopted. To do this, the tensor $Q_{ij}([\sigma \times \mathbf{r}], \mathbf{r})$, for example, is treated as if it were made up of ordinary independent vectors, $(\mathbf{r})$ and (σ), having the magnitudes ρ and (σ), respectively. This procedure is at least consistent with the above-mentioned treatment of the "allowed" matrix elements. Two of the "first forbidden" tensor interaction matrix elements estimated in this way are $\sum_i |Q_i(\beta\alpha)|^2 \approx (\beta)^2(\alpha)^2$ and $\sum_i |Q_i(\beta[\sigma \times \mathbf{r}])|^2 \approx \frac{2}{3}(\beta)^2(\sigma)^2\rho^2$. The normalized second- and higher-rank tensors may be estimated by applying step by step the identities:

$$\sum_{ij}|Q_{ij}(\mathbf{a},\mathbf{r})/2!|^2=\sum_{ij}\left|\int d\tau V^*a_ix_jU\right|^2-\tfrac{1}{2}\sum_i|Q_i([\mathbf{a}\times\mathbf{r}])|^2-\tfrac{1}{3}|Q_0(\mathbf{a}\cdot\mathbf{r})|^2,$$

$$\sum_{ijk}|Q_{ijk}(\mathbf{a},\mathbf{r})/3!|^2=\sum_{ijk}\left|\int d\tau V^*a_ix_jx_kU\right|^2-\tfrac{2}{3}\sum_{ij}|Q_{ij}([\mathbf{a}\times\mathbf{r}],\mathbf{r})/2!|^2-\tfrac{3}{5}\sum_i|Q_i((\mathbf{a}\cdot\mathbf{r})\mathbf{r})|^2$$
$$+\tfrac{1}{5}\sum_i[Q^*_i((\mathbf{a}\cdot\mathbf{r})\mathbf{r})\cdot Q_i(\mathbf{a}r^2)+\text{c.c.}]-\tfrac{2}{5}\sum_i|Q_i(\mathbf{a}r^2)|^2, \tag{17}$$

$$\sum_{ijkl}|Q_{ijkl}(\mathbf{a},\mathbf{r})/4!|^2=\sum_{ijkl}\left|\int d\tau V^*a_ix_jx_kx_lU\right|^2-\tfrac{3}{4}\sum_{ijk}|Q_{ijk}([\mathbf{a}\times\mathbf{r}],\mathbf{r})/3!|^2$$
$$-\tfrac{5}{7}\sum_{ij}|Q_{ij}((\mathbf{a}\cdot\mathbf{r})\mathbf{r},\mathbf{r})/2!|^2+\tfrac{2}{7}\sum_{ij}[Q^*_{ij}((\mathbf{a}\cdot\mathbf{r})\mathbf{r},\mathbf{r})\cdot Q_{ij}(\mathbf{a}r^2,\mathbf{r})+\text{c.c.}]/(2!)^2$$
$$-\tfrac{5}{7}\sum_{ij}|Q_{ij}(\mathbf{a}r^2,\mathbf{r})/2!|^2-\tfrac{3}{10}\sum_i|Q_i([\mathbf{a}\times\mathbf{r}]r^2)|^2-\tfrac{1}{5}|Q_0((\mathbf{a}\cdot\mathbf{r})r^2)|^2.$$

Upon applying the above method one obtains the following estimated magnitudes for the square of the normalized matrix elements:

$$|Q_n(s\mathbf{r},\mathbf{r})/n!|^2\approx N_n(s)^2\rho^{2n}$$
where
$$N_n=2^n(n!)^2/(2n)! \tag{18}$$

and (s) is the order of magnitude of the scalars (β), 1, and $(\beta\gamma_5)$ for the scalar, polar vector, and pseudo-scalar interactions, respectively.

$$|Q_n(s\boldsymbol{\sigma},\mathbf{r})/n!|^2$$
$$\approx(2n+1)/3n\cdot N_n(s)^2(\sigma)^2\rho^{2n-2}. \tag{19}$$

