both in allowed and in forbidden transitions. If this were true it would indicate that either the tensor interaction or the pseudoscalar is important to the beta-process. It would be desirable, however, to find some light nuclei which obey the seventh or ninth power laws. According to the results of reference 1 it is such nuclei that would produce spectra which differ in shape significantly from the allowed spectra.

I am indebted to Professor Eugene Paul Wigner of Princeton University and Dr. Henry S. Sommers, Jr., of Harvard for many interesting discussions related to the subject matter of this paper.

MARCH 1 AND 15, 1942 PHYSICAL REVIEW VOLUME 61

The Multiple Scattering of Fast Electrons

L. A. KULCHITSKY AND G. D. LATYSHEV
Physical-Technical Institute of the Academy of Science of the Ukrainian S. S. R., Kharkow

(Received October 31, 1941)

The method and apparatus are described which were developed for the investigation of the multiple scattering of fast electrons. The scattering of electrons with an energy of 2.25 Mev is studied in the foils of aluminum, copper, iron, molybdenum, silver, tin, tantalum, gold, and lead. The results obtained are compared with the theory of multiple scattering. For elements from Al ($Z=13$) to Sn ($Z=50$) the experimental values for the multiple scattering coincide with those theoretically predicted. For heavy elements Ta ($Z=73$), Au ($Z=79$), Pb ($Z=82$) the experimental values are 10 to 13 percent lower than calculated.

I. INTRODUCTION

THE nuclear scattering of fast electrons has been investigated in many experimental researches. In the majority of cases, the singular or elementary nuclear scattering was investigated. All these investigations showed a close agreement between the experimental values and those calculated according to Mott's theory.

The multiple scattering of electrons was studied chiefly by Sheppard and Fowler,[1] and Fowler.[2] They investigated the scattering of electrons with a mean energy of approximately 10 Mev. The spectral composition of the energy of the electrons was spread between 2 and 17 Mev. The scattering was investigated in thin lead, aluminum, and carbon plates placed in the center of a Wilson chamber. The results obtained were compared with Williams'[3] theory. This author found that the angular distribution of the scattered electrons coincides approximately with the Gauss curve, while the mean angle for the scattering is in close agreement with those theoretically predicted for light elements (Al, C). For lead, the value of the mean angle of scattering is approximately twice as small as that theoretically calculated.

M. Slawsky and H. Crane[4] (Wilson chamber method) investigated the multiple scattering of electrons with energies near 0.9 Mev. The most probable angle of scattering in these experiments was so large, that the application of the theory of multiple scattering is not warranted as the scattering was diffuse.

For scattering in Al only, Slawsky and Crane compare their experimental values with the theory of Bethe, Rose, and Smith.[5] In this case the most probable angle coincided with that theoretically calculated.

Oleson, Chao, Halpern, and Crane[6] investigated the multiple scattering of electrons with

[1] C. W. Sheppard and W. A. Fowler, Phys. Rev. **56**, 849 (1939); **57**, 273 (1940).
[2] W. A. Fowler, Phys. Rev. **54**, 773 (1938).
[3] E. J. Williams, Proc. Roy. Soc. **169**, 531 (1939); Phys. Rev. **58**, 292 (1940).
[4] M. M. Slawsky and H. R. Crane, Phys. Rev. **56**, 1203 (1939).
[5] H. A. Bethe, M. E. Rose, and L. P. Smith, Proc. Am. Phys. Soc. **78**, 573 (1938).
[6] N. L. Oleson, K. T. Chao, J. Halpern, and H. R. Crane, Phys. Rev. **56**, 482, 1171 (1939).

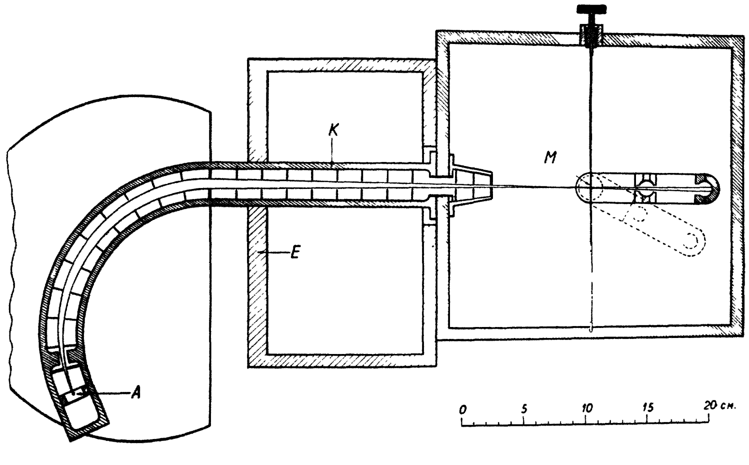


FIG. 1. General lay-out of the apparatus. Section in horizontal plane. A, the source; M, the target; K the spectrograph chamber; E, magnetic shield.

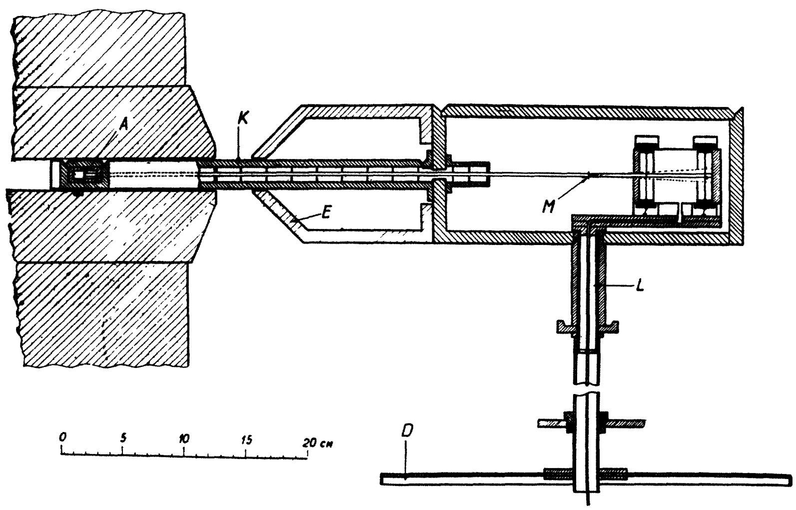


FIG. 2. General lay-out of the apparatus. Vertical section. A, source of electrons; M, target; E, magnetic shield; L, the glass-ground joint with leads to the counters and a tube to the vacuum pump; O, circular dial divided in degrees.

an energy of between 2 and 8 Mev in thin plates of lead and carbon, placed in the center of a Wilson chamber. In the comparison of experiments these authors came to the same conclusion as Fowler and Sheppard, and Fowler.

