The Angular Distribution of Positron Annihilation Radiation

ROBERT BERINGER* AND C. G. MONTGOMERY
Sloane Physics Laboratory, Yale University, New Haven, Connecticut

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The angular distribution of the two quanta which are emitted in the annihilation of a positron and an electron was observed with coincidence counters of high resolving powers both in time and angle. The direct observations are consistent with the hypothesis that the two quanta are emitted in exactly opposite directions to within at least one degree. A comparison of the counter efficiency obtained from this angular distribution curve with a direct and independent measurement of the counter efficiency for annihilation radiation shows that the half-width of a possible distribution of deviations from strict collinearity is zero to within 15 minutes of angle. It may be concluded that the momentum balance is accurately maintained by the quanta alone, and that, in general, a positron stops before it is annihilated in accord with the theoretical estimates.

INTRODUCTION

BEFORE the discovery of the positron, Dirac[1] predicted the existence of electron states of negative total energy. For free electrons, momentum considerations demanded that when such states were filled by transition from states of positive energy, two gamma-ray quanta must be simultaneously emitted. Later calculations[2] of the probability of this process of positron annihilation gave the result that positrons passing through matter would, in general, stop before being annihilated and the two quanta, each having an energy of mc^2, would be emitted in directly opposite directions. These two simultaneous quanta were first detected by Klemperer.[3] He measured coincidences between two Geiger counters placed close to the source of annihilation radiation so that the solid angle subtended by one counter at the source was nearly 2π. Better angular resolution was obtained by Alichanian, Alichanow, and Arzimovitch,[4] in whose experiments the solid angle subtended by one counter was about 0.7 steradian. From a knowledge of the angular relation between the two quanta, we can determine whether or not the positron is annihilated while in motion or whether a third body, such as a nucleus, is involved in the process. The experiments reported here were undertaken to determine this angular relation more precisely by using counters subtending a solid angle of only 0.015 steradian.

THE COINCIDENCE EXPERIMENTS

Two counters, each with an electrostatic shield and an amplifying tube, were mounted upon arms which permitted rotation about an axis parallel to the counter wires. The counters[5] had cathodes three centimeters long of seamless copper tubing 1.05 cm in diameter, sealed in a glass envelope. They were filled with a mixture of 94 percent argon plus 6 percent oxygen, to a pressure of 9 cm of mercury. A source of annihilation radiation was placed at the axis of rotation. Coincidences from the simultaneously emitted gamma-rays were recorded with a Rossi type adding and a hard tube recording circuit. The arrangement is shown schematically in Fig. 1. The coincidence circuit had a resolving time

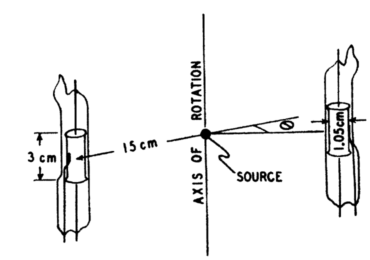

FIG. 1. Schematic arrangement of counters for observing coincidences from annihilation radiation.

* Junior Sterling Fellow, 1941–1942.
[1] P. A. M. Dirac, Proc. Roy. Soc. **A126**, 360 (1930).
[2] P. A. M. Dirac, Proc. Camb. Phil. Soc. **26**, 361 (1930); H. A. Bethe, Proc. Roy. Soc. **A150**, 129 (1935).
[3] O. Klemperer, Proc. Camb. Phil. Soc. **30**, 347 (1934).
[4] A. I. Alichanian, A. I. Alichanow, and L. A. Arzimovitch, Nature **137**, 703 (1936), and C. R. (Doklady) **1**, 287 (1936).
[5] C. G. Montgomery and D. D. Montgomery, J. Frank. Inst. **231**, 447–467, 509–545 (1941).

of 3×10^{-6} second as determined by the accidental counts registered with the counters separated by several meters, each near a radium source. A vacuum-tube pulse generator[6] was also used for this determination and gave results in substantial agreement.

Foils of Cu^{64} activated by about 10-μa hours bombardment in the 3.7-Mev deuteron beam of the Yale cyclotron were used as positron sources. They were pressed into small pellets and covered with 0.32 g/cm² of Pb, sufficient to stop the positrons. With such a source of annihilation radiation at the point of intersection of the axis and the perpendicular plane bisecting the counter cathodes, the ratios of coincidence to single counting rates were determined for various angular deviations of one counter from the line through the center of the source and the second counter. These deviations were measured with an optical lever. In all, ten sources were used from which about 800 coincidence counts were recorded. Corrections were applied to the data for all spurious coincidences. In no case was the measured natural coincidence background owing to cosmic rays and accidental counts from other sources higher than 5 percent of the observed annihilation coincidence rate, while that caused by accidental counts from the annihilation quanta themselves as calculated from the resolving time of the circuit did not exceed 10 percent.

Figure 2 shows these observations. The vertical lines attached to the points represent the probable errors of the ratios as computed from the number of counts involved in each determination. It is apparent from this figure without further consideration that the two annihilation quanta are emitted in very nearly opposite directions. We shall now proceed to discuss this more precisely.

