iron was converted into the oxide by ignition. The oxide was again dissolved and the operation repeated. Measurements on the G-M counter showed that a weak 2.2-minute period remained just as in the case of the carbon bombardment.

Nickel was also bombarded with fast neutrons, but no 2-minute period was observed. However, the production of such a period by fast neutron bombardment of oxygen in the form of sodium carbonate was confirmed.

These results indicate that the 2-minute activity reported as a result of the bombardment of iron with alpha-particles may be due to O^{15}. However, the results do not necessarily exclude the possibility of the existence of a short period in nickel, the presence of which is not evident from the above described experiments. The development of an experimental technique for the rapid removal of oxygen is necessary before the question of the existence of a short period in nickel can be settled.

It is a pleasure to acknowledge the support received from the University Development Fund and from Mr. Julius F. Stone. Valuable assistance through the W.P.A. cyclotron project No. 65-1-42-89 is also acknowledged.

APRIL 1 AND 15, 1942 PHYSICAL REVIEW VOLUME 61

Forbidden Transitions in β-decay and Orbital Electron Capture and Spins of Nuclei

R. E. MARSHAK
University of Rochester, Rochester, New York

(Received January 16, 1942)

A general formula giving minimum lifetimes for forbidden β-transitions of arbitrary order or forbidden orbital electron capture is derived. Exact Coulomb wave functions are used for the electron. It is shown that the observed electron emission of K^{40} requires Gamow-Teller selection rules. Combined with the Konopinski-Uhlenbeck result that only the tensor and vector interactions are compatible with the energy spectra of the β-rays from Na^{24}, P^{32}, and RaE, it follows that the tensor interaction alone can explain both the lifetimes and energy spectra of forbidden β-transitions. The application of the tensor interaction to K^{40} and to the other long-lived β-emitters, Rb^{87}, Lu^{176}, Be^{10}, C^{14}, and to existing data on orbital electron capture, leads to certain spin and parity predictions about parent and product nuclei—e.g., neither Be^{10} nor C^{14} can have a spin greater than $3\hbar$, the 2-Mev γ-ray from K^{40} is associated with K-electron capture to an excited state of A^{40} having even parity, etc. The stability of the known neighboring isobars and the conditions under which L-electron capture becomes more probable than K-electron capture are also discussed.

I. INTRODUCTION

EXPERIMENTS on β-radioactive nuclei have tended in recent years to confirm Gamow and Teller's[1] modification of Fermi's theory of β-decay according to which an "allowed" transition may involve a spin change of as much as one unit of $\hbar$. In particular, the strikingly large probability of the reaction $He^6 \rightarrow Li^6 + e^-$, was presumed to provide strong support for the correctness of the Gamow-Teller selection rules since the transition from He^6 with a spin zero to Li^6 with a spin one involves a spin change of one unit. The Li^6 spin has been measured but the spin of He^6 was based on the argument from nuclear theory that all nuclei, whether stable or unstable, with an even number of protons and an even number of neutrons, have spin zero. It is certainly true that all known *stable* even-even nuclei have spin zero but the extrapolation to unstable even-even nuclei has been rendered extremely dubious by the fact that certain homologues of He^6, such as Be^{10} and C^{14}, decay very slowly and cannot therefore have the same

[1] G. Gamow and E. Teller, Phys. Rev. **49**, 895 (1936); there is a good deal of indirect evidence for the Gamow-Teller selection rules [cf. especially E. P. Wigner, Phys. Rev. **56**, 519 (1939); and White, Creutz, Delsasso, and Wilson, Phys. Rev. **59**, 63 (1941)] but none can be as conclusive as the type discussed below.

spin as He^6.[2] If not *all* unstable even-even nuclei have spin zero, it becomes a little difficult to decide whether any have, and the evidence for the Gamow-Teller selection rules becomes more shaky. Thus the decay of He^6 and C^{10} would be compatible with the original Fermi selection rules if the spins of both He^6 and C^{10} are one; furthermore, the slow reactions involving Be^{10} and C^{14} could be explained by assuming even larger spins than one. It therefore becomes of interest to find some independent test of the validity of the Gamow-Teller selection rules.

A test of the Gamow-Teller selection rules is, in principle, possible if the lifetime and spin of the β-radioactive nucleus, the spin of the product nucleus, and the maximum energy of the β-ray are all known. A knowledge of all these quantities enables one to calculate a lower limit to the lifetime for a given spin change and given maximum energy of the β-ray. An interaction between the nuclear particles and the electron-neutrino field, which contains an explicit dependence on the spin (to be referred to as a Gamow-Teller type of interaction), will in general permit a change of spin of one unit more—for approximately the same lifetime—than will be allowed by an interaction containing no explicit dependence on the spin (to be referred to as a Fermi type of interaction). To put it another way, corresponding to a known spin change the minimum lifetime predicted by a Fermi type of interaction is greater—by a factor of about one thousand—than the minimum lifetime predicted by a Gamow-Teller type of interaction. If the observed lifetime lies between the Fermi and Gamow-Teller minimum values then the Fermi type of interaction is definitely excluded and the Gamow-Teller type of interaction is strongly supported. The most difficult to obtain but at the same time the crucial piece of information required to make this kind of decision is knowledge of the spin of the β-radioactive nucleus. To determine the spin of the β-emitter it is necessary that the lifetime be long compared to the experimental time and the only β-emitters satisfying this condition will be those undergoing "forbidden" transitions. This requires an extension of the theory of β-decay to a calculation of the lifetimes for forbidden transitions of arbitrary order; this calculation is carried out in Section II and the general formulae are given by Eqs. (11), (17a), (17b), and (19c).

Until very recently only the spins of the fairly abundant naturally β-radioactive nuclei Rb^{87} and Lu^{176} [3] had been measured. But now Zacharias[4] has measured the spin of the very rare naturally β-radioactive isotope K^{40}. It is to be expected that existing techniques could be employed to measure the spins of the artifically produced long-lived nuclei such as Be^{10} and C^{14}, but it will be shown that knowledge of the spins of the natural β-emitters—in particular of K^{40}—is sufficient to decide for the Gamow-Teller selection rules in β-decay. A preliminary calculation on K^{40} which did not take account of the Coulomb field for the electron has been reported on[5] but in the present paper exact wave functions are used for both the electron and neutrino. Since recent improved experiments supply convincing evidence[6] for a theory of β-decay which does not involve derivatives of the light particle wave functions, one must make a decision among five possible independent formulations—the scalar, pseudo-scalar, vector, pseudo-vector, and tensor interactions. Of these the first three are Fermi type interactions whereas the last two are Gamow-Teller type interactions. As far as lifetime calculations are concerned the last two cannot be distinguished. However in the meantime Konopinski and Uhlenbeck[7] have tried to decide among the various possible formulations on the basis of the observed energy spectra of Na^{24}, P^{32}, and RaE. They conclude that the scalar, pseudo-scalar, and pseudo-vector interactions can be eliminated. This, combined with our result that only the pseudo-vector and tensor interactions are possible, leads to the unique result that the tensor interaction is the correct one.

---

[2] For a discussion of this point, see J. R. Oppenheimer, Phys. Rev. **59**, 908 (1941).

[3] M. Heyden and H. Kopfermann, Zeits. f. Physik **108**, 232 (1938) and H. Schüler and H. Gollnow, Zeits. f. Physik **113**, 1 (1939).

[4] J. R. Zacharias, Phys. Rev. **60**, 168 (1941).

[5] R. E. Marshak, Phys. Rev. **59**, 937A (1941).

[6] For a discussion of this evidence, cf. R. D. Evans, "Introduction to the Atomic Nucleus," M. I. T. Course Notes; I am indebted to Professor Evans for placing his manuscript at the disposal of our laboratory.

[7] E. J. Konopinski and G. E. Uhlenbeck, Phys. Rev. **60**, 308 (1941); I wish to thank the authors for sending me their paper prior to publication.

In Section III the tensor interaction is applied to the known long-lived β-emitters Rb^{87}, Lu^{176}, Be^{10}, C^{14}, with half-lives of the order or greater than one thousand years, to determine the nature of the forbidden transition taking place. Table I gives the experimental data and Table II lists the theoretical lifetimes obtained under different assumptions as to the spin and parity changes of the nuclei involved. It is found, for example, that neither Be^{10} nor C^{14} can have a spin larger than $3\hbar$, that the Rb^{87} decay involves a parity change, and so on.

Finally in Section IV the tensor interaction is used to find a general formula [cf. Eq. (20)] for allowed and forbidden orbital electron capture. It is shown that for allowed transitions and sufficiently negative energy differences between initial and final nuclei, L_{I}-electron capture becomes comparable to K-electron capture. Under favorable circumstances it should be possible to observe these long-lived allowed transitions. Moreover, the presence or absence of appreciable x-radiation associated with L_{I} capture will distinguish long-lived allowed transitions from first forbidden transitions. Again, in principle, for first- or higher-order forbidden transitions and sufficiently negative energy differences between initial and final nuclei, $L_{\text{II, III}}$-electron capture becomes comparable to K capture; but it is likely that such activities will escape detection because of the small intensities involved. An attempt is made to classify the type of the transition (i.e., allowed or forbidden) for the better known long-lived unstable nuclei decaying by orbital electron capture, e.g., V^{47}, Mn^{54}, K^{40}. The stability of the known neighboring isobars is also discussed on the basis of Eq. (20).

II. THE CALCULATION OF LIFETIMES FOR FORBIDDEN β-EMISSION

It is well known from the theory of atomic spectra that the lifetime of an excited state is independent of the magnetic quantum number of the state for a given total angular momentum change and provided that the summation is taken over all possible magnetic quantum numbers of the final state. A similar rule holds for β-ray emission and we can therefore choose the magnetic quantum number of the initial state in such a way that the least amount of calculation is required. Let j_i be the spin of the initial nucleus and j_f the spin of the final nucleus. Then the maximum contribution to the transition probability comes from that part of the electron-neutrino field which has associated with it a total angular momentum $k=|j_i-j_f|$. We assume first that $j_i>j_f$; then if we choose the maximum value of m_i (equal to j_i), there is only one possible transition, namely, $j_i,\ m_i=j_i\rightarrow j_f,\ m_f=m_i-\mu$ where $\mu=k$ is the maximum magnetic quantum number associated with the field. The transition probability T may therefore be calculated in a unique way by considering the emission of an electron-neutrino pair with total angular momentum quantum number k and total magnetic quantum number $\mu=k$; the lifetime τ will be $1/T$. If $j_i<j_f$, the transition probably T found for the emission of an electron-neutrino pair with total quantum numbers k and $\mu=k$, must be multiplied by the statistical factor $(2j_f+1)/(2j_i+1)$ to give the reciprocal lifetime, i.e.,

$$1/\tau=[(2j_f+1)/(2j_i+1)]\cdot T.$$

We are interested in the shortest lifetime which is compatible with the emission of an electron-neutrino pair of total angular momentum $k=|j_i-j_f|$. This will clearly be given by the Gamow-Teller type of interaction of which there are two kinds,[8] the tensor and pseudo-vector interactions. Since Konopinski and Uhlenbeck have excluded the pseudo-vector interaction on the basis of energy spectra considerations, we perform our calculation for the tensor interaction. However, this is no real restriction since the axial vector interaction leads to essentially the same result. The tensor interaction may be written as:

$$H=G\{(\Psi^*\beta\boldsymbol{\sigma}\Phi)\cdot(\psi^*\beta\boldsymbol{\sigma}\phi)+(\Psi^*\beta\boldsymbol{\alpha}\Phi)\cdot(\psi^*\beta\boldsymbol{\alpha}\phi)\}. \quad (1)$$