As can be seen, the multiple scattering of electrons has been investigated only by the Wilson chamber method with small statistical accuracy, and with considerable averaging of energy of the scattered electrons.

The purpose of this investigation is a detailed study of the multiple scattering of electrons under precise and definite experimental conditions.

II. THE EXPERIMENTAL METHOD

The source of fast electrons was a thin-walled glass tube containing radium emanation, placed between the poles of an electromagnet at a point A (Fig. 1). By means of a magnetic field, and a system of diaphragms shown in Figs. 1 and 2 a homogeneous, narrow and almost parallel pencil of electrons was obtained. As can be seen from Figs. 1 and 2, this electronic beam, after passing the system of diaphragms, entered a large metal chamber. In the center of this chamber far from the walls, the investigated substance was placed in the form of a thin plate

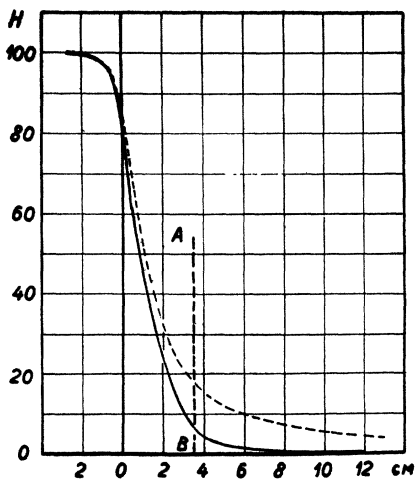


FIG. 3. Variation with distance of the stray magnetic field on the edges of the pole shoes. The dotted line—without magnetic shield; the solid line—with the shield. *AB*, position of the magnetic shield.

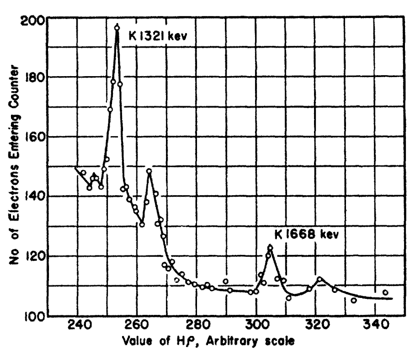


FIG. 4. Part of the natural spectrum of RaC.

(the target). The electronic beam was focused on this target and scattered by it.

The scattered electrons were registered by two G–M counters in coincident arrangement and the counters were placed in the same metal chamber, evacuated to a pressure of 10^{-2} mm Hg. The geometrical arrangement of the counters can be seen in Figs. 1 and 2.

By means of a special arrangement, the two counters could be rotated round the axis through the scattering target.

As the scattering plate was in the center of the chamber and the counters sufficiently far from one another, the electrons which could pass both counters could be emitted from a very small volume containing the scattering target. Electrons scattered by the walls of the chamber, and the other parts of the apparatus, could not pass simultaneously both counters, and therefore were not registered.

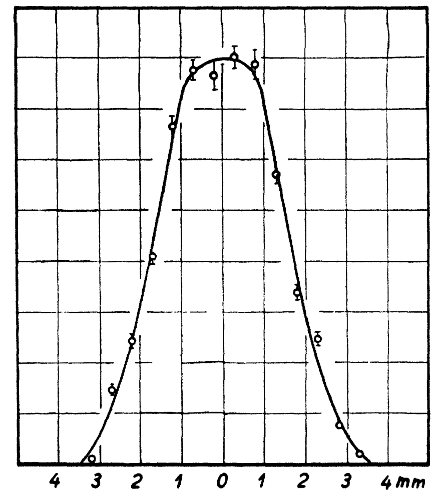


FIG. 5. Form of the electronic beam in the horizontal plane at a distance of 151 mm from the exit diaphragm.

The scattering plate was mounted on an aluminum frame, and by rotating it through 90°, the target could be placed near the upper lid of the chamber; this was done to measure the background of the counters. This measurement was very small indeed and contributed only a small part of the measured effect; the correction for the background counts had to be made only when the scattering at large angles (30–40°) was investigated.

It was impossible to measure the background counts with the raised target when investigating the scattering at small angles (0–5°); in this case the primary beam could pass unhindered into the counters, but in scattering at small angles, the background was too small in comparison with the measured effect, and its influence could be ignored.

In all investigations of the scattering of electrons, the geometry of the electronic beam, its cross section, divergence homogeneity and the influence of the edges of the diaphragms etc., are of primary importance. Because of this a great deal of time was spent in obtaining a well-defined electronic beam and for measuring its geometry. This part of the work is described in Section V.

The axis around which the counters could be rotated was connected with a large circular

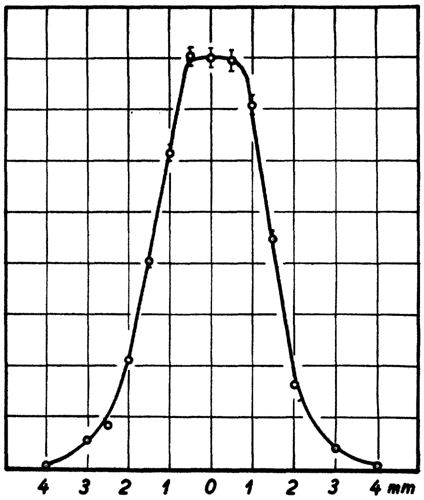


FIG. 6. Form of the electronic beam in the vertical plane at a distance of 151 mm from the exit diaphragm.

graduated dial, by means of which the angle of rotation of the counters could be ascertained.