Here, $(s)=1$ for the axial vector interaction and $(s)=(\beta)$ for the first term of $(4T)$, $(s)=(\beta\gamma_5)$ for the second term, and $(s)=(\gamma_5)$ for the second term of $(4V)$.[19] Finally, the matrix elements involving the components of the vector products, $[\boldsymbol{\sigma}\times\mathbf{r}]$ and $[\boldsymbol{\alpha}\times\mathbf{r}]=\gamma_5[\boldsymbol{\sigma}\times\mathbf{r}]$, have the magnitudes:

$$|Q_n(s[\boldsymbol{\sigma}\times\mathbf{r}],\mathbf{r})/n!|^2$$
$$\approx(n+1)/3n\cdot N_n(s)^2(\sigma)^2\rho^{2n}. \tag{20}$$

Cross products of the matrix elements of the type

$$i[Q_n(\boldsymbol{\alpha},\mathbf{r})/n!\cdot Q^*_n(\mathbf{r},\mathbf{r})/n!-\text{c.c.}]$$
and
$$[Q_n(\beta[\boldsymbol{\sigma}\times\mathbf{r}],\mathbf{r})/n!\cdot Q^*_n(\beta\boldsymbol{\alpha},\mathbf{r})/n!+\text{c.c.}],$$

appearing in C_{nV} and C_{nT}, vanish upon applying the method of evaluation described above. One would expect such interference terms to be very small on the basis that the phases of the individual terms of the sum are random; i.e., the presence of both positive and negative terms would tend to cancel each other and result in negligible values for their sum.

In estimating the sizes of the matrix elements (18), (19), and (20), the order of magnitudes of $(\beta)^2$, $(\sigma)^2$, $(\gamma_5)^2$ and $(\beta\gamma_5)^2$ must be introduced. It was assumed that $(\beta)^2=(\sigma_i)^2\approx 1$ for the usual reason that the operators β and σ_i cause the large components of the nuclear wave functions to be multiplied together. Accordingly the magnitude, $(\sigma)^2=(\sigma_1)^2+(\sigma_2)^2+(\sigma_3)^2\approx 3$, emerges. On the other hand, the operators γ_5 and $\beta\gamma_5$ mix the large and small components of the wave functions resulting in values of $(\gamma_5)^2$ and $(\beta\gamma_5)^2$ smaller by a factor of $\approx(v/c)^2$, where v is the order of magnitude of the velocity of nuclear particles. One would expect v to be about 1/10 the velocity of light. Critchfield[20] obtained $(\beta\gamma_5)^2\approx 0.1$ by assuming special (s and p) nuclear wave functions; however, his introduction of such wave functions led to a much smaller magnitude for $(\gamma_5)^2$ alone: namely, $(\gamma_5)^2\approx 10^{-5}W_0^2$. A similar small magnitude was obtained by Marshak who made use of the quantum-kinematical relation, $\int d\tau V^*\boldsymbol{\alpha}U=iW_0\int d\tau V^*\mathbf{r}U$. This yields the value, $(\alpha)^2=(\gamma_5)^2(\sigma)^2\approx W_0^2\rho^2$. This relation is applicable only if the coulomb energy differences

[19] The operator $\boldsymbol{\alpha}$, appearing in the second terms of (4), may be written $\boldsymbol{\alpha}=\gamma_5\boldsymbol{\sigma}$ where $\gamma_5=-i\alpha_1\alpha_2\alpha_3$.

[20] C. L. Critchfield, Phys. Rev. 61, 249 (1942).

between the initial and final nuclei are ignored. For most of the decays treated the coulomb energy difference is larger than the electron's maximum energy. In this paper the magnitudes, $(\beta\gamma_5)^2 \approx (\gamma_5)^2 \approx (v/c)^2$, are assumed. $(v/c)^2$ is considered as a parameter having the range of values $(v/c)^2 \approx ((W_0\rho)^2 - 0.1)$. The value of the nuclear radius, $\rho = 0.004A^{\frac{1}{3}}$, which is roughly five percent larger than that used by Marshak, is assumed in the cases treated here.