In this expression G is the coupling constant of the nuclear particle with the electron-neutrino field, Ψ and Φ are the quantized Dirac wave functions of the nuclear particle; if Ψ represents a proton, then Φ represents a neutron and

[8] Cf. H. A. Bethe and R. F. Bacher, Rev. Mod. Phys. 8, 82 (1936).

conversely. The quantities ψ, ϕ are the quantized wave functions of the electron and anti-neutrino, respectively, normalized to one particle in a sphere of unit radius, and evaluated at the position of the nuclear particle; ψ is a solution of the Dirac equation in the Coulomb field of the nucleus, and ϕ is a solution of the Dirac equation in the field-free case. The operators β, $\boldsymbol{\alpha}$ are the usual Dirac operators, and we have $\boldsymbol{\sigma}=-i\alpha_x\alpha_y\alpha_z\boldsymbol{\alpha}$. The probability of the emission of an electron with energy[9] between W and $W+dW$ is:

$$P(W)dW=\frac{2}{\pi}\sum_{(j_1l_1m_1)}\sum_{(j_2l_2m_2)}|H_{ab}|^2\frac{W}{(W^2-1)^{\frac{1}{2}}}dW. \quad (2)$$

The first summation is over the angular momentum quantum numbers of the electron, the second over the angular momentum quantum numbers of the neutrino; H_{ab} is the matrix element of a transition for the interaction (1) taken over the nuclear volume. Since the de Broglie wave-length of the electron and neutrino are large compared to nuclear dimensions, this matrix element can be evaluated in successive approximations corresponding to increasingly larger values of the total angular momentum of the electron-neutrino pair. For a given spin change, $k=|j_i-j_f|$ of the nucleus, by far the largest contribution to the transition probability will come from the part of (1) containing the four-component operator $\beta\boldsymbol{\sigma}$; therefore we first neglect the second term in the expression (1). We consider the single matrix element:

$$H_{ab}=\int d\tau(\Psi_b{}^*\beta\boldsymbol{\sigma}\Phi_a)\cdot(\psi^*\beta\boldsymbol{\sigma}\phi). \quad (3)$$

It is convenient to use only the positive energy solutions of the Dirac equation; if we use an ordering between solutions of positive and negative energy which is both Lorentz invariant and invariant with respect to all reflections, we may write for H_{ab}:

$$H_{ab}=\int d\tau(\Psi_b{}^*\beta\boldsymbol{\sigma}\Phi_a)\cdot(\psi C\beta\boldsymbol{\sigma}\phi)^*. \quad (4)$$

The operator C can be written as $\beta\alpha_y$[10] and the ψ, ϕ are now positive energy solutions of the following forms:[11]

Type (a) : $j=l+\frac{1}{2}$

$$\left.\begin{aligned}\psi_1&=i\left(\frac{l-m+\frac{3}{2}}{2l+3}\right)^{\frac{1}{2}}Y_{l+1,\,m-\frac{1}{2}}f_l(r),\\ \psi_2&=i\left(\frac{l+m+\frac{3}{2}}{2l+3}\right)^{\frac{1}{2}}Y_{l+1,\,m+\frac{1}{2}}f_l(r),\\ \psi_3&=\left(\frac{l+m+\frac{1}{2}}{2l+1}\right)^{\frac{1}{2}}Y_{l,\,m-\frac{1}{2}}g_l(r),\\ \psi_4&=-\left(\frac{l-m+\frac{1}{2}}{2l+1}\right)^{\frac{1}{2}}Y_{l,\,m+\frac{1}{2}}g_l(r).\end{aligned}\right\} \quad (5a)$$

Type (b) : $j=l-\frac{1}{2}$

$$\left.\begin{aligned}\psi_1&=i\left(\frac{l+m-\frac{1}{2}}{2l-1}\right)^{\frac{1}{2}}Y_{l-1,\,m-\frac{1}{2}}f_{-l-1}(r),\\ \psi_2&=-i\left(\frac{l-m-\frac{1}{2}}{2l-1}\right)^{\frac{1}{2}}Y_{l-1,\,m+\frac{1}{2}}f_{-l-1}(r),\\ \psi_3&=\left(\frac{l-m+\frac{1}{2}}{2l+1}\right)^{\frac{1}{2}}Y_{l,\,m-\frac{1}{2}}g_{-l-1}(r),\\ \psi_4&=\left(\frac{l+m+\frac{1}{2}}{2l+1}\right)^{\frac{1}{2}}Y_{l,\,m+\frac{1}{2}}g_{-l-1}(r).\end{aligned}\right\} \quad (5b)$$

The functions f, g are defined in reference 11 and involve confluent hypergeometric functions of (pr) where p is the electron momentum. The neutrino wave functions are of exactly the form (5a), (5b) except that the f's and g's are now Bessel functions and the energy is equal to the momentum which is denoted by q. Thus we have:

$$\left.\begin{aligned}f_l&=(\pi q/2r)^{\frac{1}{2}}J_{l+3/2}(qr),\\ g_l&=-(\pi q/2r)^{\frac{1}{2}}J_{l+\frac{1}{2}}(qr),\end{aligned}\right\} \quad (6a)$$

$$\left.\begin{aligned}f_{-l-1}&=(\pi q/2r)^{\frac{1}{2}}J_{l-\frac{1}{2}}(qr),\\ g_{-l-1}&=(\pi q/2r)^{\frac{1}{2}}J_{l+\frac{1}{2}}(qr).\end{aligned}\right\} \quad (6b)$$

[9] All quantities are given in relativistic units: unit of energy is mc^2, of time is $\hbar/mc^2$, and of length is $\hbar/mc$; m is the electron mass.

[10] The operator C is different from the $\delta=\beta\alpha_z\alpha_x$ originally used by Fermi (Zeits. f. Physik **88**, 161 (1934)); C satisfies the conditions: (a) $\boldsymbol{\alpha}^*C=C\boldsymbol{\alpha}$, (b) $\beta^*C=-C\beta$, whereas δ satisfies condition (a) but requires a positive sign in (b). In the standard representation of the Dirac matrices C must be used instead of δ. Cf. W. Pauli, Rev. Mod. Phys. **13**, 203 (1941).

[11] Cf. M. E. Rose, Phys. Rev. **51**, 484 (1937); we use his notation throughout. This also follows the notation of Konopinski and Uhlenbeck (cf. reference 7 above).

If the neutrino rest mass m_0 were not taken as zero the f functions would have an additional factor $[(\epsilon-m_0)/\epsilon]^{\frac{1}{2}}$ and the g functions a factor $[(\epsilon+m_0)/\epsilon]^{\frac{1}{2}}$ where ϵ is the total energy measured in units of mc^2 and m_0 the rest mass measured in the same units. Moreover, it is to be remembered that all wave functions are normalized to one particle in a sphere of unit radius. Since $qr \ll 1$, we shall always use the asymptotic expansions of the Bessel functions for small values of the argument, namely:

$$J_{n+\frac{1}{2}}(z) \simeq \left(\frac{2}{\pi}\right)^{\frac{1}{2}} \frac{z^{n+\frac{1}{2}}}{1\cdot 3\cdots(2n+1)}. \tag{6c}$$

We shall also expand the hypergeometric functions and always keep the first non-vanishing term in (pr). These approximations are extremely good in all cases in which we are interested.

We can now calculate $P(W)dW$ for the various forbidden transitions, but as an illustration of the method we shall first calculate the transition probability when the electron-neutrino pair has a total angular momentum quantum number $k=1$ and a total magnetic quantum number $\mu=1$. If we then consider terms involving only the lowest powers of (pr) and (qr) and substitute for $(\psi c\beta\sigma\phi)^*$ the appropriate expressions, namely:

$$A_x=(\psi C\beta\sigma_x\phi)^*=i(-\psi_1\phi_3+\psi_2\phi_4-\psi_3\phi_1+\psi_4\phi_2)^*,$$

$$A_y=(\psi C\beta\sigma_y\phi)^*=-(\psi_1\phi_3+\psi_2\phi_4+\psi_3\phi_1+\psi_4\phi_2)^*,$$

$$A_z=(\psi C\beta\sigma_z\phi)^*=i(\psi_1\phi_4+\psi_2\phi_3+\psi_3\phi_2+\psi_4\phi_1)^*,$$

the double summation in (2) reduces to two terms, namely, when

$$\text{(a)}\quad \begin{cases} j_1=\frac{1}{2}, & l_1=1, \quad m_1=\frac{1}{2}, \\ j_2=\frac{1}{2}, & l_2=0, \quad m_2=\frac{1}{2}, \end{cases}$$

and when

$$\text{(b)}\quad \begin{cases} j_1=\frac{1}{2}, & l_1=0, \quad m_1=\frac{1}{2}, \\ j_2=\frac{1}{2}, & l_2=1, \quad m_2=\frac{1}{2}. \end{cases}$$

Consider the combination (a); if the electron wave functions (5b) corresponding to $j_1=\frac{1}{2}$, $l_1=1$, $m_1=\frac{1}{2}$ and the neutrino functions (6a) corresponding to $j_2=\frac{1}{2}$, $l_2=0$, $m_2=\frac{1}{2}$ are inserted, we get for the operators:

$$A_x=i[-iY^{*2}{}_{00}(\pi q/2r)^{\frac{1}{2}}f_{-2}(pr)J_{\frac{1}{2}}(qr)] = +Y^2{}_{00}\pi^{\frac{1}{2}}qf_{-2}(pr),$$

$$A_y=-iA_x,$$

$$A_z=0.$$

It is seen that we have only retained the terms coming from $\psi_1\phi_3$ and used (6c); the other combinations either vanish or bring in products of higher order Bessel functions (such as $J_{3/2}(qr)\sim(qr)^{\frac{3}{2}}$ instead of $J_{\frac{1}{2}}(qr)\sim(qr)^{\frac{1}{2}}$) and higher order hypergeometric functions which may be neglected. In a similar fashion combination (b) gives

$$A_x=i[+iY^{*2}{}_{00}(\pi q/2r)^{\frac{1}{2}}g_0(pr)J_{\frac{1}{2}}(qr)] = -Y^2{}_{00}\pi^{\frac{1}{2}}qg_0(pr),$$

$$A_y=-iA_x,$$

$$A_z=0.$$

These expressions must be substituted into (4) and then H_{ab} inserted into (2) with the result:

$$P(W)dW=\frac{G^2}{4\pi^3}\left|\int d\tau\Psi_b{}^*\beta\left(\frac{\sigma_x-i\sigma_y}{\sqrt{2}}\right)\Psi_a\right|^2(g_0{}^2+f_{-2}{}^2)\frac{q^2W}{p}dW. \tag{7}$$

