The Intensities of the Hard and Soft Components of Cosmic Rays as Functions of Altitude and Zenith Angle

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The intensities of the hard and of the soft component of cosmic rays have been recorded by a coincidence method as a function of zenith angle, at the four elevations 259, 1616, 3240, and 4300 meters, and the errors which may occur in the measurements have been studied. The absolute directional intensities have been calculated, as well as the integrated intensities at the various elevations. A comparison has been made between the variation of the hard component and the variation of the soft component with depth. The nature of the variation of the hard component with zenith angle and altitude suggests that there may be appreciable production of mesotrons at low altitude.

INTRODUCTION

In view of the discussions and hypotheses which have been presented recently concerning the nature and origin of the soft and hard components of cosmic rays,[1–9] it seems to be of importance to determine unambiguously the intensities of these components as functions of altitude and zenith angle. During the last summer, therefore, measurements were carried out at Ithaca, New York (elevation 259 m), and at Denver, Echo Lake, and Mt. Evans, Colorado (elevations 1616, 3240, and 4300 m, respectively), with a counter telescope directed at various zenith angles, in an effort to obtain both the directional and total intensities at each elevation.

The directional intensities were calculated on the assumption that all of the multiple coincidences obtained were due to the passage of single particles through the counters, although an effort was also made to ascertain the errors which are involved in such an assumption. No attempt was made to distinguish sharply between mesotrons and electrons, but rather the distinction was between particles capable of penetrating only the counter walls and those capable of penetrating also a certain thickness of lead.

DESCRIPTION OF THE APPARATUS

The apparatus used consisted of six identical Geiger-Müller counters, supported in a straight line by a light wooden frame (see Fig. 1) which could be tipped at any desired zenith angle. The separation between the axes of the extreme counters was 36.4 cm. The counters were made of brass, and were of the self-quenching type, containing 9 percent alcohol and 91 percent argon with a total pressure of 11 cm. The internal diameter was 4.24 cm, the length of the collecting wire was 20 cm, and the thickness of the counter walls was 1 mm. Between the counters were light, removable wooden shelves on which 2 cm of lead could be placed, while above the top counter was a similar shelf on which 3 cm of lead could be placed. The lead absorber was cut so that it had the same width as the counters, in order to reduce the coincidences due to scattering of particles into the beam.

The apparatus was housed in a station wagon, the roof of which was composed mostly of wooden slats approximately $\frac{1}{4}$ inch thick. Care was taken that no dense part of the station wagon or of nearby buildings or mountains should be in the solid angle of the counter telescope at any of the zenith angles used. The four zenith angles at which measurements were made were 0°, 29°, 46°, and 56° to the south, the axes of the counters being always in the

[1] W. Heisenberg and H. Euler, Naturwiss. **17**, 1 (1938).
[2] B. Rossi, Phys. Rev. **57**, 469 (1940).
[3] E. Nelson, Phys. Rev. **58**, 771 (1940).
[4] G. Bernardini, B. N. Cacciapuoti, B. Ferretti, O. Piccioni, and G. C. Wick, Phys. Rev. **58**, 1017 (1940).
[5] L. W. Nordheim, Phys. Rev. **59**, 554 (1941).
[6] M. Schein, W. P. Jesse, and E. O. Wollan, Phys. Rev. **59**, 614 (1941).
[7] J. F. Carlson and M. Schein, Phys. Rev. **59**, 840 (1941).
[8] G. Cocconi, Phys. Rev. **60**, 532 (1941).
[9] B. Rossi and K. Greisen, Phys. Rev. **60**, 121 (1942).

east-west direction. The starting potentials of the counters were checked daily and all counters were operated at exactly the same voltage above the starting potential.

The recording circuit contained a coincidence-anticoincidence arrangement which was not of an unusual type and will not be described.