III. APPARATUS

1. The Magnet

The cross section of the pole-shoes of the electromagnet was a segment of a circle (see Fig. 1). The gap between the pole-shoes was 23 mm. To diminish the fringing flux, a magnetic shield was placed at a distance of 35 mm from the pole-shoes, exactly at the point where the electronic beam leaves the region of the magnetic field.

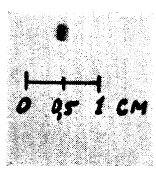


FIG. 7. Photogram of the electronic beam spot in the plane of the target.

The distribution of the magnetic field near the pole-shoes without the magnetic shield is given in Fig. 3 (dashed curve). The solid curve shows the distribution of the magnetic field with the shield. The current for the electromagnet was taken from a storage battery. The strength of the magnetic field was determined by the usual method, with a search coil and a ballistic galvanometer.

2. The Electronic Beam. Diaphragms

The geometrical shape of the electronic beam was determined by several diaphragms placed inside the apparatus. The height of all the diaphragms was 2.5 mm. In the region of the magnetic field the electrons described an arc of 115°, the radius of this arc being equal to 121 mm. The holes in the diaphragms arranged on the surface perpendicular to the direction of the magnetic field, subtended an angle of 2.1° from the axial line of the beam.

After leaving the region of the magnetic field, the electronic beam was focused on the scattering substance (target) placed at a distance of 225 mm from the magnet.

Along the calculated geometrical path of the electronic beam, several diaphragms were placed (see Figs. 1 and 2). All diaphragms were made of aluminum to diminish the scattering. All the inside walls were lined with aluminum for the same purpose. There were three defining diaphragms. The first formed an electronic source, the second was placed at the boundary of the region of the magnetic field, where the path of the electron is transformed from the arc of a circle into a straight line. The size of the third and last diaphragm was 1.5×2.5 mm^2. These three diaphragms were made accurately according to the calculated form of the electronic beam; other diaphragms were a little wider.

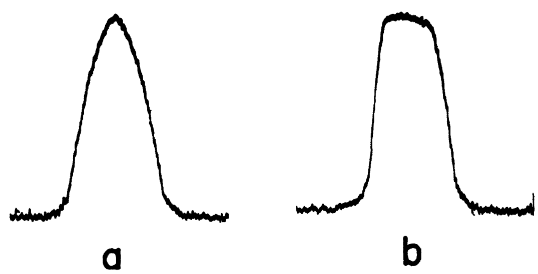


FIG. 8. Microphotograms of the electron beam spot: (a) in the horizontal plane, (b) in the vertical plane.

3. The Target

The substance used as a target is in the form of a thin plate. It was fastened to an aluminum frame by two silk threads, and by rotating at 90° could be raised to the upper lid of the apparatus. The target was 73 mm from the last

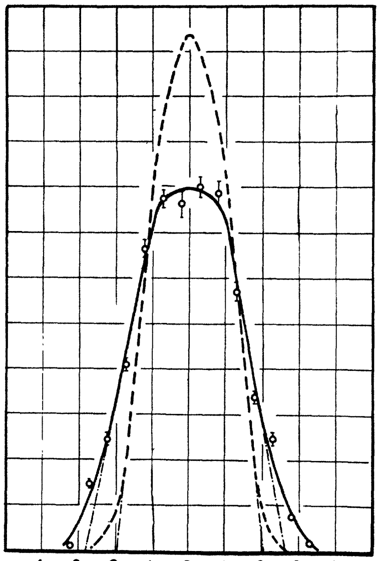


FIG. 9. Normalized curves of the electron beam in the horizontal plane measured by the G-M counter and by microphotogram method. The dash-point curve gives the extrapolation of the linear parts of the curves.

diaphragm and 37.5 mm from the entrance window of the first counter. The area of the targets was 20×20 mm². The thickness was chosen in such a way, that a half-width of the Gauss curve for scattering electrons was approximately equal to 10° for all elements.

4. The Counters

The diameter and the length of the cylindrical parts of the G–M counters were, respectively, equal to 13 mm and 45 mm. The distance between the axes of the counters was 47.5 mm. The first counter had two windows, the second only one, the size of the windows being 3.6×4.5, 5.3×6.7, and 8.6×10.5 mm consecutively; the first figure gives the width, and the second the height.

The first window was covered by a silvered celluloid film; the next two by aluminum foil 10 micron thick.

The central wires of the counters were made of steel wire 0.2 mm in diameter.

The G–M counters were placed inside a metal chamber evacuated to 10^{-2} mm Hg, and could be rotated round the target by means of a glass-ground joint, the same ground joint being used also for evacuation of the counters (13 cm Hg). All the wire belts from the counters to the amplifier passed through the same ground joint.

The potential of the counters was 2000 volts. A metal dial 458 mm in diameter and divided into 360 degrees was fastened to the ground joint.

5. The Amplifier

The amplifier was built according to the scheme developed by M. S. Kosodaev and G. D. Latyshev.[7] A description is given elsewhere.

6. The Glass-Ampoules

The glass tubes filled with radon (the ampoules) were 0.5–0.7 mm in diameter and 5–7 mm in height; the wall-thickness equalled 0.1 mm and the strength of the radon source was between 100–200 mC.

7. The Apparatus. Focusing the Electric Beam

A general scheme of the apparatus is given in Figs. 1 and 2. The apparatus, as a whole, could be shifted by means of micrometrical screws, and in this way the path of the electrons between the pole-shoes could be changed.

The source of electrons (a point "*A*" in Fig. 1) could also be shifted along the path of the electrons. By means of these two movements the electronic beam could be focused on the surface of the target; after the beam was well-focused the apparatus was securely fixed and lagged from outside with lead for shielding the counters from γ-rays.