The functions g_0 and f_{-2} are evaluated at the nuclear radius R and to a good approximation we have:

$$g_0{}^2+f_{-2}{}^2=2p^2F_0(Z,W)\left(\frac{1+s_0}{2}\right)$$

where

$$F_0(Z,W)=\frac{4}{\Gamma^2(1+2s_0)}(2pR)^{2s_0-2}e^{\pi\alpha ZW/p}\left|\Gamma\left(s_0+\frac{i\alpha ZW}{p}\right)\right|^2,$$

and $s_0=(1-\alpha^2Z^2)^{\frac{1}{2}}$. The quantity α is the fine structure constant, and R is the nuclear radius measured in units of $\hbar/mc$; also $F(0, W_0)=1$. For the lifetime $\tau_T^{(0)}$ we may write:

$$\frac{1}{\tau_T^{(0)}}=\frac{1}{\tau_0}|M_0|^2B_0(Z, W_0); \quad \tau_0=\frac{2\pi^3}{G^2}, \tag{8}$$

where

$$M_0=\int d\tau\Psi^*_b\left(\frac{\sigma_x-i\sigma_y}{\sqrt{2}}\right)\Psi_a, \tag{8a}$$

and

$$B_0(Z, W_0)=\int_1^{W_0}(W^2-1)^{\frac{1}{2}}W(W_0-W)^2F_0(Z, W)\left(\frac{1+s_0}{2}\right)dW. \tag{8b}$$

W_0 is the maximum energy of the electron. If the initial state of the nucleus has a spin j_i smaller than the spin j_f of the final state, the theoretical lifetime is smaller by a factor $(2j_i+1)/(2j_f+3)$. In the matrix element M_0 the operator $(\sigma_x-i\sigma_y)/\sqrt{2}$ which corresponds to $(x-iy)/\sqrt{2}$ in atomic spectra refers to the nucleus alone and can be considered now as the usual combination of Pauli two-component spin operators[12] which has the effect of lowering the order of a spherical harmonic by one. When there is a nuclear spin change of one unit in an allowed transition the nuclear wave functions will contain a spherical harmonic of the first order and $(\sigma_x-i\sigma_y)/\sqrt{2}$ will lower it to one of zero order; this zero-order harmonic combines with one of the zero-order harmonics contributed by the electron and neutrino wave functions which do not appear explicitly in (8b) because they have been absorbed into the constant in (8). The matrix element M_0 is of the order of magnitude unity, becoming an order of magnitude smaller for heavier nuclei (cf. Section III below), provided the parities of the initial and final states are the same.

If the parities of the initial and final states are different, the matrix element M_0 will vanish; to get a spin change of one unit it will then be necessary to include the terms in A_x, A_y, A_z which had previously been neglected and moreover to include the contributions from the "small" term in (1). The transition probability arising from the "small" $\beta\boldsymbol{\alpha}$ term can easily be calculated in the manner outlined above. The same two angular momentum combinations (a) and (b) occur as before and we get:

$$P_1(W)dW=\frac{G^2R^2}{4\pi^3}|M'_0|^2(g_0^2+f_{-2}^2)\frac{q^2W}{p}dW; \quad M'_0=\int\frac{d\tau}{R}\Psi_b^*\beta\left(\frac{\alpha_x-i\alpha_y}{\sqrt{2}}\right)\Psi_a. \tag{9a}$$

The quantity R is again the nuclear radius measured in units of $\hbar/mc$. It is somewhat more troublesome to calculate the contribution to the transition probability of the "second"-order terms in A_x, A_y, and A_z since one must take account of several more possible combinations of angular momentum quantum numbers of the electron and neutrino. However, it turns out that the two combinations,

(a′) $j_1=\frac{1}{2},\quad l_1=0,\quad m_1=\frac{1}{2},$
$j_2=\frac{1}{2},\quad l_2=0,\quad m_2=\frac{1}{2},$

(b′) $j_1=\frac{1}{2},\quad l_1=1,\quad m_1=\frac{1}{2},$
$j_2=\frac{1}{2},\quad l_2=1,\quad m_2=\frac{1}{2},$

give the terms which, having the most sensitive dependence on the nuclear charge, dominate all the others for $2Z/W_0>1$. The major part of the transition probability due to the "second"-order terms in A_x, A_y, A_z is therefore:

$$P_2(W)dW=\frac{G^2}{4\pi^3}|M''_0|^2(f_0^2+g_{-2}^2)\frac{q^2W}{p}dW; \quad M''_0=\int\frac{d\tau}{R}\Psi_b^*\left[\frac{(\boldsymbol{\sigma}\times\mathbf{r})_x-i(\boldsymbol{\sigma}\times\mathbf{r})_y}{\sqrt{2}}\right]\Psi_a. \tag{9b}$$

There are also the cross terms, but these need not be considered since we shall be interested only

[12] This is true in the non-relativistic approximation for the nuclear particles—which is certainly justified; in the same approximation β may be replaced by 1. In the future the Pauli spin-operator $\boldsymbol{\sigma}$ will replace $\beta\boldsymbol{\sigma}$ in all nuclear matrix elements.

in applications for which (9a) is considerably larger than (9b) or conversely; at most, the minimum lifetimes will be too long by a factor of two.

In order to compute minimum lifetimes on the tensor interaction with unfavorable parity change, it is necessary to estimate maximum values of (9a) and (9b). It might be thought that an essential uncertainty is introduced into the lifetimes by the presence of more than one kind of nuclear matrix element. This is certainly true for *actual* lifetimes but is not the case for the *minimum* lifetimes which can be predicted by the theory and which is all that concerns us. An estimate of the maximum value (9a) may be had by treating the $\beta\boldsymbol{\alpha}$ in M'_0 as a velocity vector.[13] We find $M'_0 \lesssim W_0$ (cf. reference 7). To get an estimate of (9b) we expand $(f_0^2+g_{-2}^2)$ in powers of (αZ); the most important term in this expansion (for $2Z/W_0>1$) can be written in the form $\frac{1}{4}\alpha^2Z^2(g_0^2+f_{-2}^2)$. Now the matrix element

$$M''_0=\int\frac{d\tau}{R}\Psi_b^*\left[\frac{(\boldsymbol{\sigma}\times\mathbf{r})_x-i(\boldsymbol{\sigma}\times\mathbf{r})_y}{\sqrt{2}}\right]\Psi_a$$

is of order 1; therefore the ratio of (9a) to (9b) is $4W_0^2R^2/\alpha^2Z^2$ if we insert the maximum values of M'_0 and M''_0. Since we are interested only in minimum lifetimes corresponding to a spin change of one unit with parity change we can then use (9a) when $4W_0^2R^2/\alpha^2Z^2\gg 1$. We get:

$$\frac{1}{\tau_T^{(0)\prime}}=\frac{1}{\tau_0}R^2|M'_0|^2B_0(Z,W_0). \tag{10a}$$

When $4W_0^2R^2/\alpha^2Z^2\ll 1$, we can neglect (9a) and use (9b), and we obtain:

$$\frac{1}{\tau_T^{(0)\prime\prime}}=\frac{1}{\tau_0}\left(\frac{\alpha^2Z^2}{4}\right)|M''_0|^2B_0(Z,W_0). \tag{10b}$$

Equation (10b) holds for $\alpha Z\ll 1$ while (10a) holds for any Z. In both (10a) and (10b) $B_0(Z, W_0)$ is the same function which occurs in (8).

The lifetimes on the tensor interaction—both for favorable and unfavorable parity change—for the various forbidden transitions, may be calculated in a similar fashion, by taking $k=|j_i-j_f|$, $\mu=k$. A first forbidden transition permits a spin change of two units and requires a parity change, a second forbidden transition permits a spin change of three units and requires that the parity remain unchanged, and so on. In general, if n is the order of the forbidden transition and the parity change is favorable—parity change for n odd and no parity change for n even—then the maximum allowable spin change is $(n+1)$ units; if the parity change from initial to final nuclear state is unfavorable, then the spin change $(n+1)\hbar$ will be associated with a forbidden transition of order $(n+1)$. We first calculate the lifetimes corresponding to the case of favorable parity change. For this type of forbidden transition, each additional order involves one more combination of angular momentum quantum numbers of the electron-neutrino pair and therefore two more terms in the double summation (2) since the roles of the electron and neutrino may be interchanged. Table III lists the pairs of quantum number triplets entering into each forbidden transition: each number triplet represents (j, l, m) in the order given. The lifetime $\tau_T^{(n)}$ on the tensor interaction for the nth forbidden transition corresponding to a spin change of $(n+1)$ units and favorable parity change is found to be:

$$\frac{1}{\tau_T^{(n)}}=\frac{1}{\tau_0}\frac{|M_n|^2}{[1\cdot 3\cdots(2n+1)]^2}R^{2n}B_n(Z,W_0). \tag{11}$$

In (11) R is again the nuclear radius measured in units of $\hbar/mc$; M_n is of order of magnitude unity

[13] This is not quite correct since $\boldsymbol{\alpha}$ not $\beta\boldsymbol{\alpha}$ is the current on the Dirac theory; however, the estimate is close enough for our purposes. In the vector interaction $\boldsymbol{\alpha}$ enters without β so that the estimate given there is more rigorous (cf. 19a).

(cf. Section III below), and is given by:

$$M_n=4\pi\int d\tau\Psi^*_b\left(\frac{\sigma_x-i\sigma_y}{\sqrt{2}}\right)\Psi_a\left(\frac{r}{R}\right)^n Y^*_{nn}Y_{00}. \tag{11a}$$

It is interesting to note that only one kind of nuclear matrix element arises for each forbidden transition. This follows from the fact that other matrix elements which appear containing combinations of spherical harmonics of the form $Y^*_{mm}\cdot Y^*_{n-m,\,n-m}$ may all be expressed as known constants times $Y^*_{nn}Y_{00}$. The functions $B_n(Z, W_0)$ depend on the nuclear charge Z, the maximum energy of the β-ray W_0, and the order of the transition. The formula (11) holds for arbitrary n but we only give $B_n(Z, W_0)$ for the first three orders:

$$B_1(Z, W_0)=\frac{1}{2}\int_1^{W_0}\frac{W}{p}q^2dW\left[(g_0^2+f_{-2}^2)+\frac{9(g_1^2+f_{-3}^2)}{R^2}\right], \tag{12a}$$

$$=\int_1^{W_0}pWq^2dW\left[q^2F_0\left(\frac{1+s_0}{2}\right)+p^2F_1\left(\frac{2+s_1}{4}\right)\right], \tag{12b}$$

$$B_2(Z, W_0)=\frac{1}{2}\int_1^{W_0}\frac{W}{p}q^2dW\left[q^4(g_0^2+f_{-2}^2)+\frac{30q^2(g_1^2+f_{-3}^2)}{R^2}+\frac{225(g_2^2+f_{-4}^2)}{R^4}\right], \tag{13a}$$

$$=\int_1^{W_0}pWq^2dW\left[q^4F_0\left(\frac{1+s_0}{2}\right)+\frac{10}{3}q^2p^2F_1\left(\frac{2+s_1}{4}\right)+p^4F_2\left(\frac{3+s_2}{6}\right)\right], \tag{13b}$$

$$B_3(Z, W_0)=\frac{1}{2}\int_1^{W_0}\frac{W}{p}q^2dW\left[q^6(g_0^2+f_{-2}^2)+\frac{63q^4(g_1^2+f_{-3}^2)}{R^2}+\frac{1575q^2(g_2^2+f_{-4}^2)}{R^4}+\frac{11025(g_3^2+f_{-4}^2)}{R^6}\right], \tag{14a}$$

$$=\int_1^{W_0}pWq^2dW\left[q^6F_0\left(\frac{1+s_0}{2}\right)+7q^4p^2F_1\left(\frac{2+s_1}{4}\right)+7q^2p^4F_2\left(\frac{3+s_2}{6}\right)+p^6F_3\left(\frac{4+s_3}{8}\right)\right]. \tag{14b}$$

In (12a), (13a), and (14a) the different f's and g's are exact Coulomb functions of the electron for the continuous spectrum and are explained in reference 11. We have written out (12a)–(14a) because of their usefulness later for the calculation of the probability of orbital electron capture. In deriving (12b), (13b), and (14b) we have made use of the connecting formula which holds for $pR\ll 1$, namely:

$$[g^2_n+f^2_{-(n+2)}]=\frac{2(p^2)^{n+1}R^{2n}}{[1\cdot 3\cdots(2n+1)]^2}F_n(Z, W)\left[\frac{(n+1)+s_n}{2(n+1)}\right].$$

The s_n's are functions of Z: $s_n=[(n+1)^2-\alpha^2Z^2]^{\frac{1}{2}}$. The F_n's are functions of Z, W and the order of the transition:

$$F_n(Z, W)=\frac{[(2n+2)!]^2}{(n!)^2\Gamma(1+2s_n)}(2pR)^{2(s_n-n)-2}e^{\pi\alpha ZW/p}\left|\Gamma\left(s_n+\frac{i\alpha ZW}{p}\right)\right|^2. \tag{15}$$

The lifetime given in (11) is strictly correct if the spin of the β-radioactive nucleus is greater than the spin of the final nucleus. If this is not true, then in accordance with the argument given at the beginning of this section, the right-hand side of (11) should be multiplied by $(2j_f+1)/(2j_i+1)$.

