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Scattering of Polarized X-Rays by Ferromagnetic Substances

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It is shown that if high frequency polarized x-rays are scattered by some substance, the total intensity of the scattered rays should be altered by anything leading to special orientations of the electron spins of the scattering electrons. Some experiments are described in which attempts were made to detect the effect on intensity due to changes in spin orientations in ferromagnetic substances when these substances are magnetized. No effects were observed. It is shown that an attempt to reconcile the results of these experiments with those of gyromagnetic experiments leads to orientations of the spin giving the spin magnetic quantum numbers $+1/2$ and $-1/2$.

INTRODUCTION

SEVERAL attempts have been made to detect some effect of an applied magnetic field on reflections of x-rays by ferromagnetic substances.[1] These experiments have not been successful. No changes have been observed in position or intensity of the diffraction maxima. A change should have been observed if magnetization results in special orientations of the planes of the orbital electrons, but not if magnetization is caused by electron spin. The fact that gyromagnetic experiments on ferromagnetics always yield a value of approximately 2 for the Landé g factor indicates that ferromagnetism is associated with the spin. The present paper deals with the problem from a new approach in which polarized x-rays were used, and a phenomenon was used that should be sensitive to spin orientations.

It is well known that Thomson's classical theory of x-ray scattering predicts complete polarization for x-rays scattered at 90° to the primary beam by free electrons. This theory was modified by the quantum-mechanical treatment of Klein and Nishina in 1928. Nishina[2] showed that there should be an unpolarized component, the relative intensity of which varies approximately as the square of the frequency of the primary x-rays. The existence of an unpolarized component of the order of magnitude predicted has been shown experimentally.[3]

By taking electron spin into account, A. H. Compton[4] has shown that classical electrodynamics also predicts an unpolarized component that is proportional to ν^2. The unpolarized, or magnetic, scattering arises in classical theory from a precessional acceleration of the spin magnetic moment vector due to the torque on it of the magnetic vector of the x-ray wave. If γ is the magnetic moment of the electron and H the magnetic vector of the x-ray wave, this torque is

$$T=\gamma\times H. \tag{1}$$

If the Y axis be taken as the direction of propagation of the primary wave and the YZ plane be chosen to include OP, the direction of the scattered ray, Compton showed that the classical magnetic scattering of an electron is

$$I_s'=I_0\frac{4\pi^2\nu^2\gamma^4}{r^2c^4p^2}\sin^2\xi\sin^2\theta, \tag{2}$$

where I_0 is the intensity and ν the frequency of the primary wave, p is the angular momentum of the electron, ξ is the angle between γ and H, and θ is the angle between OP $(=r)$ and T. Averaging this expression over all possible orientations of the spin vector gave

$$I_s=I_0\frac{4}{3}\frac{\pi^2\nu^2\gamma^4}{r^2c^4p^2}[1+\sin^2\varphi\cos^2\alpha], \tag{3}$$

where φ is the angle between OP and the Y axis and α is the angle between H and the Z axis.

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The appearance of $\sin^2\xi$ and $\sin^2\theta$ in (2) shows that the magnetic scattering by a single electron depends sharply on the orientation of the spin vector. Equation (3) would not be expected to apply if the spins were not oriented at random, but were oriented so as to have a strong component in some definite direction. Gyromagnetic experiments indicate that magnetization of ferromagnetic bodies may result in just such orientations. Some experiments have recently been done by the writer with the purpose of attempting to determine whether magnetic scattering of polarized x-rays by ferromagnetic substances is influenced by an applied magnetic field.

EXPERIMENTAL PROCEDURE

The conventional experimental procedure was used to produce and study the polarized x-rays. The primary rays were directed toward a block of carbon and a beam of rays scattered by it at 90° to the primary beam was again scattered, this time by the ferromagnetic substance under investigation. Intensity measurements of the tertiary rays were made at a point P in a direction at 90° to both the primary and secondary rays. According to the classical theory of Thomson, the intensity in this direction ought to be zero. Actually there are three reasons why it is not zero. The first reason is geometrical in nature. Since the scattering substances do not have zero dimensions, some rays that have not been scattered at exactly 90° will arrive at P. Secondly, some rays will be scattered more than once in each scatterer, and some of the multiply scattered rays will reach P. In the third place some magnetically scattered rays that are unpolarized will reach P. The magnetic scattering is negligible when the x-ray tube is operated at low voltages but becomes appreciable at voltages of 200 kilovolts or higher.