Experience showed that the focusing was not spoiled during the changing of the radon sources. By means of a special screw, the distance between the pole-shoes could be increased and point "*A*" brought outside. After changing the radon sources the pole-shoes could be screwed to exactly the same position.

IV. THE MEASUREMENTS OF THE ENERGY OF ELECTRONS

Under the conditions in our experiments the most precise determination of the energy of electrons was procured by the investigation of the natural β-spectrum of RaC. A part of the

[7] A. I. Alikhanov, G. D. Latyschev, J. Exper. Theor. Phys. **10**, 9–10 (1940).

energy distribution of the electrons of natural β-spectrum of RaC is given in Fig. 4. On this curve the abscissae represent the deflections of the ballistic galvanometer proportional to the $H\rho$. The ordinates represent the number of electrons entering the counter in the unit-time at different values of $H\rho$. The curve in Fig. 4 is given without the recalculation for equal intervals of $H\rho$.

In these investigations of the natural β-spectrum, the counters were oriented at zero-angle in the direction of the electronic beam; the target was raised to the lid of the apparatus. Because of this, the electronic beam entered the windows of both counters. The energy of the entering electrons was changed by the variation of the magnetic field. The maximum on Fig. 4 designated by K 1321 kev represents the transition of the conversion of electrons of K shell of RaC′ from the excited level of the nucleus RaC′ equal to 1414 kev; the maximum 1399 kev represents the transition of the conversion of electrons of K shell produced by the x-rays of energy 1761 kev.

It is easy to see that the position of these maxima given on Fig. 4 does not coincide precisely with the values of the energy (or $H\rho$) written near each maximum. The electrons loose some energy inside the walls of the glass ampoules in which the radon is encased and because of this, all the maxima must be more or less displaced to the region of smaller energies.

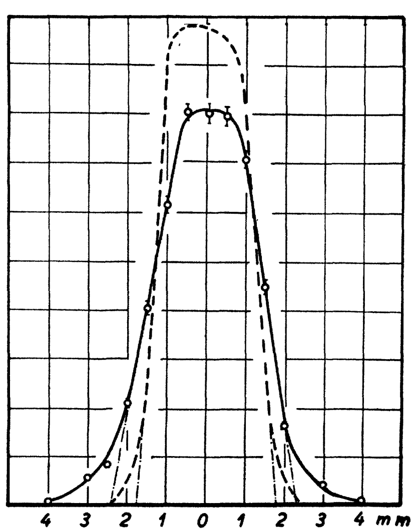


FIG. 10. Normalized curves of the electron beam in the vertical plane measured by the G-M counter and by microphotogram method. The dash-point curve gives the extrapolation of the linear parts of the curves.

The energy of conversion electrons (K 1321 and L 1399 kev) from the K and L shells of the atoms of RaC′, differ only by 78 kev, i.e., only by 3 percent, and because of this the energy loss must be the same. The maxima are displaced by almost the same values. (In the region of the energies of electrons 1.5 Mev the energy loss of electrons changes very slowly with the energy.)

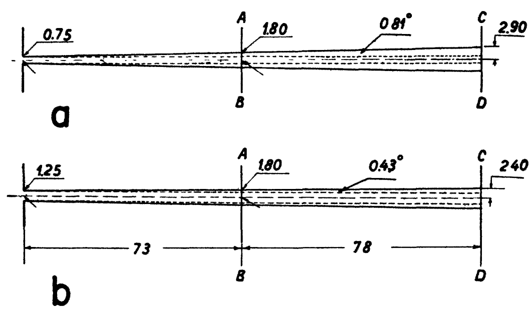


FIG. 11. The divergence of the electron beam. (a) In the horizontal plane. (b) In the vertical plane. The length scale is thrice diminished in comparison with the breadth. Dimensions are given in mm.

Therefore, knowing the difference H between the conversion electrons of the K and L shells of the RaC′ atom, from the excited energy level 1414 kev of the same atom, and knowing the difference in the deflections of the ballistic galvanometer, one can graduate it in H units.

In our experiments the energy of electrons which undergo the multiple scattering equals 2250 kev.

V. THE ELECTRONIC BEAM

We studied the electronic beam at two points: (1) at a distance of 151 mm from the point of emergence of the electrons from the last diaphragm, and (2) at the point where the target was plaçed at a distance of 73 mm from the last diaphragm.

At the first point, the beam was investigated by means of a G–M counter. A special counter was constructed which had a long narrow entrance slit, the width of which was 0.4 mm and its length 25 mm. By a special arrangement the counter could be shifted in a horizontal direction, the slit of the counter being vertical.

TABLE I. Scattering of electrons whose energy is 2.25 Mev. The angles are given in degrees. α_m is given according to Williams' theory. $\theta=\sqrt{\pi\alpha_m}$ is the half-width of the theoretical Gauss curve. θ_{th} is the half-width of the theoretical Gauss curve with the correction for the finite width of the beam. θ_{exp} is the half-width of the experimental Gauss curve. The last column shows the percent deviation of the experimental half-width value from the theoretical value.

Element	Z	A	$\sigma\frac{\text{mg}}{\text{cm}^2}$	δ	M	α_m	$\theta=\sqrt{\pi\alpha_m}$	θ_{th}	θ_{exp}	% deviation
I	II	III	IV	V	VI	VII	VIII	IX	X	XI
Al	13	26.97	26.6	2.00	60.2	5.36	9.5	9.85	9.50	−3.7
Fe	26	55.84	15.4	2.10	41.4	5.39	9.55	9.90	9.60	−3.0
Cu	29	63.57	17.15	2.31	46.8	6.05	10.7	11.05	10.40	−3.0
Mo	42	96.00	12.4	2.31	36.6	5.87	10.4	10.75	10.25	−4.6
Ag	47	107.88	11.55	2.34	35.1	5.90	10.45	10.80	10.20	−3.7
Sn	50	118.7	17.40	2.37	34.2	5.96	10.55	10.90	10.65	−2.3
Ta	73	181.4	8.9	2.45	28.7	6.02	10.65	11.00	9.85	−10.5
Au	79	197.2	8.9	2.54	29.4	6.72	11.10	11.40	9.20	−13.1
Pb	82	207.2	7.9	2.42	26.1	5.91	10.5	10.85	9.70	−10.6

The experimental results are given in Fig. 5. The abscissae represent the distances in mm from the middle of the beam, the ordinates, the number of electrons entering the counter in the unit time. This curve represents the distribution of the energy of electrons in the beam in the plane normal to the direction of the magnetic field.