It is worth comparing our results with those of Konopinski and Uhlenbeck (cf. reference 7). Since we are interested in the lifetime associated with the maximum possible spin change and favorable parity change, we would use the irreducible tensor of the second rank B_{ij} (cf. reference 7 for notation) for the first forbidden transition, the tensor of the third rank S_{ijk} for the second forbidden, and a tensor of the fourth rank S_{ijkl} for the third forbidden transition. The first two have already been worked out and give the lifetimes:

$$\frac{1}{\tau^{(1)}}=\frac{1}{\tau_0}\cdot\frac{1}{2^2\cdot 3}\sum_{ij}|\beta_{ij}|^2B_1(Z, W_0), \tag{16a}$$

$$\frac{1}{\tau^{(2)}}=\frac{1}{\tau_0}\cdot\frac{1}{2^3\cdot 3^3\cdot 5}\sum_{i,j,k}|S_{ijk}|^2B_2(Z, W_0). \tag{16b}$$

The tensor S_{ijkl} leading to a possible spin change of four units can also be worked out[14] and turns out to be

$$S_{ijkl}=\sigma_{(i}x_jx_kx_{l)}-\frac{1}{14}\delta_{(ij}\{3r^2\sigma_kx_{l)}+6(\boldsymbol{\sigma}\cdot\mathbf{r})x_kx_{l)}\}+\frac{3}{35}\delta_{(ij}\delta_{kl)}(\boldsymbol{\sigma}\cdot\mathbf{r})r^2 \tag{16c}$$

A calculation of the lifetime on the lines of the first and second forbidden transitions leads to the expression

$$\frac{1}{\tau^{(3)}}=\frac{1}{\tau^{(0)}}\frac{1}{2^6\cdot 3^5\cdot 5^2}\sum_{ijkl}|S_{ijkl}|^2B_3(Z, W_0). \tag{16d}$$

The B_n's are the same functions of energy and nuclear charge which we found in (11)—as they should be—but the matrix elements and coefficients are considerably different. This is not surprising because the various tensors B_{ij}, S_{ijk}, S_{ijkl} do not give the single selection rules, ± 2, ± 3, ± 4, respectively, but they also lower spin changes. In order to calculate a lifetime which could be compared with experiment it would be necessary to separate each tensor into parts corresponding to the various spin changes (e.g., B_{ij} into parts corresponding to 0, ± 1, ± 2), and then each of these parts into the various "magnetic quantum number" components. It is precisely this separation which the use of wave functions in spherical coordinates for both electron and neutrino accomplishes and we shall therefore compute lifetimes using (11).

If the parity change associated with the nth forbidden transition is incompatible with the actual parity difference between initial and final nuclear states, one must apply the same sort of reasoning which led to (10a) and (10b). One finds from the "small" $\beta\boldsymbol{\alpha}$ in the tensor interaction the following lifetime:

$$\frac{1}{\tau_{T^{(n)\prime}}}=\frac{1}{\tau_0}R^{2n+2}\frac{|M'_n|^2}{[1\cdot 3\cdots 2n+1]^2}B_n(Z, W_0), \tag{17a}$$

where

$$M'_n=4\pi\int\frac{d\tau}{R}\Psi^*_b\beta\left(\frac{\alpha_x-i\alpha_y}{\sqrt{2}}\right)\Psi_a\left(\frac{r}{R}\right)^nY^*_{nn}Y_{00}\quad\text{is of order } W_0.$$

One gets from the higher order terms in A_x, A_y, and A_z the lifetime:

$$\frac{1}{\tau_{T^{(n)\prime\prime}}}=\frac{R^{2n}}{\tau_0}\left(\frac{\alpha^2Z^2}{4}\right)\frac{|M''_n|^2}{[1\cdot 3\cdots(2n+1)]^2}\bar{B}_n(W_0), \tag{17b}$$

where

$$M''_n=4\pi\int\frac{d\tau}{R}\Psi_b^*\left[\frac{(\boldsymbol{\sigma}\cdot\mathbf{r})_x-i(\boldsymbol{\sigma}\cdot\mathbf{r})_y}{\sqrt{2}}\right]\Psi_a\left(\frac{r}{R}\right)^nY^*_{nn}Y_{00}\quad\text{is of order unity.}$$

---

[14] I am indebted to Professor G. E. Uhlenbeck for advice on the application of the "tensor" method to the third forbidden transition, and for a very interesting conversation on the relative advantages of the "tensor" and "spherical harmonic" methods.

In (17a) the B_n's are the same functions as those which appear in (11). The functions $B_n(W_0)$ which enter into (17b) are somewhat different; their definitions for the first three orders ($\alpha Z \ll 1$) are:

$$\bar{B}_1(W_0)=\int_1^{W_0} pWq^2dW\left[q^2+\frac{p^2}{4}\right], \tag{18a}$$

$$B_2(W_0)=\int_1^{W_0} pWq^2dW\left[q^4+\frac{5}{6}q^2p^2+\frac{p^4}{9}\right], \tag{18b}$$

$$\bar{B}_3(W_0)=\int_1^{W_0} pWq^2dW\left[q^6+\frac{7q^4p^2}{4}+\frac{7}{9}q^2p^4+\frac{p^6}{16}\right]. \tag{18c}$$

Roughly speaking, one should use (17a) to estimate the minimum lifetime corresponding to an nth forbidden transition with unfavorable parity change when $(W_0^2R^2/4\alpha^2Z^2)[B_n(Z, W_0)/B_n(W)]>1$. When the other inequality holds one should use (17b).

According to Konopinski and Uhlenbeck[7] the vector interaction between nuclear particles and the electron-neurino field is also compatible with the accurately known energy spectra of forbidden β-emitters. Since we wish to decide between the vector and tensor theories of β-decay, it is necessary to compute lifetimes for the various forbidden transitions on the basis of the vector interaction. It turns out that as far as lifetimes are concerned, the vector interaction for an nth forbidden transition is essentially equivalent to an $(n-1)$ forbidden transition on the tensor interaction with unfavorable parity change. It must be remembered that an allowed transition on the vector interaction permits no spin change and requires that the parity remain unchanged, a first forbidden transition permits a spin change of one unit and requires parity change, and so on. If we relabel the lifetime to represent the new selection rules so that a maximum spin change of n units is permitted by an nth forbidden transition, we may write for the lifetime $\tau_F^{(n)}$ derived from the "small" α term in the vector interaction:

$$\frac{1}{\tau_F^{(n)\prime}}=\frac{1}{\tau_0}R^{2n}|G'_n|^2D'_n(Z, W_0), \tag{19a}$$

where

$$G_n'\doteq 4\pi\int\frac{d\tau}{R}\Psi^*_b\left(\frac{\alpha_x-i\alpha_y}{\sqrt{2}}\right)\Psi_a\left(\frac{r}{R}\right)^{n-1}Y^*_{n-1,\,n-1}Y_{00}\quad\lessapprox W_0,$$

$$D'_n\equiv B_{n-1}(Z, W_0)\qquad\text{[cf. Eq. (17a)]}$$

The lifetime $\tau_F^{(n)\prime\prime}$ derived from the fourth component of the vector interaction is:

$$\frac{1}{\tau_F^{(n)\prime\prime}}=\frac{1}{\tau_0}R^{2n-2}\frac{\alpha^2Z^2}{4}|G''_n|^2D''_n(Z, W_0). \tag{19b}$$

The matrix element G'' is not the same as in (17b) but instead is given by:

$$G''_n=4\pi\int\frac{d\tau}{R}\left(\frac{x-iy}{\sqrt{2}}\right)\left(\frac{r}{R}\right)^{n-1}Y^*_{n-1,\,n-1}Y_{00}\quad\lessapprox 1,$$

$$D''_n\equiv\bar{B}_{n-1}(W_0).\qquad\text{(cf. (17b))}$$

Again there are cross terms of the form $i(G'_nG''_n+\text{c.c.})$ but it has been checked that these terms are smaller than (19a) when $\alpha^2Z^2/4W^2_0R^2>4/n$ and less than (19b) when $\alpha^2Z^2/4W^2_0R^2<n/4$ so that we need not consider them here. In the case of the tensor interaction with unfavorable parity change a rule was given for finding minimum lifetimes with (17a) or (17b). A similar rule applies

for the vector interaction: when $(W^2{}_0R^2/4\alpha^2Z^2)(D'_n/D''_n)>1$ one should take (19a), and when the other inequality holds one should use (19b). However approximately $G'_n \approx W_0G''_n$ (cf. reference 7) so that we may combine (19a) and (19b) to get a "total" lifetime $\tau_F{}^{(n)}$:

$$\frac{1}{\tau_F{}^{(n)}}=\frac{|G''_n|^2}{\tau_0}\left[W_0{}^2R^2B_{n-1}(Z,\,W_0)+\frac{\alpha^2Z^2}{4}\bar{B}_{n-1}(W_0)\right]. \tag{19c}$$

Equation (19c) can be used without any restriction (except that $\alpha Z \ll 1$) to find a lower limit to the lifetime for a given spin change $n\hbar$ on the vector interaction.

III. COMPARISON WITH EXPERIMENT

In Table I we give the observational data on the five well-known long-lived β-emitters, the spins of the initial and final nuclei where known, the lifetimes, and the measured maximum energies of the β-rays; for the latter two quantities the limits of uncertainty are given wherever they exist. If no such limits are given the figures listed are not in error by more than 20 percent. In Table I the spins of Ca^{40}, B^{10}, and Hf^{176} which are listed have not been measured directly but are inferred with almost complete certainty from nuclei of similar structure whose spins have been measured. This is especially true of Ca^{40} which can be considered to consist of an integral number of α-particles; all similar nuclei—He^4, C^{12}, O^{16}, S^{32}—on which measurements have been made, possess a spin of zero. B^{10} is about equally certain since it fits into the sequence H^2, Li^6, N^{14}, all of which have been found to have a spin of one unit; moreover, its magnetic moment fits into the sequence of decreasing magnetic moments corresponding to the above sequence of nuclei, if its spin is one. Hf^{176} is not so certain, the only evidence being that it is a stable nucleus of even atomic weight and even charge and should possess a spin zero.