In deriving the formulas (11), it was tacitly assumed that transitions involving a spin change, $\Delta J = |J_i - J_f|$, could lead to $2\Delta J + 1$ degenerate magnetic states of the electron plus neutrino. This is in effect what is assumed when the sum over the quantum numbers m is performed as indicated in Eq. (3). Marshak has shown that if $J_i > J_f$ only one value of the electron-neutrino total magnetic quantum number, $\mu = k = \Delta J$ is permitted. $k = \Delta J$ is the value of the electron-neutrino total angular momentum that gives the largest contribution to the transition probability. Larger values of k that are possible yield probabilities smaller by a factor of $\approx (W_0\rho)^2$ and are ignored by both methods of treatment. Consequently half-lives calculated from Eq. (1) should be multiplied by $2\Delta J + 1$ if $J_i > J_f$.

The relation between the nuclear matrix elements (18), (19), and (20), and those used by Marshak (constructed from spherical harmonics) was investigated. It was found that the half-life formulas were identical if Marshak's normalized matrix elements $|M_n|^2$, $|M_{n-1}'|^2$, and $|M_{n-1}''|^2$ were evaluated by the averaging process just discussed. Performing these averages we find the values $|M_n|^2 \approx |M_{n-1}'|^2 \approx 1$ for $(\gamma_5)^2 \approx (W_0\rho)^2$, $(\sigma)^2 = 3$. Those matrix elements containing the components of $[\boldsymbol{\sigma} \times \mathbf{r}]$ are slightly smaller. By averaging, the values, $|M_{n-1}''|^2 \approx (n+1)/(2n+1)$, were obtained.

Aside from the difference in form of the nuclear matrix elements a more serious difference between Marshak's calculations and those presented here appears in the cases of unfavorable parity change from the tensor or polar vector interaction. He ignores the terms linear in $(\alpha Z/\rho)$ and independent of $(\alpha Z/\rho)$ which appear in the formula (16) for M_ν, and completely ignores L_ν and N_ν, on the basis that $(\alpha Z) \ll 1$. This approximation is good only as long as the maximum kinetic energy of the electron is very small in comparison to the coulomb energy difference $(\alpha Z/\rho)$. The energy dependent factors multiplying $(\alpha Z/\rho)$ in M_ν and N_ν contain the energy to one degree higher than in the factor multiplying $(\alpha Z/\rho)^2$. In the case of Rb^{87} ($W_0 - 1 = 0.26$, $(\alpha Z/\rho) = 15.2$) the error introduced is insignificant; however, for K^{40} ($W_0 - 1 = 1.4 - 2.6$, $(\alpha Z/\rho) = 10.1$) dropping all except the terms in $(\alpha Z/\rho)^2$ increases the calculated "fourth forbidden" half-life by a factor of 2 on the basis of Marshak's values of ρ and (α). The discrepancy is aggravated when the larger values of ρ and (α) used in this paper are applied. (See Table I.)

IV. COMPARISON WITH EXPERIMENT

The theoretical half-lives of RaE, P^{32}, K^{40}, and Rb^{87} listed in Table I were calculated by numerical integration of Eq. (1) for the tensor and polar vector interactions. The values $(6)10^3$ sec. and $(3)10^3$ sec. were assumed for $(2\pi^3 \ln 2/G^2)$ for the tensor and polar vector interactions, respectively.[14] The calculated half-lives listed for the cases of unfavorable parity change correspond to the range of values, $(v/c)^2 = [0.1 - (W_0\rho)^2]$. The shape of the electron energy distribution is very sensitive to changes in (v/c) as can be seen from the calculated average electron energies. The spin and parity change for each transition is listed in the first column. The $(f_{nX}t_{\exp})$ values listed in the last column give the order of magnitude of $(2\pi^3 \ln 2/G^2)$ that would yield the experimental half-life.

RaE

Previous to the work of Konopinski and Uhlenbeck[21] it was generally supposed that the half-life of RaE could be accounted for by the "first forbidden" approximation. However, these authors have shown that the distribution in energy of the emitted β-rays could be predicted only by assuming $\Delta J = (2 \rightarrow 0)$, "no," which requires the "second forbidden" approximation.