By means of another arrangement the counter could be moved in a vertical direction, the slit of the counter being horizontal. The experimental results are given in Fig. 6. This curve gives the distribution of electrons in the plane parallel to the direction of the magnetic field and normal to the direction of the electronic beam.

At the second point, the electronic beam was investigated by the photographic method. Instead of the target a photographic plate was placed inside the apparatus. In Fig. 7 is shown a photographic tracing left by an electronic beam impinging on the photo-plate (target).

Figure 8 represents the microphotogram of this tracing. The abscissae give the distances from the center of the spot; the ordinates give the blackening of the plate, which in each point is proportional to the number of electrons impinging on this point.

Figure 8a gives the distribution of the intensity of the electronic beam in the horizontal plane; the same distribution for the vertical plane is given in Fig. 8b.

The small "tails" in the curves in Figs. 5, 6, 8a, and 8b are produced by the electrons scattered by the last diaphragm. From these curves the number of scattered electrons can be evaluated. Indeed by extrapolation of the linear parts of the curves, we can determine the area of the "tails." Dividing the sum of the areas of two "tails" (on the right and the left of the curve) by the whole area of the curve, we obtain the percent number of electrons scattered by the edges of the diaphragm. This calculation shows that in our experiments only 2–3 percent of all the electrons in the beam are scattered by the edges of the last diaphragm.

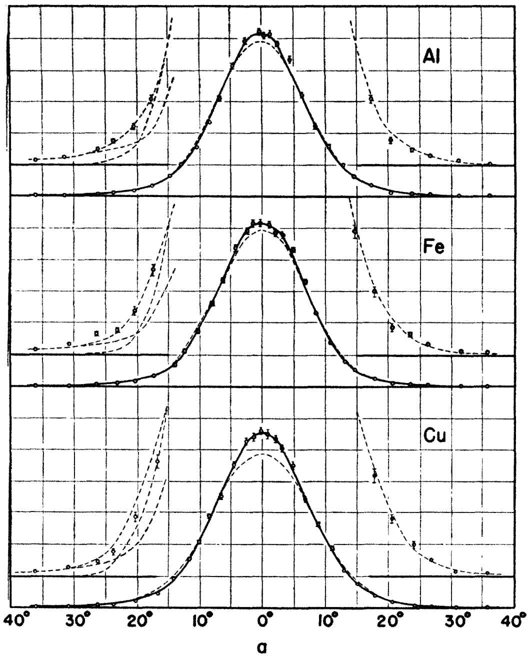
For the evaluation of the divergence of the electronic beam, the curves in Figs. 5 and 8a, which give the distribution of electrons in the horizontal plane, and Figs. 6 and 8b, which give the intensity distribution in the vertical plane, were normalized in pairs. These normalized pairs are given in Figs. 9 and 10.

By extrapolating the linear parts of the curves in Fig. 9 to the axis of the abscissae, we found that the breadth of the beam in the horizontal plane, at a distance of 151 mm from the last diaphragm, equalled 5.8 mm and 3.6 mm at a distance of 73 mm from the same diaphragm.

The breadth of the beam in the horizontal plane was 1.5 mm. The experimentally found shape of the beam in the horizontal plane is represented in Fig. 11a.

We can conclude, therefore, that the angle between the inclined path of the electrons and the horizontal axis of the beam does not exceed 0.8°. (The same angle was calculated by considering all the possible geometrical paths of electrons in the apparatus, limited by the given locations of the diaphragms, and was found to equal 0.9°.)

The height of the split of the last diaphragm in the vertical plane was 2.5 mm. From the data



of Fig. 10, the height of the cross section of the electronic beam equals 4.8 mm at a distance of 151 mm from the diaphragm and 3.6 mm from the same diaphragm.

Figure 11b gives the shape of the beam in the vertical plane.

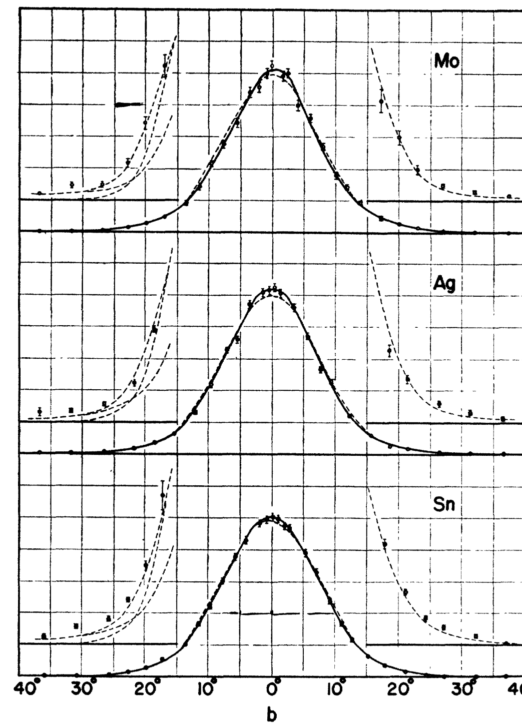
As previously, we can conclude that the divergence of the path of electrons from the horizontal axis in the vertical plane does not exceed 0.4° (the same angle calculated from consideration of all the possible paths limited by the given arrangement of diaphragms, was found equal to 0.32°).