To compute numerical values for the theoretical lifetimes it is necessary to know τ_0. The most consistent procedure is to compute τ_0 by applying Eq. (8) to the fast reaction: ${}_2He^6 \to {}_3Li^6 + e^-$. This must be an allowed transition on the basis of a Gamow-Teller type of interaction which permits a spin change of one unit. If we insert the values for $\tau^{(0)}$ and W_0 which are 1.14 sec. and 7.9, respectively, and evaluate $|M_0|^2$ in accordance with Grönblom's method[15] we find for the maximum value of $\tau_0 = 7\times10^3$ sec. Account has been taken of the factor that the transition probability must be multiplied by $(2j_f+1)/(2j_i+1)$—which is 3 in the present case—since the initial spin is less than the final spin. The value for τ_0—which is a maximum value—is three times as large as Gronblom's value because of a somewhat different definition of τ_0. This maximum value of τ_0 must be close to the correct value since the initial and final wave functions cannot be very different. Even if the value of τ_0 had to be reduced because $|M_0|^2<2$, the arguments of Nordheim and Yost[16] show that the matrix elements for heavier elements and forbidden transitions would have to be reduced even more, thereby effectively increasing τ_0. We can therefore regard τ_0 as really a *minimum*.

TABLE I. Observational data, references given in text.

Reaction	Spin of initial nucleus	Spin of final nucleus	Observed lifetime in years (half-life $\div \log_e 2$)	Maximum energy (in mc^2)
${}_{19}K^{40} \to {}_{20}Ca^{40} + e^-$	$4\hbar$	$0\hbar$	2.0×10^9	$W_0=2.4\text{–}3.6$
${}_{37}Rb^{87} \to {}_{38}Sr^{87} + e^-$	$3/2\hbar$	$9/2\hbar$	$1\text{–}2\times10^{11}$	$W_0=1.3\text{–}1.5$
${}_{71}Lu^{176} \to {}_{72}Hf^{176} + e^-$	$\geq 7\hbar$	$0\hbar$	1.0×10^{10}	$W_0=1.4$
${}_4Be^{10} \to {}_5B^{10} + e^-$	?	$1\hbar$	$0.5\times10^3\text{–}10^5$	$W_0=2.2$
${}_6C^{14} \to {}_7N^{14} + e^-$	?	$1\hbar$	$10^3\text{–}10^5$	$W_0=1.3$

Table II lists values of the minimum theoretical lifetimes for the various long-lived β-emitters under different assumptions as to the spin and parity changes and in the case of K^{40}, for the upper and lower limits of the maximum energy of the β-ray. Column 2 lists the nuclear radii for the five nuclei; these values were derived using the formula $R\hbar/mc = 1.5\times10^{-13}A^{\frac{1}{3}}$ (cf. Wigner, reference 1). Column 3 lists the

[15] Cf. B. O. Gronblom, Phys. Rev. **56**, 508 (1939): instead of using the operator $(Q_1\sigma_{z_1}+Q_2\sigma_{z_2})$ as he does, we must use the operator $(Q_1/\sqrt{2})(\sigma_{x_1}-i\sigma_{y_1})+(Q_2/\sqrt{2})(\sigma_{x_2}-i\sigma_{y_2})$ which, operating on his wave function ψ, gives $\beta(1)\beta(2)(p(1)n(2)-p(2)n(1))$; it turns out that $|M_0|^2$ is 2 in agreement with his result.

[16] L. W. Nordheim and F. L. Yost, Phys. Rev. **51**, 942 (1937).

spin change, and column 4 lists values of the minimum lifetimes derived on the basis of equation (11)—i.e., the tensor interaction with favorable parity change. The quantity $|M_n|^2$ has in all cases been set equal to unity to get these minimum lifetimes. In principle the maximum value of $|M_n|^2$ is the isotopic number of the β-radioactive nucleus—2 in the case K^{40}, Be^{10}, C^{14}, 13 for Rb^{87}, 34 for Lu^{176}. However, it would be a mere formality to use these larger values and then to state that the matrix elements are actually much smaller because the transitions are not within the same supermultiplet.[17] Moreover, the inclusion of the factor 2 for K^{40} would not alter the fundamental conclusion to which we are led, namely, that the vector interaction predicts too long a lifetime. Column 5 of Table II contains—with one exception—values of the minimum lifetimes based on the tensor interaction with unfavorable parity change; the appropriate equation (17a) or (17b) is used and again the square of the matrix element is taken equal to unity. The exception noted is K^{40} for which the minimum lifetime, $\tau_V^{(4)}$, listed in brackets in column 5 is derived from the vector interaction using (19c) with $|G'_n|^2=1$ and $\tau_0=2.3\times10^3$.[18] In columns 6 and 7 of Table II values of the functions $B_n(Z, W_0)$ and $B_n(0, W_0)$ are given so that the effect of the Coulomb field due to different Z may be seen; also in column 6 the functions $\bar{B}_n(W_0)$ are listed. The $B_n(Z, W_0)$ were computed by numerical integration taking account of the Coulomb correction factors $F_0\cdots F_3$ *throughout* the specified energy intervals; $B_n(0, W_0)$ and $B_n(W_0)$ were also computed by numerical integration.[19] It is worth noting that the influence of the Coulomb field is no greater for the various forbidden transitions on the tensor interaction with favorable parity change than for allowed transitions. We shall discuss the five nuclei separately.

$_{19}\mathbf{K}^{40}$

Two theoretical lifetimes are given—corresponding to $W_0=3.6$ (representing a maximum kinetic energy of 1.3 Mev for the emitted electron) and $W_0=2.4$ (representing a maximum kinetic energy of 0.7 Mev). The higher value is more recent and trustworthy having been found by Henderson,[20] using the standard absorption method in conjunction with a thin walled counter in a concentric arrangement. He claims that there are some potassium β-rays which are more energetic than the 1.10 Mev RaE β-rays and that the maximum energy is 1.3 ± 0.1 Mev. The older measurements made by Bocciarelli[21] and by Anderson and Neddermeyer[22] gave the lower value 0.7 Mev but are less reliable. Since Henderson finds only one group of β-rays and no evidence has ever been found for γ-rays in the energy region up to 1.3 Mev,[23] the transition must take place from the ground state of K^{40} with spin $4\hbar$ to the ground state of Ca^{40} with spin zero. With a spin change of four units and with $W_0=3.6$ the tensor (or pseudo-vector) interaction predicts a minimum lifetime of 5.8×10^7 yr. If the parity change was unfavorable, i.e., if the parity of the ground state of K^{40} is the same as Ca^{40}, the minimum lifetime would be 2×10^{10} yr. If the original Fermi theory of β-decay (i.e., vector interaction) were correct, the minimum theoretical lifetime is 7×10^9 yr. This must be a real lower limit since our

---

[17] Cf. Wigner, reference 1; as Wigner has pointed out, the only transitions within the same supermultiplet occur for positron emitters like C^{11}, N^{13}, etc., and for the light nuclei decaying rapidly with β-emission like He^6, C^{10}, etc. All other transitions are "unfavored" so that the matrix elements must be relatively small compared to the "favored" transitions.

[18] The significance of the smaller τ_0 (by a factor 3) is that τ_0 was evaluated from the reaction $He^6\rightarrow Li^6+e^-$ assuming that it involved a spin change of one unit and was therefore an allowed transition on the tensor interaction; this brought in a factor 3 in the transition probability. If the vector interaction is, for the moment assumed to be correct, the reaction $He^6\rightarrow Li^6+e^-$, since it is as fast as reactions which are known to involve no spin change, would still have to be allowed. This would imply a spin zero for He^6 and therefore a $0\rightarrow0$ transition. However, there is now only one possible transition, namely $j_i=0$, $m_i=0\rightarrow j_f=0$, $m_f=0$ and the factor $(2j_f+1)/(2j_i+1)=3$ no longer enters. Since $|G'_0|=|\int d\tau\Psi^*_b(Q_1+Q_2)\Psi_a|^2=2$ (cf. reference 15) and all other terms are identical, the "vector" τ_0 is three times smaller than the "tensor" τ_0.

[19] I am indebted to Messrs. William Pratt and Herbert York of the College of Arts and Sciences, University of Rochester, for these computations.

[20] W. J. Henderson, Phys. Rev. **55**, 238A (1939); I am indebted to Dr. Henderson for correspondence on his measurements.

[21] D. Bocciarelli, Atti accad. Lincei **17**, 830 (1933).

[22] C. D. Anderson and S. H. Neddermeyer, Phys. Rev. **45**, 653L (1934).

[23] A 2-Mev γ-ray emitted by K^{40} has been found by Gray and Tarrant (cf. Proc. Roy. Soc. **143**, 695 (1934)); since there are only 3γ-quanta per 100 β-rays, the γ-ray emission cannot be correlated with the β-emission and is to be associated with orbital electron capture by K^{40} to form A^{40}. Cf. Section IV below.

TABLE II. Minimum theoretical lifetimes in years.

β-emitter	Nuclear radius in units of $\hbar/mc$	Spin change	Lifetimes on tensor interaction; favorable parity change	Lifetimes on tensor interaction; unfavorable parity change	Energy functions $B_n(Z, W_0)$ and $\bar{B}_n(W_0)$	$B_n(0, W_0)$
$_{19}K^{40}$	1.30×10^{-2}	$4\hbar$	$\tau_T^{(3)}=1.3\times10^{10}(W_0=2.4)$		$B_3(19, 2.4)=4.2\times10^1$	$B_3(0, 2.4)=2.3\times10^1$
		$4\hbar$	$\tau_T^{(3)}=5.8\times10^7(W_0=3.6)$	$[\tau_V^{(4)}=7\times10^9(W_0=3.6)]$	$B_3(19, 3.6)=9.6\times10^3$	$B_3(0, 3.6)=5.5\times10^3$
		$4\hbar$		$\tau_T^{(3)\prime}=2\times10^{10}(W_0=3.6)$	$\bar{B}_3(3.6)=1.2\times10^3$	
$_{37}Rb^{87}$	1.69×10^{-2}	$3\hbar$	$\tau_T^{(2)}=3.5\times10^6$		$B_2(37, 1.5)=7.2\times10^{-2}$	$B_2(0, 1.5)=1.3\times10^{-2}$
		$3\hbar$		$\tau_T^{(2)\prime\prime}=6\times10^9$	$\bar{B}_2(1.5)=2.2\times10^{-3}$	
$_{71}Lu^{176}$	2.13×10^{-2}	$4\hbar$	$\tau_T^{(3)}=2\times10^{11}$		$B_3(71, 1.4)=1.5\times10^{-1}$	$B_3(0, 1.4)=2.5\times10^{-3}$
$_4Be^{10}$	0.82×10^{-2}	$1\hbar$		$\tau_T^{(0)\prime}=0.4$	$B_0(2.2)=0.20$	
		$2\hbar$	$\tau_T^{(1)}=2.8\times10^1$		$B_1(4, 2.2)=1.1$	$B_1(0, 2.2)=1.0$
		$2\hbar$		$\tau_T^{(1)\prime}=8.7\times10^4$	$B_1(2.2)=0.5$	
		$3\hbar$	$\tau_T^{(2)}=4.3\times10^6$		$B_2(4, 2.2)=2.7$	$B_2(0, 2.2)=2.6$
$_6C^{14}$	0.92×10^{-2}	$1\hbar$		$\tau_T^{(0)\prime}=1.0\times10^1$	$B_0(1.3)=3.5\times10^{-3}$	
		$2\hbar$	$[\tau_T^{(1)}=2.2\times10^4]$		$B_1(6, 1.3)=1.1\times10^{-3}$	$B_1(0, 1.3)=0.9\times10^{-3}$
		$2\hbar$		$\tau_T^{(1)\prime}=1.7\times10^8$	$B_1(1.3)=2.6\times10^{-4}$	

TABLE III. Quantum number triplets.