[21] The results for RaE presented here are based on the numerical calculations performed by Konopinski and Uhlenbeck. I am greatly indebted to Dr. Konopinski for access to the numerical tables of the RaE correction factors. The nuclear radius, $\rho = 0.027$, used in paper II for RaE is also used here in estimating the magnitudes of the nuclear matrix elements.

It remained to be shown that the half-life could also be obtained in this approximation.

For an element as heavy as RaE the formulas (14) are required. Using the estimated values of the matrix elements given by Eqs. (19) and (20), one obtains in the "first" and "second forbidden" approximations:

$$f_{1T}(3.3,\,83)\approx(7.9)10^{-2}, \tag{21}$$

$$f_{2T}(3.3,\,83)\approx[4.1+79(v/c)^2]10^{-3}, \tag{22}$$

$$f_{2V}(3.3,\,83)\approx[8.2+79(v/c)^2]10^{-3}. \tag{23}$$

The half-life given by the "first forbidden" calculation (21) is too small by a factor of only $\frac{1}{5}$. Thus the evidence of the calculated half-life alone does not eliminate the possibility of accounting for the RaE decay in the "first forbidden" approximation. However, the "second forbidden" formulas (22) and (23) yield calculated half-lives equally satisfactory. Moreover, the fact that the shape of the energy distribution can be reproduced accurately only by the "second forbidden" approximation, indicates that by either Gamow-Teller or Fermi selection rules the transition involves no change of parity with a spin change of $\Delta J=(2\rightarrow 0)$. The value of (v/c) that yields the best reproduction of the electron energy distribution according to the tensor interaction is $(v/c)\approx 0.16$. Using this value of (v/c) one obtains $(f_{2T}t_{\text{exp}})=(2\pi^3\ln 2/G^2)\approx(2.6)10^3$ sec. as compared to the value, $\approx(6)10^3$ sec., appropriate for "allowed" transitions.[14]

A slightly better energy distribution can be obtained by including the cross products of the matrix elements as is done in paper II. There the ratio, $Q_{ij}(\beta\boldsymbol{\alpha},\mathbf{r})/Q_{ij}(\beta[\boldsymbol{\sigma}\times\mathbf{r}],\mathbf{r})\approx-5.8$, was used in order to obtain the best spectrum. On the other hand, this procedure is equivalent to a different evaluation of the nuclear matrix element, $Q_{ij}(\boldsymbol{\alpha},\mathbf{r})$. If one assumes the ordinary vector $(\boldsymbol{\alpha})$ perpendicular to $(\mathbf{r})$ and $(\boldsymbol{\sigma})$ and of magnitude $(\alpha)=(\sigma)(v/c)$, one may obtain such a non-vanishing interference term. The ratio used in paper II corresponds to the magnitude, $(v/c)=0.16$, which yields an (ft) value of $(6.4)10^3$ sec.

Although the inclusion of the interference term is thus seen to give an excellent account of the observed half-life and energy distribution, its inclusion is not essential. The fact that the cross term contributes more than half of the probability of electron emission, when the ratio, -5.8, is assumed, perhaps indicates that its inclusion is not to be preferred over the simpler evaluations that lead to vanishing interference terms.

P^{32}

The best reproduction of the observed energy spectrum of P^{32} was obtained in II by adjusting the ratio of the nuclear matrix elements, $Q_{ij}(\beta\boldsymbol{\alpha},\mathbf{r})/Q_{ij}(\beta[\boldsymbol{\sigma}\times\mathbf{r}],\mathbf{r})\approx-2.2$, and including the cross term. Using the same method of evaluation of these matrix elements as mentioned in the previous paragraph, one obtains for $\Delta J=(2\rightarrow 0)$, "no" and $W_0=4.37$[22] a theoretical half-life too large by a factor of 100. The observed half-life of P^{32} is 14.07 ± 0.01 days.[23]

An equally good reproduction of the energy distribution can be obtained with the simpler assumptions used in evaluating the matrix elements. [See Eqs. (19) and (20).] For the tensor and polar vector interactions one obtains:

$$f_{1T}(4.37,\,15)\approx(1.1)10^{-2}, \tag{24}$$

$$f_{2T}(4.37,\,15)\approx[1.9+(1.1)10^3(v/c)^2]10^{-5}, \tag{25}$$

$$f_{2V}(4.37,\,15)\approx[3.9+(1.1)10^3(v/c)^2]10^{-5}. \tag{26}$$

The best energy distribution is given by $C_{2T}F_0pWq^2dW$ when $(v/c)\approx 0.029$ is assumed, which is considerably less than the minimum value $W_0\rho$ used in Table I. However, the half-life predicted is ≈ 200 times too large. In order to obtain the correct half-life, one must assume $(v/c)=0.67$ in (25), which is unreasonably large.