From the distribution curve of the natural β-spectrum of RaC, given in Fig. 4, we can find the spectral composition of the beam impinging on the scattering foil. Indeed the breadth of the conversion maximum K 1321 kev, determined from Fig. 4, is equal to $\Delta H\rho = 5$ scale divisions of the deflection of the ballistic galvanometer, and is therefore equal to $5/251 \sim 2$ percent.

From this we can conclude that our electronic beam consists of electrons with an energy of 2250 ± 23 kev.

VI. THE EXPERIMENTAL RESULTS

We investigated the angular distribution of the scattered electrons with an energy of 2250



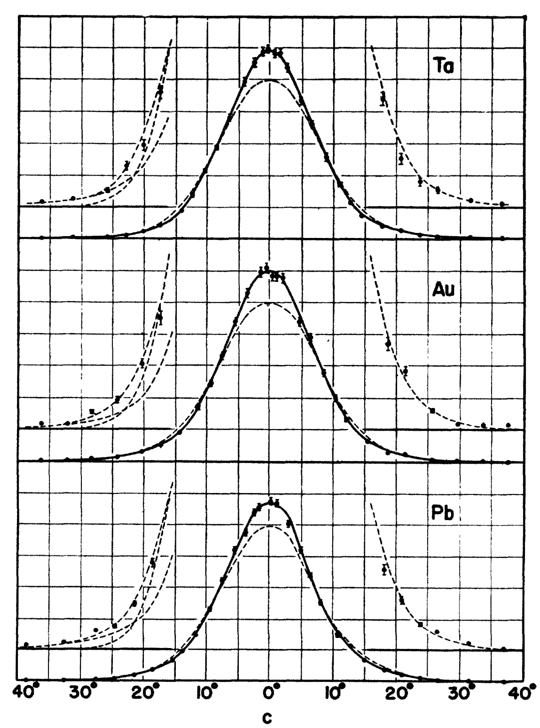


FIG. 12. a, b, c. Scattering of electrons in various elements. Experimental values are indicated by circles. The abscissae represent the angle of scattering, the ordinates the number of electrons scattered. The "tails" of the curves are magnified eight times in separate curves. Dotted curves represent theoretical values.

kev in the foils of Al, Fe, Cu, Mo, Ag, Sn, Ta, Au, and Pb. The thickness of the scattering foils (Table I) was chosen in such a way, that the half-width values of the Gauss curve would be equal for all the elements studied. The absolute value of the half-width was chosen to be equal to $\sim 10°$. The calculations show that at this value of the half-width, the theory of multiple scattering is applicable.

The experimental results are given in Fig. 12, the experimental values being designated by circles. The abscissae on these curves represent the angles of the scattering in degrees, the ordinates, the number of electrons. The experiments were carried on in a range of angles up to 35–40°. For angles larger than 40°, the number of electrons was too small.

The experimental points on the "tails" of the curves are given for clearness in the magnified scale ($\times 8$). The statistical fluctuations in the experimental values are represented in Fig. 12 by vertical dashes.

In our experiments the error in the determination of the half-width (the angle at which the intensity of the scattered electrons is diminished by "e" times) of the experimental curve does not exceed 3–4 percent.

VII. THE COMPARISON OF EXPERIMENTAL RESULTS WITH THE THEORY OF MULTIPLE SCATTERING

1. The Theoretical Formulae

The experimental results were compared with the theory of multiple scattering given by Williams.[3]

To date all the experiments carried out for the investigation of multiple scattering, were performed by the Wilson chamber method. In this method the projection of the path of the electron on the plane parallel to the bottom of the cloud chamber is usually measured. Because of this, all Williams' formulae are calculated for this case.

Williams' theory permits the calculation of the mean arithmetical value of the projection of the angle of deviation, and the angular distribution of the projections of the tracks of the scattered electrons.

According to Williams' theory of the multiple scattering in the foils, the angular distribution of the electrons scattered at small angles must be Gaussian. In the scattering at larger angles, the single scattering described by the Rutherford formula, predominates. In the intermediate region of the angles, these two distributions are superimposed.

The probability that the projection of the path of the electron lies between α and $\alpha+d\alpha$ is:

$$\rho(\alpha)d\alpha = g(\alpha)d\alpha + S(\alpha)d\alpha, \qquad (1)$$

where $g(\alpha)$ is Gaussian, and $S(\alpha)$ is Rutherford's distribution of the scattered electrons.

Williams introduces two angles φ_2 and α_{mS}. When the electrons are scattered at angles less than φ_2, their distribution is purely Gaussian. When the scattering angles equal α_{mS}, the multiple scattering equals Rutherford's scattering. For angles larger than α_{mS}, the Gauss curve for the multiple scattering swiftly diminishes.

The Gauss distribution is given by:

$$g(\alpha) = \text{Const.}\ \exp\left[-\frac{\alpha^2}{\pi\bar{\alpha}_m{}^2}\right], \qquad (2)$$

where $\bar{\alpha}_m$ is the mean arithmetical value of the projection of the angle of deviation, and equals

$$\bar{\alpha}_m = \left(\bar{\alpha} - \frac{\pi}{\varphi_2}\right) : \left(1 - \frac{\pi}{2\varphi_2{}^2}\right). \qquad (3)$$

Here α is the mean arithmetical value for the projection of the angle of the deviation from the general distribution $P(\alpha)$ (Eq. (1))

$$\bar{\alpha} = 0.80(\log_e M)^{\frac{1}{2}} + 1.45, \qquad (4)$$

where M is a value which characterizes the average number of collisions of electrons with the nuclei and equals

$$M = \frac{2\pi Z^{4/3}{}_{\text{eff}} N t \hbar^2}{1.75^2 m_0{}^2 \beta^2 c^2} = 1850 Z^{4/3}{}_{\text{eff}} \frac{\sigma}{A} \frac{1}{\beta^2}, \qquad (5)$$

in which A is the atomic weight of the substance, σ is the surface density in g/cm^2, and $Z_{\text{eff}} = (Z^2+Z)^{\frac{1}{2}}$ the value of the effective nuclear charge.

As a condition for the multiple scattering of the electrons we have $M \gg 1$.