The relative intensities of the three types of scattering to P can be estimated roughly. The "geometrical" scattering can be calculated from the dimensions of the apparatus. Such a calculation for the apparatus used shows that this scattering should be about 0.012 I' where I' is the intensity of the tertiary rays at a point P' ($OP'=OP$) in a direction at 90° to the secondary rays but parallel to the primary rays. The intensity due to multiple scattering can only be estimated experimentally. Following a method used by Compton and Hagenow,[5] this was done by using different thicknesses of the scattering substances and noting the effect on the polarization obtained. The results obtained indicated that the intensity reaching P by multiple scattering was about 0.036 I'. The magnetic scattering to P can be calculated from either Compton's or Nishina's formula if the wave-length of the primary x-rays is known. Throughout all the experiments the x-ray tube was operated at 220 kilovolts and the primary rays filtered through 0.7 mm of copper and 0.5 mm of aluminum. Under such circumstances, it would seem reasonable to assume that the effective wave-length is about 0.10 angstrom. If this value be used in Nishina's formula, we get for the magnetic scattering to P the intensity 0.038 I'. A result of the same order of magnitude would be obtained if Compton's formula were used. If we add the intensities for the three types of scattering, we see that the magnetic scattering accounts for about 44 percent of the total that should reach P.

The ferromagnetic substance being studied was cut into a thin slab and mounted so that each of its ends was in contact with one of the pole pieces of a powerful electromagnet. Lead was used to shield the pole pieces from the x-rays. The substances studied consisted of several samples of iron and permalloy. A Geiger-Müller counter was used to determine the relative intensities of the tertiary rays with the magnet off and then on.

It was found in all the experiments that magnetizing the sample did not make the slightest detectable difference in the intensity of the rays scattered to P. This was true regardless of whether the sample was magnetized parallel or perpendicular to the magnetic vector of the polarized x-rays. No high degree of accuracy can be claimed for the intensity measurements, but it is believed that a change of 2 or 3 percent in intensity would not have escaped detection.

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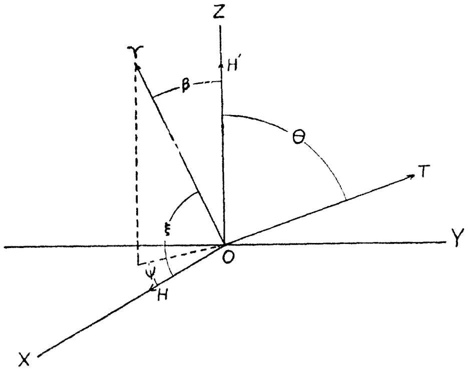

FIG. 1. Orientation of x-ray beam, sample, and applied field.

DISCUSSION OF THE RESULTS

When the sample is magnetized parallel to the magnetic vector of the x-rays, the magnetic scattering ought to vanish if all electron spins should be oriented parallel to the field. This, as we have seen, would cause a reduction of about 44 percent in the intensity at P. When the sample is magnetized perpendicular to the magnetic vector, a calculation based on Compton's results shows that orientation of the spins parallel to the field should cause the total intensity at P to be practically doubled. Such ideal orientations were, of course, not expected. It would seem reasonable, however, to expect some change in intensity if ferromagnetism is associated with special spin orientations as indicated by gyromagnetic experiments.

Assuming that the scattering electrons of ferromagnetic substances do take up special orientations when the sample is magnetized, we shall now inquire as to whether any such orientations are possible for which the scattering is the same as for random orientations. We shall indicate by β any angle that the spin axes may assume with the field such that the scattering is unaltered, and then determine what values β may have. The computations will be carried out for the case in which the applied field is perpendicular to the magnetic vector of the x-rays. The same values of β are obtained when computations are made for the field parallel to the magnetic vector.

In Fig. 1 the polarized x-rays are considered to move along the Y axis with the magnetic vector in the XY plane. The sample is at the origin and the applied field H' is along the Z axis which is also the direction of observation of the scattered rays. The spin vector γ is assumed to make a constant angle β with the applied magnetic field. The angles ξ and θ of (2) are shown in the figure. Since T is perpendicular to both γ and H, it lies in the YZ plane. In order to compute the scattering for a constant value of β, we must compute the average value of $\sin^2\xi \sin^2\theta$ in Eq. (2) for this value of β. This may be done in the following manner.