The "first forbidden" half-life listed in Table I is too small by a factor of only $\frac{1}{2}$, while f_{2V} and f_{2T} give half-lives at least two and four times too large, respectively. The energy distribution for these three alternative cases is practically the same; it deviates somewhat from the observed spectra.[22] In view of the fact that the energy distribution favors only slightly the "second forbidden" approximation the evidence of the half-life indicates that P^{32} is probably a

[22] J. L. Lawson, Phys. Rev. 56, 131 (1939).
[23] D. Mulder, G. W. Hoeksema, and G. J. Sizoo, Physica 7, 849 (1940).

"first forbidden" transition according to Gamow-Teller selection rules with $\Delta J=(2\rightarrow 0)$, the transition being from the P^{32} state of odd parity to the ground state of S^{32} which has even parity.

K^{40}

The known spin change[3] of K^{40} requires at least the "third forbidden" approximation if Gamow-Teller selection rules are to be preferred to Fermi rules. In substantial agreement with the calculations of Marshak,[9] it was found that the observed half-life,[24] $t_{exp}=(4.5\pm 0.9)10^{16}$ sec., could be given by the "third forbidden" tensor interaction if the maximum energy has approximately the value $W_0\approx 2.4$, used here. On the other hand, if the end-point energy[10] amounts to as much as $3.6\,mc^2$, the "third forbidden" half-life calculated is too small by a factor $\approx 1/50$, and the "fourth forbidden" tensor or polar vector interactions yield results even better than the "third forbidden" approximation with $W_0=2.4$.

The average energy quoted by Bramley and Brewer,[24] $\bar{E}=0.35$ Mev, is in good agreement with the "third forbidden" value calculated with $W_0=2.4$. For $W_0=3.6$ the average energy according to the "fourth forbidden" tensor or polar vector interactions is at least 0.5 Mev which is considerably higher than previously reported.

Until more precise values of the end-point and average electron energies are determined, the argument in favor of Gamow-Teller selection rules presented by the K^{40} decay will remain somewhat doubtful.

[24] A. Bramley and A. K. Brewer, Phys. Rev. **53**, 502 (1938).

Rb^{87}

The low maximum energy[25] of the Rb^{87} electrons, $W_0=1.26$, accounts for its very long half-life. Experimental determinations of the half-life vary somewhat; $(2-4)10^{18}$ sec. is reported by Hemmendinger and Smythe,[12] while $(6-12)10^{18}$ sec. is the value found by Hahn, Strassmann, and Walling.[11] The latter authors indicate that the shorter half-life, $(6)10^{18}$ sec., is probably the best.

The "third forbidden" approximations yield half-lives of the correct order of magnitude, while the "second forbidden" calculations result in a value too small by a factor of 1/1000. It should be noticed that the average kinetic energy of the decay electrons corresponding to the best calculated half-life is roughly $\frac{1}{3}$ their maximum kinetic energy.

V. CONCLUSIONS

For the four cases treated, the tensor interaction seems consistently to give the best results. The good agreement with experimental determinations obtained for RaE can, of course, be given equally well by the polar vector interaction. However, the polar vector interaction fails to account for the half-life of P^{32}. It likewise is inadequate for K^{40} if the lower end-point energy of K^{40} is more nearly correct than the larger value reported by Henderson.

The author wishes to thank Dr. E. J. Konopinski under whose valuable guidance this work was done, for suggesting the lifetime problems treated, and for criticizing the work in preparation.

[25] W. F. Libby and D. D. Lee, Phys. Rev. **55**, 245 (1939).