First forbidden: $n=1$	3/2, 2, 3/2	3/2, 1, 3/2		
($k=2$, $\mu=2$)	1/2, 0, 1/2	1/2, 1, 1/2		
Second forbidden: $n=2$	5/2, 3, 5/2	5/2, 2, 5/2	3/2, 2, 3/2	
($k=3$, $\mu=3$)	1/2, 0, 1/2	1/2, 1, 1/2	3/2, 1, 3/2	
Third forbidden: $n=3$	7/2, 4, 7/2	7/2, 3, 7/2	5/2, 3, 5/2	5/2, 2, 5/2
($k=4$, $\mu=4$)	1/2, 0, 1/2	1/2, 1, 1/2	3/2, 1, 3/2	3/2, 2, 3/2

calculations are quite exact, and since all theoretical considerations point to a matrix element much smaller than unity with a correspondingly longer lifetime. The scalar and pseudo-scalar theories of β-decay predict even longer minimum lifetimes. These values are to be compared with the observed[24] lifetime, known to 20 percent, of 2.0×10^9 yr. The vector interaction and *à fortiori* the scalar and pseudo-scalar interactions are therefore excluded, and we must conclude that only the Gamow-Teller type of interaction (tensor or pseudo-vector) is compatible with the observations. This conclusion can be made certain by a very accurate determination of the maximum energy of the β-rays, although Henderson's measurements seem good. When the above result is combined with Konopinski and Uhlenbeck's result (cf. reference 7), that only the tensor and vector interactions can explain the observed energy spectra of Na^{24}, P^{32}, and RaE, we can say that the *tensor theory* of β-decay alone is capable of explaining the lifetimes and energy spectra of forbidden β-emitters. The remainder of our discussion presupposes the correctness of the tensor theory.

If the maximum energy of the potassium β-ray is 1.3 Mev then $|M_3|^2\cong0.03$; this low value would not be contrary to expectations

[24] Cf. A. Bramley and A. K. Brewer, Phys. Rev. **53**, 502 (1938); the electron intensity is measured as a function of the time so that the lifetime corresponds to the β-emission alone (cf. reference 23).

because of the high order of the transition and the not inappreciable atomic weight of potassium (cf. references 16 and 17). On the other hand, it would follow that a maximum energy of 0.7 Mev is definitely ruled out since the predicted minimum lifetime even on the tensor theory is then considerably larger than the observed value, namely 1.3×10^{10} yr. Hence the theory itself would cast doubt on the correctness of the older energy measurements. The tensor theory also predicts a parity change; since the parity of the ground state of Ca^{40} is presumably even, the ground state of K^{40} has odd parity.[25]

${}_{37}\mathbf{Rb}^{87}$

The minimum lifetime predicted for rubidium by the tensor theory of β-decay with the most favorable parity change, i.e., no parity change since $n=2$ corresponds to the observed spin change of 3 units (cf. Heyden and Kopfermann, reference 3), is 3.5×10^{6} yr. We have used $W_0=1.5$, since experiments giving higher energy values are usually more reliable.[26] The observed lifetime is $1-2\times10^{11}$ yr.[27] To bring the theoretical and observed lifetimes into agreement, it is necessary to assume that $|M_2|^2 \simeq 2\times10^{-5}$; this value is too small. If the parity of the ground state of Rb^{87} is different from the parity of the ground state of Sr^{87}, a choice of $|M''_2|^2 \simeq 0.04$ will raise the minimum lifetime of 6×10^{9} yr. to the observed value. This seems quite reasonable so that it is likely that the Rb^{87} β-emission involves a parity change.

${}_{71}\mathbf{Lu}^{176}$

The radioactive isotope of lutecium[28] is interesting because of its very large spin, greater than or equal to $7\hbar$ (cf. Schüler and Gollnow, reference 3). If correct—which is far from certain in view of the complicated pattern and rather small abundance—this is the largest spin which has thus far been measured for any nucleus. Lu^{176} decays to Hf^{176} which is a stable nucleus of even atomic weight and even charge and doubtless has a spin of zero. The maximum energy of the β-ray is 215 ± 15.kilovolts and the observed lifetime is 1×10^{11} yr.[29] The minimum lifetime (on the tensor theory) for a spin change of four units is already as large (cf. Table II) as the observed value. Since it is unlikely that the matrix element $|M_3|^2$ is close to one for this transition, it is probable that only a spin change of three units occurs. Consequently Lu^{176} must decay to an excited state of Hf^{176}, which has a spin of at least $3\hbar$ and probably $4\hbar$. This excited state should emit one or more γ-rays and if it goes to the ground state of Hf^{176}, thereby emitting one γ-ray, should have a rather long life. No one has as yet looked for these γ-rays. If it should turn out that there is no evidence for an excited state of Hf^{176} it may be that the measurement of the spin of Lu^{176}—which was difficult to make—is in error; in this case if the β-decay is to the ground state of Hf^{176}, the tensor theory would predict a spin of $4\hbar$ or possibly $3\hbar$ for Lu^{176}.

${}_{4}\mathbf{Be}^{10}$

It is hopeless to try to account for the long lifetime (greater than 500 years)[30] of Be^{10} if the ground state is assured to have spin $0\hbar$ and the same parity as the ground state of B^{10}. Even if the parity of the states of the two nuclei is different, the lifetime would be of the order of a year or possibly 10 yr., supposing that $|M'_0|^2 \sim 0.1$. A spin change of two units leads to a minimum lifetime of about 30 yr. if there is a parity change and to a minimum lifetime of 8.7×10^{4} yr. if there is no parity change. To attain the observed lifetime of at least 500 yr. it would be necessary that $|M_1|^2$ is less than 0.06. This seems somewhat too small for such a light nucleus so that it is probable that the ground state of Be^{10} has a spin of $3\hbar$—since B^{10}

[25] Professor D. Inglis has kindly informed the writer that the application of the harmonic oscillator potential model to K^{40} would lead to odd parity for the ground state; unfortunately this model also predicts a positive magnetic moment for K^{40} in disagreement with Zacharias' measurement (cf. reference 4). However, it is not to be expected that a nucleus as heavy as K^{40} would possess much "shell structure."

[26] Cf. Muhlhoff, Ann. d. Physik **399**, 205 (1930), and reference 6.

[27] A. Hemmendinger and W. R. Smythe, Phys. Rev. **51**, 1052 (1937).

[28] By means of a mass spectrograph, Mattauch and Lichtblau (Zeits. f. Physik **111**, 514 (1938)) settled conclusively that the naturally radioactive isotope is Lu^{176} and not Lu^{177} as had previously been thought (cf. Heyden and Wefelmeier, Naturwiss. **26**, 612 (1938)).

[29] W. F. Libby, Phys. Rev. **56**, 21 (1939).

[30] Cf. E. Pollard, Phys. Rev. **57**, 241 (1940); and E. M. McMillan and S. Ruben, unpublished (quoted in S. Ruben and M. D. Kamen, Phys. Rev. **59**, 349 (1941)).

has a spin of one unit[31]—and the same parity as the ground state of B^{10}. A more accurate determination of the lifetime can decide whether there is a parity change or not. The minimum lifetime predicted for a spin change of three units and the most favorable parity change (i.e., no parity change) is 4.3×10^6 yr.; this is incompatible with present observations so that a spin of $4\hbar$ for Be^{10} is ruled out.[32]

${}_6C^{14}$

The maximum energy of the β-ray from C^{14} is considerably less than that from Be^{10}, namely 0.15 Mev[33] instead of 0.60 Mev. The product nucleus N^{14} has a spin of one.[34] If the C^{14} ground state has a spin zero and a parity different from that of the N^{14} ground state, the predicted lifetime would be at least 10 yr. This lifetime would be increased to the lower limit given in Table I provided that $|M_0|^2\sim0.01$—which is rather small. If the spin of C^{14} were $3\hbar$, and a parity change takes place, the minimum theoretical lifetime would be 2.2×10^4 yr.—which falls within the range of values given by Ruben and Kamen. A spin of $4\hbar$ for C^{14} is certainly excluded as is a spin $3\hbar$ with the same parity as N^{14}. Improved experiments on the lifetime can decide between spin $0\hbar$ (or $2\hbar$) and spin $3\hbar$ for C^{14}; e.g., if the lifetime actually turns out to be close to the lower limit of about 10^3 yr. then the spin $3\hbar$ will be excluded and the spin $0\hbar$ (or spin $2\hbar$) will be favored. Of course direct measurements of the spin of C^{14} (and of the other artificially produced long-lived β-emitter Be^{10}) would supply a powerful check on the whole tensor theory of β-decay.

IV. FORBIDDEN ORBITAL ELECTRON CAPTURE

It is only necessary to make several obvious changes in the theory of the tensor interaction as given in Section II to derive a general formula for the lifetimes of nuclei which decay by orbital electron capture with arbitrary spin and parity change. The fundamental difference is, of course, that instead of the emission of an electron-neutrino pair into the continuum, an electron in a definite discrete energy state in the atom is absorbed by the nucleus, and a neutrino is emitted with the available kinetic energy. The original nucleus changes over to a nucleus with a positive charge one unit less and the product nucleus may be left either in an excited or the ground state. In the equations discrete Coulomb wave functions replace the Coulomb wave functions for the continuum which had previously been used for the electron, and the integration over the electron energy spectrum reduces to a single term.

In an allowed transition on the tensor interaction only an s electron[35] ($j=\frac{1}{2}$, $l=0$) will be captured with any appreciable probability. This represents what is usually called K capture[36] although, as it will turn out, $L_{\rm I}$ electrons—which also have $j=\frac{1}{2}$, $l=0$—will under favorable circumstances compete strongly with K-electron capture. In an allowed transition the maximum allowable spin change is $1\hbar$. For a first forbidden transition not only will s electrons be captured but also p electrons: under certain conditions—which will be specified later—p-electron capture becomes more probable than s-electron capture; a nuclear spin change of two units is permitted in a first forbidden transition. In the case of a second forbidden transition d electrons can be

---

31 Cf. Millman, Kusch, and Rabi, Phys. Rev. **56**, 165 (1939).

32 E. P. Cooper and E. C. Nelson, Phys. Rev. **58**, 1117 (1940), have shown that it is impossible to explain a spin of $4\hbar$ for Be^{10} with the Hartree model of the nucleus. It is likely that it is just as impossible to explain a spin of $3\hbar$ for Be^{10} even with tensor forces. The difficulty of reconciling current nuclear theory (proton-neutron symmetry, etc.) with the difference in structure between long-lived Be^{10} and short-lived C^{10} still remains.