The direction cosines of T are

$$0,\ \sin\theta,\ \cos\theta,$$

and those of γ are

$$\cos\xi,\ \pm(1-\cos^2\xi-\cos^2\beta)^{\frac{1}{2}},\ \cos\beta.$$

Since T is perpendicular to γ

$$\pm\sin\theta\ (\sin^2\xi-\cos^2\beta)^{\frac{1}{2}}+\cos\theta\ \cos\beta=0.$$

This gives

$$\begin{aligned}\sin^2\theta\ \sin^2\xi-\sin^2\theta\ \cos^2\beta&=\cos^2\theta\ \cos^2\beta\\&=\cos^2\beta-\sin^2\theta\ \cos^2\beta,\end{aligned}$$

or

$$\sin^2\theta\sin^2\xi=\cos^2\beta.$$

Thus we see that $\sin^2\theta\ \sin^2\xi$ has the constant value $\cos^2\beta$.

To get the scattering for this orientation of the spin we simply substitute $\cos^2\beta$ for $\sin^2\xi\ \sin^2\theta$ in (2). We shall write the result as

$$I_s'=4K\cos^2\beta, \tag{4}$$

where K represents a constant factor.

To compute the scattering in the Z direction of the polarized rays for random orientations, we can either use Eq. (3) by noting that $\alpha=90°$ and $\varphi=90°$, or we can average (4) for all values of β. We could do the latter by evaluating the following integral

$$I_s=\frac{1}{4\pi}\int_0^{\pi}\int_0^{2\pi}4K\sin\beta\cos^2\beta d\psi d\beta.$$

Either method gives

$$I_s=\frac{4}{3}K. \tag{5}$$

We now obtain the possible values of β by equating the scattering from (4) and (5) which leads to the expression

$$\cos\beta = \pm\tfrac{1}{3}\sqrt{3}. \quad (6)$$

We thus see that there are two orientations of the spins which give the same scattering as random orientations. These values of β are of special interest because they are just the angles necessary to give the spin magnetic quantum numbers $+\frac{1}{2}$ and $-\frac{1}{2}$ that are demanded by the theory of spectra. It is of further interest to note that they are arrived at here by classical electrodynamics in an attempt to reconcile experimental results.

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The Discharge Mechanism of Fast G-M Counters from the Deadtime Experiment

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The deadtime of a self-quenching counter is an insensitive time, of the order of 10^{-4} second, which occurs between the time when a counter registers a count and the time at which it has recovered sufficiently to register another count. A simple method of measurement of this time is shown. From a theory developed to explain this deadtime phenomenon it is possible to compute times associated with the deadtime. The check between theory and experiment is very good. The theory involves the formation of a positive ion space charge sheath about the wire of the counter, this sheath expanding to the cylinder. Not only is it possible to show the internal action of a counter by the deadtime experiment, but also it is possible to study the lengthwise spread of that discharge. It was found that the discharge spread throughout the length of the counter, but the spread could be stopped by a small glass bead on the wire. This discovery led to the construction of a directional Geiger counter.

I. INTRODUCTION

A. Historical

SINCE the publication of the original works on G-M counters in 1928 and 1929 by Geiger and Müller,[1] there have been numerous articles concerned with construction techniques, operating properties and theories of discharge of G-M counters. In 1936 the value of the G-M counter as a laboratory instrument was greatly increased by the design of the Neher-Harper[2] extinguishing circuit. Around 1935 and 1936, independently in several laboratories, the fast, or self-quenching, G-M counter was discovered and used. In construction, the fast counters differed from the usual G-M counter only in the addition of alcohol vapor or some other organic vapor to the gas. Many of the operating properties of fast counters were investigated by Trost.[3] The properties of the fast counters differed from those of the usual type G-M counter, here termed a slow counter. In 1940, based primarily on experimental measurements by Ramsey,[4] Montgomery and Montgomery[5] formulated a consistent theory of G-M counter action.

In this present investigation of fast counter action, the most illustrative experiment is that showing the deadtime phenomenon. The deadtime technique herein developed can be used to study the lengthwise spread of the discharge within the G-M counter tube.

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