The angle φ_2, which enters in (3) and the physical meaning of which was considered above, equals:

$$\varphi_2 = 5.1(\log_e M)^{\frac{1}{2}} - 4.0. \quad (6)$$

The Rutherford distribution $S(\alpha)$ in Williams' theory has a form:

$$\begin{aligned} S(\alpha) &= 0 \quad \text{for} \quad \alpha < \varphi_2 \\ S(\alpha) &= \pi/\alpha^3 \quad \text{for} \quad \alpha > \varphi_2. \end{aligned} \quad (7)$$

The angle α_{mS} at which the value of the Gaussian distribution equals Rutherford's, is

$$\alpha_{mS} = 5.1\bar{\alpha}_m - 4.00. \quad (8)$$

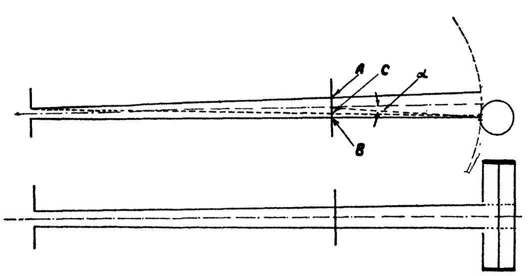


FIG. 13. Scheme of the electron beam investigation for calculating its finite breadth. AB—electron spot on the target. CB—section of the spot from which the electrons enter the slit of the counter, when turned at the angle α.

In all these formulae, the angles are in the units of δ. To convert into degrees, it is necessary to multiply them by δ.

$$\delta = 12.7\left(\frac{\sigma}{A}\right)^{\frac{1}{2}} \frac{Z_{\text{eff}}}{\beta^2 W}, \quad (9)$$

where W is the whole energy of the electron in Mev.

Contrary to the Wilson chamber experiments, where the experimentally found angular distribution is a projection of three-dimensional tracks on the bottom plane of the chamber, in our experiments a proper angular distribution of electrons in the space was investigated.

For the Gauss distribution, the half-widths of the distribution curve (the angle at which the intensity diminishes in "e" times) is the same for the distribution proper, and for its projection.

Because of this it was possible to compare theory with the values of the half-width of the experimentally found curves.

The half-width value for the Gauss curve equals

$$\theta = \pi^{\frac{1}{2}}\bar{\alpha}_m. \quad (10)$$

2. Correction for the Finite Dimensions of the Electronic Beam

All these formulae are valid if the beam impinging on the target is infinitesimally thin. In our experiments the beam was a definite thickness. Because of this a correction for the finite dimensions of the beam had to be made. For this, to the exit window of the counter a diaphragm was fastened with a narrow slit (0.3×4.5 mm). The intensity of the electronic beam was measured at different angular positions of the counter (which was changed by rotating the counter); the frame with the scattering target was raised. This investigation was done with the first counter only.

In this case a definite angular position of the counter is connected with a narrow region BC of the whole cross section of the beam AB impinging on the plate (see Fig. 13). The photograph of the spot left by the electronic beam is given in Fig. 7. The full height of the beam was caught by the counter.

It must be mentioned that the results of this experiment are given in Fig. 14. The abscissae are the angular positions of the counter, the ordinates, the number of electrons.

The whole area of this curve was divided into parts of 1° each, as is shown in Fig. 14. For each section a theoretical curve was constructed, normalized proportionally to the area of the section. The middle point of each section was shifted several degrees according to the position of the section in Fig. 14.

All the ordinates of this curve were added together. As a result a Gauss curve was found the half-width value of which was much larger than the previous one.

This correction for the finite dimensions of the beam did not exceed 3–4 percent in our experiments.

3. The Comparison of the Experimental Results with Williams' [3] Theory

The constructed theoretical curves were normalized with the experimental curves in Fig. 12, dashed lines represent the theoretical curves, and solid lines those found experimentally.

The values of half-width (i.e. the values of the angles at which the intensity of scattered

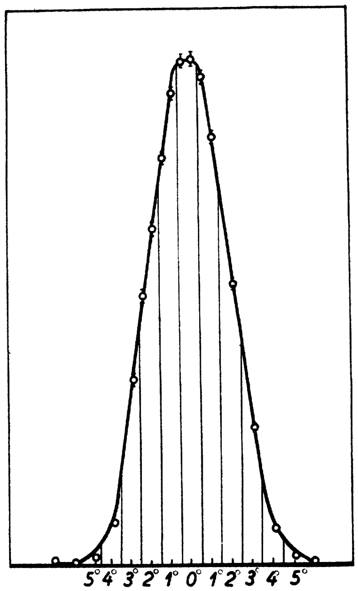


FIG. 14. Curve of the electron beam taken with the rotation of the counter round the target axis. On the axis of the abscissae the angle through which the counter turns are plotted; on the axis of the ordinates the number of electrons is plotted.

electrons is diminished by "e" times given in Table I columns IX and X) were found from the experimental and theoretical curves (Fig. 12).

The analysis of these results and the curves given in Fig. 12, show that the discrepancy between the experimental results and Williams' theory is not very great. The experimental value of the half-width is smaller than the theoretical value by only 3–5 percent.

As the atomic number of the scatterer is increased, this discrepancy also increases and for Ta, Au, and Pb reaches 10–13 percent.

The discrepancy has a systematical character which increases with the increase of the atomic number. This can be seen particularly clearly in Fig. 15, where the dependence of $\theta(\sigma/A)^{-\frac{1}{2}}$ on Z is given. Such graphical representation is possible in our case, because the thickness of the scattering foils was chosen so that the half-width of the scattering θ varies in all cases only between small limits (from 9.8 to 11.4 degrees) and the value "M" under the logarithm in Eq. (4) changes only slightly. The dashed curve in Fig. 15 gives the theoretical dependence of $\theta_B(\sigma/A)^{-\frac{1}{2}}$ from Z, and the circles and the solid curve give the experimentally found dependence.