THE COUNTER EFFICIENCY

It is evident that if the annihilation quanta are all emitted in exactly opposite directions, the ratio of the coincidence rate to the counting rate of a single counter at $\theta=0$ is equal to the efficiency of the counter for gamma-radiation of this energy, since if n quanta per second are passing through each counter, the single counting rate is ϵn, where ϵ is the efficiency, and the coincidence rate is $\epsilon^2 n$. Thus a determination of counter efficiency by an independent experiment will allow us to draw more precise conclusions. Knowing the number of positrons emitted per sec. from a Cu^{64} source, we can easily determine the efficiency for the annihilation radiation by observation of the single counting rate when the source has been wrapped with enough material to stop all positrons. One assumes that all

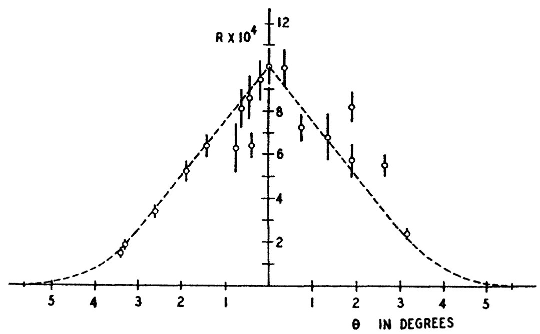

FIG. 2. Observations of coincidences from annihilation radiation. The ordinates are the ratios of the coincidence counting rates to the counting rates of a single counter at angles given by the abscissae. The broken lines represent what is expected if the two quanta are emitted in exactly opposite directions.

positrons are annihilated with the emission of two quanta, which is very nearly the case[2] for Cu^{64} positrons; and that the whole counter cathode area is sensitive.

An activated Cu foil was fastened to the central electrode of a small ionization chamber which could be evacuated. The current between the source and the thick brass shell of the chamber was measured with a vacuum-tube electrometer. By taking the ratio of the number of electrons to positrons from Cu^{64} as 1.6,[7] the number of positrons emitted per sec. may be calculated when correction has been made for those positrons which do not escape from the Cu foil but nevertheless give rise to annihilation quanta. This correction was made by graphical integration of the electron and positron spectra given by Townsend.[8] The mean value of three determinations of the efficiency was $(10\pm1)\times10^{-4}$. The probable error has been estimated in

[6] C. G. Montgomery, W. E. Ramsey, D. B. Cowie, and D. D. Montgomery, Phys. Rev. **56**, 635 (1939).

[7] S. N. Van Voorhis, Phys. Rev. **50**, 895 (1936); C. V. Strain, Phys. Rev. **54**, 1021 (1938).

[8] A. A. Townsend, Proc. Roy. Soc. **A177**, 357 (1941).

TABLE I. Calculated value of the maximum counting rate ratio as a function of the half-width of $D(\varphi)$.

Half-width, $1/b$	0	1	10′	20′	1°
R at $\theta=0$	ϵ	0.98ϵ	0.89ϵ	0.81ϵ	0.61ϵ

such a way as to allow for uncertainties in the positron-electron ratio and in the energy spectra of the particles. This value is in good agreement with other measurements of the efficiency of similar counters for 0.5-Mev gamma-rays.[9]

DISCUSSION

We are now in a position to evaluate more precisely the results of the coincidence experiment. If the two quanta are emitted exactly collinearly, the cone described by one counter at a point source limits the region in which the second counter may register coincidences between collinear quanta. For this case, the variation of the counting ratio R with the angle θ measured from the second counter to the in-line position of source and counters is given by $R=\epsilon(1-\theta/2\alpha)$, where ϵ is the counter efficiency for the quanta and 2α the cone angle measured about the axis. Plotting the counting ratio *vs.* θ, one gets two straight lines having a common intercept on the R axis at the efficiency and cutting the θ axis at $\pm\alpha$, which for counters of 1.05 cm sensitive width 15 cm from the axis is 4.0°.

For the sources used, which were roughly spherical and 4 mm in diameter, the expected variation of counting ratio with θ for collinear quanta is given by a much more complicated expression but is not markedly different from that of the ideal point source. The broken line in Fig. 2 is calculated for a plane source of 4-mm extension with a uniform surface density of isotropic collinear quantum pair emitters. The behavior of this function is almost that for the point source model for values of θ where the whole source contributes coincidences. The tails start beyond $\theta=2.5°$ where this condition is no longer satisfied. Furthermore, the ordinate at $\theta=0$ is 0.996ϵ as compared with ϵ for the point source case. The agreement with the observations is excellent and we may conclude that within limits of, say, one degree, the two quanta are emitted in exactly opposite directions.

We may determine these limits more precisely by a simple consideration. Let us suppose that directions of emission of the two quanta make an angle $(\pi-\varphi)$, the frequency of occurrence of a given φ being represented by some distribution function such as

$$D(\varphi)=\frac{b/\pi}{1+b^2\varphi^2}.$$

This function has a maximum at $\varphi=0$, falls to half this maximum value at $\varphi=\pm 1/b$, and is normalized to have a unit area. The calculated value of the counting rate ratio R at $\theta=0$ is a very sensitive function of the half-width $1/b$. Table I shows several of the calculated values.

If we say that the maximum value of R from Fig. 2 is $(10\pm1)\times10^{-4}$, and use the measured value of the counter efficiency we get $R/\epsilon=1.0\pm0.14$ and $1/b=0\pm15'$. The half-width is, of course, essentially positive, and we may conclude from these arguments that the two quanta are on the average collinear to within 15 minutes of angle.

Any lack of collinearity in the two quanta must be explained either as a scattering of the gamma-rays after their emission or must represent some momentum other than that of the quanta. This monentum may be either the momentum of the positron or the electron before annihilation or the momentum gained by a third body, an atomic nucleus for example, during the annihilation process. The experiments described above show, however, that this momentum is extremely small. If the deviation from collinearity is as much as 1°, the corresponding momentum is only that of a 9-kev electron, a deviation of 15′ corresponds to the momentum of an electron of less than 1-kev energy.

[9] J. V. Dunworth, Rev. Sci. Inst. **11**, 167 (1940); F. Norling, Phys. Rev. **58**, 277 (1940); N. Feather and J. V. Dunworth, Proc. Roy. Soc. **A168**, 566 (1938).