33 S. Ruben and M. D. Kamen, Phys. Rev. **59**, 349 (1941).

34 Ornstein and Van Wijck, Zeits. f. Physik **49**, 315 (1928).

35 We are assuming that each individual electron moves in a screened Coulomb field and obeys the Dirac equation corresponding to this field. On this model the K shell contains two s electrons with $j=\frac{1}{2}$, $l=0$, the L shell contains two $L_{\rm I}$ s electrons with $j=\frac{1}{2}$, $l=0$, two $L_{\rm II}$ p electrons with $j=\frac{1}{2}$, $l=1$, four $L_{\rm III}$ p electrons with $j=3/2$, $l=1$, and so on for the higher shells. The energy of the bound states of a Dirac electron depends on the principal quantum number n and $(j+\frac{1}{2})$, where j is the total angular momentum quantum number. However, the $(L_{\rm II}-L_{\rm III})$ splitting is less than the $(L_{\rm I}-L_{\rm II})$ splitting (due to screening) for $Z<60$ (cf. H. E. White, *Introduction to Atomic Spectra*, pp. 314–323). In order to enable the theory to be compared more easily with experiment, we therefore take the $L_{\rm II}$ and $L_{\rm III}$ p electrons together and only give results for $Z<60$. Moreover, we group all the electrons in each shell together as far as the energy is concerned using Slater's screening constants [cf. Phys. Rev. **36**, 51 (1930)], i.e., $W_K=\left[1-\frac{\alpha^2(Z-0.30)^2}{2}\right]mc^2$, $W_L=\left[1-\frac{\alpha^2(Z-4.15)^2}{8}\right]mc^2$, etc.

36 Cf. C. Möller, Physik. Zeits. Sowjetunion **11**, 9 (1937).

TABLE IV. K and L Dirac wave functions in a Coulomb field.

Z	R	g_K^2	$g_{L_I}^2$	$f_{L_{II}}^2$	$g^2_{L_{III}/R^2}$
10	1.06×10^{-2}	1.6×10^{-3}	4.2×10^{-5}	1.8×10^{-9}	3.6×10^{-6}
20	1.33×10^{-2}	1.4×10^{-2}	8.2×10^{-4}	6.7×10^{-7}	7.2×10^{-5}
30	1.56×10^{-2}	5.6×10^{-2}	4.5×10^{-3}	1.2×10^{-5}	3.2×10^{-4}
40	1.75×10^{-2}	1.6×10^{-1}	1.4×10^{-2}	8.4×10^{-4}	9.7×10^{-3}
60	2.04×10^{-2}	9.5×10^{-1}	1.0×10^{-1}	2.3×10^{-3}	4.7×10^{-3}

captured, the maximum nuclear spin change is three units, and so on. We calculate the lifetimes associated with these different kinds of transitions.

The lifetime $\tau_c^{(n)}$ (the subscript indicates capture) for the nth forbidden transition on the tensor theory with favorable parity change is:

$$\frac{1}{\tau_c^{(n)}}=\frac{1}{\tau_0}\frac{|M_n|^2}{[1\cdot3\cdots(2n+1)]^2}\times R^{2n}B_c^{(n)}(Z_{\text{eff}},W_0),\quad(20)$$

where τ_0 is the decay-constant as before and equal to 7×10^3 sec., R is the nuclear radius measured in units of $\hbar/mc$, M_n is the nuclear matrix element given by (11a) and $B_c^{(n)}(Z_{\text{eff}},W_0)$ depends on the order of the transition n, on Z_{eff}, the effective nuclear charge for the captured electron and on W_0, the energy difference (in units of mc^2) between the parent and product nucleus. We have evaluated $B_c^{(n)}$ for an allowed transition and the first three forbidden transitions with the result:

$$B_c^{(0)}=\frac{\pi}{2}\{(W_0+W_K)^2n_Kg_K^2+(W_0+W_L)^2[n_{L_I}g^2{}_{L_I}+n_{L_{II}}f^2{}_{L_{II}}]\},\quad(20a)$$

$$B_c^{(1)}=\frac{\pi}{2}\left\{(W_0+W_K)^4n_Kg_K^2+(W_0+W_L)^4[n_{L_I}g^2{}_{L_I}+n_{L_{II}}f^2{}_{L_{II}}]+9(W_0+W_L)^2\frac{n_{L_{III}}g^2{}_{L_{III}}}{R^2}\right\},\quad(20b)$$

$$B_c^{(2)}=\frac{\pi}{2}\left\{(W_0+W_K)^6n_Kg_K^2+(W_0+W_L)^6[n_{L_I}g^2{}_{L_I}+n_{L_{II}}f^2{}_{L_{II}}]+30(W_0+W_L)^4\frac{n_{L_{III}}g^2{}_{L_{III}}}{R^2}\right\},\quad(20c)$$

$$B^{(3)}=\frac{\pi}{2}\left\{(W_0+W_K)^8n_Kg_K^2+(W_0+W_L)^8[n_{L_I}g^2{}_{L_I}+n_{L_{II}}f^2{}_{L_{II}}]+63(W_0+W_L)^6\frac{n_{L_{III}}g^2{}_{L_{III}}}{R^2}\right\}\quad(20d)$$

In (20a) to (20d) $n_K\le2$ is the number of electrons in the K shell, $n_{L_I}\le2$ is the number of electrons in the L shell with $(j=\frac{1}{2},\,l=0)$, $n_{L_{II}}\le2$ is the number of electrons in the L shell with $(j=\frac{1}{2},\,l=1)$, and $n_{L_{III}}\le4$ is the number of electrons in the L shell with $(j=\frac{3}{2},\,l=1)$. We have only included the terms which involve electrons in the K and L shells since it is very doubtful that capture of electrons in the M or higher shells can ever be detected experimentally. The g_K, g_{L_I}, $g_{L_{III}}$ functions are the "large" radial Dirac wave functions in a Coulomb field corresponding[37] to the quantum number sets $(\bar{n}=1, j=\frac{1}{2}, l=0)$, $(\bar{n}=2, j=\frac{1}{2}, l=0)$, $(\bar{n}=2, j=\frac{3}{2}, l=1)$, respectively. They are given by:

$$g_K{}^2=\frac{(1+W_K)}{2\Gamma(2s_0+1)}(2\alpha Z_{\text{eff}})^3\exp\left[-(2\alpha Z_{\text{eff}}R)\right](2\alpha Z_{\text{eff}}R)^{2s_0-2},\quad(21a)$$

$$g^2{}_{L_I}=\frac{(2s_0+1)(2+2s_0)^{\frac{1}{2}}(1+W_L)}{4\Gamma(2s_0+1)((2+2s_0)^{\frac{1}{2}}+1)}\left(\frac{2\alpha Z_{\text{eff}}}{(2+2s_0)^{\frac{1}{2}}}\right)^3\exp\left[-\frac{2\alpha Z_{\text{eff}}R}{(2+2s_0)^{\frac{1}{2}}}\right]\left(\frac{2\alpha Z_{\text{eff}}R}{(2+2s_0)^{\frac{1}{2}}}\right)^{2s_0-2}\quad(21b)$$

$$g^2{}_{L_{III}}=\frac{R^2(1+W_L)}{2\Gamma(2s_1+1)}(\alpha Z_{\text{eff}})^3\exp\left[-\alpha Z_{\text{eff}}R\right](\alpha Z_{\text{eff}}R)^{2s_1-4},\quad(21c)$$

[37] Cf. H. A. Bethe, Handbuch der Physik, 24/2, p. 316; $\bar{n}$ is the principal quantum number.

with[38]

$$s_0=(1-\alpha^2Z^2_{\text{eff}})^{\frac{1}{2}},\quad W_K=1-\frac{\alpha^2Z^2_{\text{eff}}}{2},\quad s_1=(4-\alpha^2Z^2_{\text{eff}})^{\frac{1}{2}},\quad W_L=1-\frac{\alpha^2Z^2_{\text{eff}}}{8}.$$

In (21a), $Z_{\text{eff}}=Z-0.30$ and in (21b) and (21c) $Z_{\text{eff}}=(Z-4.15)$. The function $f_{L_{\text{II}}}$ which appears in (20a) to (20d) is the "small" component of the Dirac wave function for ($\bar{n}=2$, $j=\frac{1}{2}$, $l=1$), and is to a good approximation given by:

$$f^2{}_{L_{\text{II}}}=\tfrac{3}{16}\alpha^2Z^2_{\text{eff}}g^2{}_{L_{\text{I}}}. \tag{21d}$$

Values of $g_K{}^2$, $g^2{}_{L_{\text{I}}}$, $f^2{}_{L_{\text{II}}}$, $g^2{}_{L_{\text{III}}/R^2}$ for representative Z's are given in Table IV, with

$$R(\hbar/mc)=1.5\times10^{-13}A^{\frac{1}{3}}.$$

To compute theoretical lifetimes for K or L electron capture on the basis of (20) it is necessary to know M_n and W_0. The value for M_n could be taken from β-emitters in the same region of the periodic table and the same order of transition. When positron emission takes place W_0 can be determined from the maximum energy of the positron. If there is no positron emission only an upper bound of 0.5 Mev can be placed on W_0 since few if any existing measurements of thresholds (for the production of the unstable nuclei decaying by K or L capture) and masses (of the stable product nuclei) are sufficiently accurate to determine W_0. We have, therefore, computed the minimum lifetimes for $W_0=1$ corresponding to absence of positron emission. These lifetimes for different Z are listed in Table V for the allowed, and the first, second, and third forbidden transitions assuming pure K capture. The matrix element M_n in (20) is in each case taken equal to one and, furthermore, it is assumed that the parent nucleus has a larger spin than the product nucleus. If the spin of the parent nucleus is less than the spin of the product nucleus, the right-hand side of (20) should be multiplied by $(2j_f+1)/(2j_i+1)$ where j_i, j_f as before, are the spins of the parent and product nuclei, respectively. Some applications of this table to experiment are given below.

An examination of the allowed transition formula (20a) reveals that for sufficiently negative W_0, L_{I} capture becomes as probable as K capture. We have listed the values of W_0 for which this occurs, i.e., when $(W_0+W_K)^2g_K{}^2=(W_0+W_L)^2g_{L_{\text{I}}}{}^2$, for different Z in column 2 of Table VI and in column 3 the corresponding minimum lifetimes. These lifetimes are to be compared with columns 2 and 3 of Table V; it is seen that they are considerably larger than the lifetimes in column 2 and smaller than the lifetimes in column 3 (when $Z>35$) for a first forbidden transition. For nuclei with exceptionally long lives—which are of the order of the first forbidden values—which decay by orbital electron capture unaccompanied by positron emission, it would be interesting to look for x-rays associated with transitions to the L shell. If the β_4-line (in x-ray terminology) has an intensity comparable with the $K\alpha_1$ line, the transition is allowed; if it is completely absent, the transition is first forbidden. The decay of Fe^{55} with a lifetime 5–10 yr.[39] may be a case in point but no one has looked for the x-rays.

A similar analysis can be made for the first forbidden transition: it is seen from (20b) that again for sufficiently negative W_0 (though

TABLE V. Minimum lifetimes for K capture with $W_0=1$.

Z	$\tau_c^{(0)}$ (sec.)	$\tau_c^{(1)}$ (sec.)	$\tau_c^{(2)}$ (sec.)	$\tau_c^{(3)}$ (sec.)
10	3.4×10^5	6.8×10^8	3.8×10^{14}	4.3×10^{20}
20	3.8×10^4	4.9×10^8	1.8×10^{13}	1.2×10^{19}
30	9.8×10^3	9.1×10^7	2.4×10^{12}	1.2×10^{18}
40	3.4×10^3	2.5×10^7	5.2×10^{11}	2.1×10^{17}
60	5.8×10^2	3.1×10^6	4.8×10^{10}	1.4×10^{16}

TABLE VI. Minimum lifetimes for L capture with $W_0=1$.