As can be seen from these curves, there is a systematic deviation of the experimental values from the theoretical values, which lies outside possible experimental error. For light elements this deviation is of the order 4 percent, for the heavier elements it reaches 10–13 percent.

4. The Comparison with the Goudsmit and Saunderson[8] Theory

We compared our results with the theory of multiple scattering given by Goudsmit and Saunderson. As can be seen from columns IV and VII of Table II, our results are in perfect agreement with this theory for elements Al, Fe, Cu, Mo, Ag, and Sn. (The deviation does not exceed 1 percent; see Fig. 15.) For heavy elements the experimentally found scattering is smaller than that theoretically calculated by 10–13 percent. This discrepancy lies outside possible experimental errors.

5. Comparison with the Bethe, Rose, and Smith[5] Theory

In column VI of Table II are given the theoretical values of Gauss half-widths, calcu-

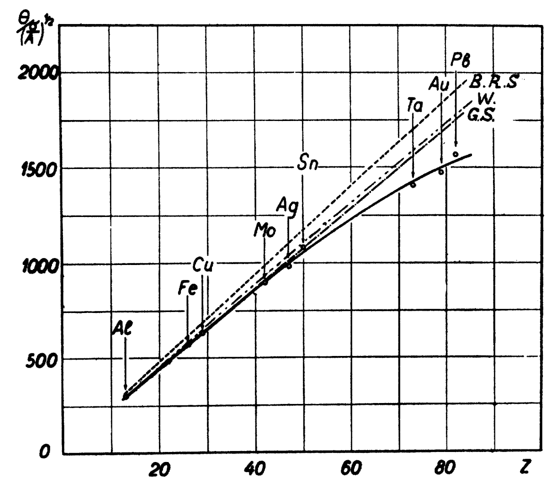


FIG. 15. On the axis of abscissae the atomic number Z is plotted; on the axis of the ordinates the values $\theta(\sigma/A)^{-\frac{1}{2}}$ are plotted. By the circles and solid line curve the experimental results are depicted. The dash-dotted curve is calculated according to the Goudsmit and Saunderson theory. The dot-dot-dashed curve is calculated according to Williams' theory. The dotted curve is calculated according to the theory of Bethe-Rose-Smith.

[8] S. Goudsmit and J. L. Saunderson, Phys. Rev. **58**, 37 (1940); **57**, 24 (1940).

TABLE II. Scattering of electrons whose energy is 2.25 Mev. The angles are given in degrees. θ_{exp} is the half-width value of the experimental Gauss curve. $\theta_{G.S.}$ is the half-width value according to the theory of Goudsmit-Saunderson. θ_w is the half-width of the Gauss curve according to Williams' theory. $\theta_{B.R.S.}$ is the half-width of the Gauss curve according to the theory of Bethe, Rose, and Smith.

Element I	Z II	θ_{exp} III	$\theta_{G.S.}$ IV	θ_w V	$\theta_{B.R.S.}$ VI	% deviation of θ_{exp} from $\theta_{G.S.}$ VII	% deviation of θ_{exp} from θ_w VIII	% deviation of θ_{exp} from $\theta_{B.R.S.}$ IX
Al	13	9.50	9.40	9.85	9.95	+1.1	−3.7	−4.5
Fe	26	9.60	9.60	9.90	10.30	0.0	−3.0	−7.2
Cu	29	10.40	10.50	11.05	11.35	−0.9	−3.0	−8.4
Mo	42	10.25	10.35	10.75	11.25	−1.0	−4.6	−8.9
Ag	47	10.20	10.30	10.80	11.45	−1.0	−3.7	−10.9
Sn	50	10.65	10.65	10.90	11.55	0.0	−2.3	−7.8
Ta	73	9.85	10.95	11.00	11.85	−10.1	−10.5	−16.9
Au	79	9.90	11.35	11.40	12.25	−12.8	−13.1	−19.0
Pb	82	9.70	10.85	10.85	11.80	−10.6	−10.6	−17.8

lated according to the theory of Bethe, Rose, and Smith; in column III the experimental values. In column IX of the same table are given the percent deviations of the theoretical values from the experimental values. It can be clearly seen that the experimental values are always smaller than those theoretically calculated (see Fig. 15).

For aluminum, the half-width of the experimental curve of the scattering is 4.5 percent smaller than the theoretical value; as the atomic number increases, the deviation smoothly increases and for lead reaches 18 percent.

VIII. CONCLUSION

1. The apparatus and experimental method we developed was found exceedingly convenient for studying the multiple scattering of electrons in foils.

2. By this method the multiple scattering of electrons with an energy of 2.25 Mev, was investigated in Al, Fe, Cu, Mo, Ag, Sn, Ta, Au, and Pb foils.

3. From the experimental curves for the scattering, the half-width values of the Gauss curve were found with an accuracy of 3–4 percent.

4. The comparison of the experimental values of the half-width, with theory, showed that for elements from Al ($Z=13$) to Sn ($Z=50$) there is complete agreement with the Goudsmit-Saunderson theory, and the deviation does not exceed 1 percent; comparison with Williams' theory for the same elements shows that the experimental values are 3–4 percent smaller than in theory.

5. For heavy elements Ta ($Z=73$), Au ($Z=79$), Pb ($Z=82$) the experimental values for the half-width are smaller by 10–18 percent than those calculated by Williams and from the Goudsmit-Saunderson theory. This deviation is beyond all possible experimental errors.

6. The comparison of experimental results with the theory of Bethe, Rose, and Smith, shows that the experimentally found values for the half-width of the scattering curves are smaller than those calculated for all investigated elements. For aluminum this deviation is 4.5 percent, as the atomic number increases this deviation also increases and reaches 18 percent for lead.

In conclusion we would like to express our thanks to Dr. A. I. Alikhanov for consultation and to Dr. L. A. Artsimovitch for the discussion of results.

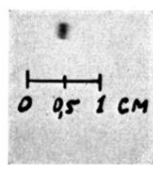


FIG. 7. Photogram of the electronic beam spot in the plane of the target.