Z	W_0 (for $\frac{L_{\text{I}}}{K}=1$)	$\tau_c^{(0)}$ (sec.)	W_0 (for $\frac{L_{\text{I}}}{K}=1$)	$\tau_c^{(1)}$ (sec.)	W_0 (for $\frac{L_{\text{II, III}}}{K}=1$)	$\tau_c^{(1)}$ (sec.)
10	−0.99	1.3×10^{10}	−0.99	1.9×10^{18}	−0.79	2.9×10^{12}
20	−0.99	1.1×10^{9}	−0.98	5.1×10^{17}	−0.67	3.8×10^{11}
30	−0.97	5.4×10^{8}	−0.95	1.6×10^{15}	−0.62	4.5×10^{10}
40	−0.93	9.1×10^{6}	−0.91	4.0×10^{13}	−0.58	9.8×10^{9}
50	−0.90	2.5×10^{6}	−0.85	1.4×10^{12}	−0.56	3.4×10^{9}
60	−0.86	6.0×10^{5}	−0.77	7.7×10^{10}	−0.53	1.3×10^{9}

[38] Cf. end of footnote 35.

[39] Cf. J. L. Livingood and G. T. Seaborg, Phys. Rev. **55**, 1268 (1939), and S. Van Voorhis, private communication, quoted in Livingood and Seaborg (see reference 42 below).

smaller in absolute magnitude than was true for allowed transitions) L_{I} capture becomes as probable as K capture. The values of W_0 for which this occurs, i.e., when $(W_0+W_K)^4 g_K^2 = (W_0+W_L)^4 g_{L_{\mathrm{I}}}^2$ and the corresponding minimum lifetimes are listed in columns 4 and 5 of Table VI. In addition, for values of W_0 even less close to -1, $L_{\mathrm{II,III}}$ capture—we consider the capture of L_{II} and L_{III} electrons together because the $L_{\mathrm{II}}-L_{\mathrm{III}}$ splitting is so small (cf. reference 35)—becomes as probable as K capture. These values of W_0, i.e., when

$$(W_0+W_K)^4 g_K^2 = \left[(W_0+W_L)^4 f_{L_{\mathrm{II}}}^2 + 18(W_0+W_L)^2 \frac{g_{L_{\mathrm{III}}}^2}{R^2}\right]$$

and the corresponding minimum lifetimes are listed in columns 6 and 7 of Table VII. From purely statistical arguments some nuclei should exist for which the $L_{\mathrm{II,III}}$ capture becomes important for a first forbidden transition, but of course the measurement will be difficult to carry out because of the long lifetimes. It is for this reason that we have not given the conditions for appreciable L capture in the case of higher order forbidden transitions; experimental verification is almost out of the question.

Throughout the preceding discussion we have assumed that the transitions take place in accordance with the tensor interaction and with favorable parity change. If the parity change required by the particular allowed or forbidden transition is at variance with the actual parity change which the unstable nucleus must undergo, the lifetime for the same spin change will be considerably increased. A repetition of the arguments which led to (17a, b) and (19a, b) would lead to similar formulae for orbital electron capture. However it does not seem worthwhile to write down these expressions. A rough estimate of the effect on the lifetime of unfavorable parity change in the case of pure K capture can always be obtained by multiplying the value one gets from (20) by $(W_0+W_K)^2R^2$.

The experimental material which could test some of the above conclusions is rather meager. It is necessary to use nuclei for which the lifetimes are long and for which positrons are either completely absent or present as a known fraction of the x-radiation accompanying the orbital electron capture. One of the best studied reactions which falls into this category is:[40] ${}_{23}V^{47}+e_K\rightarrow{}_{22}Ti^{47}$ for which no positrons at all are observed; moreover, no γ-rays are observed so that the decay must be to the ground state of Ti^{47}. The observed lifetime is $1.7/\log_e \pm 0.2 = 2.4\pm 0.2$ yr. If this decay were an allowed transition with maximum allowable spin change $1\hbar$ and no parity change—the minimum theoretical lifetime according to Table V would be 0.3 days. This value is based on the assumption that $|M_0|^2=1$ and $W_0=1$; if we choose $|M_0|^2 \sim 0.1$, as seems reasonable for an allowed transition in this atomic range, then the theoretical lifetime will be increased to the correct value 2.4 yr. provided that $W_0=-0.88$. This value of W_0 is not close enough to -1 to make L_{I}-electron capture comparable to K capture. The vanadium decay might also be explained by assuming that the maximum allowable spin change is $1\hbar$ but that a parity change takes place. The minimum theoretical lifetime with $W_0=1$, $|M_0|^2=1$—using the remark at the end of the last paragraph—would be 1 yr., which is barely reconcilable with experiment. A first forbidden transition with a spin change of $2\hbar$ and parity change seems excluded; as can be seen from Table V, the minimum theoretical lifetime for $W_0=1$, $|M_1|^2=1$ is then 7 yr.

A situation similar to that of vanadium occurs in the case of the reaction[41] ${}_{25}Mn^{54}+e_K\rightarrow{}_{24}Cr^{54}$ No positrons have been observed although a γ-ray of energy 0.85 Mev has been detected The lifetime is 1.25 ± 0.05 yr. The excited state of Cr^{54} to which Mn^{54} decays probably has a

TABLE VII. Data on isobaric pairs.

Isobaric pair	Spins (in units of $\hbar$)	Theoretical lifetime for $W_0=1$ (in yr.)	W_0 for which theoretical lifetime is 10^{12} yr. (in units of mc^2)
${}_{27}Co^{57}\rightarrow{}_{26}Fe^{57}$	—	—	—
${}_{19}K^{40}\rightarrow{}_{18}A^{40}$	$4\rightarrow 0$	3×10^{10}*	-0.3*
${}_{45}Rh^{101}\rightarrow{}_{44}Rn^{101}$	—	—	—
${}_{49}In^{113}\rightarrow{}_{48}Cd^{113}$	$9/2\rightarrow\frac{1}{2}$	4×10^{11}	$+0.6$
${}_{50}Sn^{115}\rightarrow{}_{49}In^{115}$	$?\rightarrow 9/2$	1×10^{10}	$+0.1$
${}_{71}Lu^{176}\rightarrow{}_{70}Yb^{176}$	$\geqslant 7\rightarrow 0$	$>10^{12}$	≤ 1
${}_{76}Os^{187}\rightarrow{}_{75}Re^{187}$	$?\rightarrow 5/2$	1	-0.8

* The value given assumes a spin $1\hbar$ and even parity for the excited state (cf. text).

[40] Walke, Williams, and Evans, Proc. Roy. Soc. **A171** 360 (1939).

[41] J. J. Livingood and G. T. Seaborg, Phys. Rev. **54** 391 (1938).

spin $1\hbar$ since the ground state of Cr^{54}—which is an even-even stable nucleus—must have a spin zero. If Mn^{54} has the same parity and a spin which does not differ by more than $1\hbar$ from the spin of the excited state of Cr^{54}, then with $|M_0|^2 \sim 0.1$ the theoretical lifetime will be brought into agreement with the observed lifetime of 1.25 yr. by choosing $W_0 = -0.85$. If the parity of Mn^{54} is different from the parity of the excited state of Cr^{54}, the minimum theoretical lifetime for $W_0 = 1$, $|M_0|^2 \approx 1$ is 0.9 yr. It is probable that the transition is allowed and that no parity change takes place.

The few other long-lived nuclei which are thought to decay by K-electron capture,[42] such as Fe^{55}; Cl^{36}, etc., are not very well investigated and so will not be considered here. However, it is interesting to apply the theory to some of the known neighboring stable isobars. All the known neighboring stable isobars—taken from Livingood and Seaborg's article[42]—are listed in column 1 of Table VII with the spins where measured, or predicted with reasonable certainty, given in column 2. The pairs (K^{40}, A^{40}) and (Lu^{176}, Yb^{176}), are also listed although one member of each pair, namely K^{40} and Lu^{176}, are electron emitters.

In column 3 are given the theoretical lifetimes for K-electron capture on the assumption that (a) $W_0 = 1$—since positrons are absent, (b) $|M_n|^2 = 0.01$—which is a reasonable guess for highly forbidden transitions, (c) the nuclei decay to the ground state of the parent nuclei, and (d) the parity change is favorable. Values are not given for the pairs (Co^{57}, Fe^{57}), (Rh^{101}, Ru^{101}) since the spins of neither nucleus of the pair is known. A value for the theoretical lifetime is given for the pair (Os^{187}, Re^{187}) under the assumption that Os^{187} has the maximum known spin for an even-odd nucleus, namely, $9/2\hbar$. Similarly a value is given for the pair (Sn^{115}, In^{115}) when we assume that Sn^{115} has a spin of $\frac{1}{2}\hbar$. Since no evidence has been found that the first member of any of the pairs of nuclei listed in Table VII decays by K-electron capture to the second member (with the exception of K^{40} which is discussed in more detail below), we must assume that the lifetimes are greater than say 10^{12} yr. We have therefore tabulated in column 4 the values of W_0 for which the theoretical lifetime attains this value. We see that there is a scattering as would be expected from statistical considerations.

The case of K^{40} is of particular interest as has already been remarked. In Section III (cf. reference 23) it was pointed out that K^{40} emits a homogeneous 2-Mev γ-ray and that about 3 quanta are emitted per 100 disintegration electrons. Furthermore, it followed that this γ-ray could not be correlated with the energy levels of the Ca^{40} nucleus formed in the electron disintegration. It will be shown that the assumption that the γ-ray is associated with K-electron capture to an excited state of A^{40} leads to reasonable results. Suppose the 2-Mev γ-ray is due to a radiative transition from an excited state of A^{40} to its ground state; it follows that there would be sufficient energy available for positron decay of K^{40} to the ground state of A^{40}. Since the spin of the ground state of A^{40} is zero, the positron emission would be third forbidden and formula (11)[43] would predict an observable intensity—about as large as the observed electron intensity of 23 per sec. per gram of potassium—provided the parity of the ground state of A^{40} were even (i.e., different from K^{40} which is taken as odd). Since no positrons are observed the parity of the A^{40} must be odd. The excited state of A^{40} would then presumably be even and have a spin $1\hbar$. The emission of the 2-Mev γ-ray would then be associated with a K-electron capture by K^{40} having a spin $4\hbar$ and odd parity to an excited state of A^{40} with spin $1\hbar$ and even parity. This would be a second forbidden transition with unfavorable parity change. The transition probability would be according to (20):

$$1/\tau_c^{(2)} = 1.3\times 10^{-16}|M_2|^2(W_0+W_K)^8 \text{ sec.}^{-1}$$

to be compared with the observed value 5×10^{-19} sec.$^{-1}$. If we choose $|M_2|^2 \sim 0.01$, then $W_0 \sim -0.3$ which is quite reasonable.

In conclusion, it is a pleasure to thank Professor V. F. Weisskopf for many helpful discussions and advice generously given.

[42] These are listed in J. J. Livingood and G. T. Seaborg, Rev. Mod. Phys. **12**, 30 (1940); a more up-to-date table by Dr. Dessauer of our laboratory has also been very useful.

[43] Equation (11) can be applied to positron emission if $(-Z)$ replaces Z throughout.