PRESENTATION OF DATA

With the apparatus described above, the following measurements were made:

(a) Sixfold coincidences with all lead absorber in place. The rate at which such coincidences occur will be denoted by N. The total absorber thickness in this condition, including both the lead, the counter walls, and the wood between and above the counters, was equivalent to 167 g/cm² of lead. Since there are very few

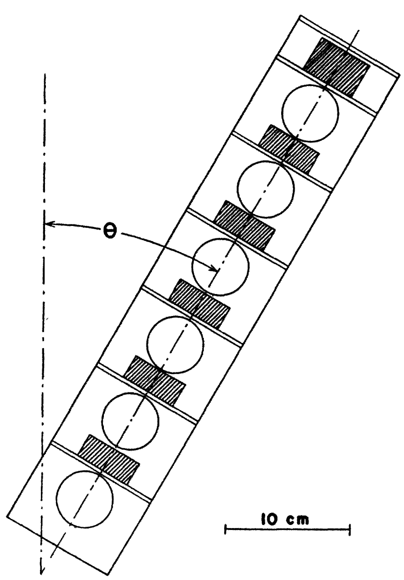

FIG. 1.

electrons capable of penetrating this thickness of lead, N is proportional to the number of mesotrons with momentum above 3.0×10^8 ev/c which arrive in the solid angle of the counter telescope.[10]

TABLE I. Intensities of soft and hard components as functions of altitude and zenith angle.

Altitude (meters)	Depth in g/cm²	Zenith angle 0°	29°	46°	56°
259	1007	$N+n=2.86\pm0.021$	2.10 ± 0.026	1.24 ± 0.020	0.73 ± 0.015
		$N=2.27\pm0.021$	1.67 ± 0.024	1.08 ± 0.020	0.65 ± 0.015
		$n=0.59\pm0.030$	0.43 ± 0.035	0.16 ± 0.028	0.08 ± 0.021
		$n'=0.55\pm0.034$		0.42 ± 0.038	
		$n+n'=1.14\pm0.045$		0.58 ± 0.047	
1616	857	$N+n=3.94\pm0.024$	2.89 ± 0.066	1.74 ± 0.044	1.04 ± 0.032
		$N=2.85\pm0.067$	2.25 ± 0.056	1.33 ± 0.036	0.85 ± 0.027
		$n=1.09\pm0.071$	0.64 ± 0.087	0.41 ± 0.057	0.19 ± 0.042
		$n'=1.02\pm0.12$		0.53 ± 0.10	
		$n+n'=2.12\pm0.14$		0.94 ± 0.12	
3240	708	$N+n=6.53\pm0.075$	4.56 ± 0.057	2.56 ± 0.049	1.45 ± 0.030
		$N=4.13\pm0.062$	2.92 ± 0.048	1.97 ± 0.043	1.18 ± 0.028
		$n=2.40\pm0.10$	1.64 ± 0.075	0.59 ± 0.065	0.27 ± 0.041
		$n'=2.57\pm0.17$		1.02 ± 0.12	
		$n+n'=4.97\pm0.19$		1.62 ± 0.13	
4300	616	$N+n=9.18\pm0.093$	6.64 ± 0.079	3.79 ± 0.059	2.14 ± 0.044
		$N=5.19\pm0.068$	4.15 ± 0.061	2.54 ± 0.049	1.56 ± 0.038
		$n=3.98\pm0.11$	2.49 ± 0.10	1.25 ± 0.077	0.58 ± 0.058
		$n'=5.41\pm0.25$		2.18 ± 0.14	
		$n+n'=9.39\pm0.27$		3.43 ± 0.16	

(b) Sixfold coincidences with the lead absorber removed; this rate will be denoted by $N+n$. The difference n between the coincidence rates with and without the lead absorber is proportional to the number of particles with sufficient energy to penetrate the counter walls and the wooden shelves (12 g/cm² brass plus 1.7 g/cm² wood) but with insufficient energy to penetrate 167 g/cm² of lead. This number includes electrons with energy above approximately 2×10^7 ev, and also those mesotrons which are stopped by 167 g/cm² of lead ("slow" mesotrons).

(c) Twofold coincidences between the extreme counters, with the same absorber as in the measurement of $N+n$.

(d) Twofold coincidences between the extreme counters, with the inactive counters and the shelves removed. In both of the measurements (c) and (d) the number of showers and chance coincidences recorded is the same. Therefore the difference between the counting rates (d) and (c) represents the number of particles capable of penetrating two counter walls (2.3 g/cm² of brass) but not capable of penetrating

[10] Because of the finite dimensions of the counters, not all particles traverse the counters perpendicularly to the axes of the counters; therefore the effective absorber thickness is larger than 167 g/cm². By a treatment very similar to that used below in calculating the directional intensities (page 216), one can show that the average absorber thickness is given by

$$\bar{x}=x_0\frac{\int_0^{\tan^{-1}a/l}(a-l\tan\varphi)\cos^3\varphi d\varphi}{\int_0^{\tan^{-1}a/l}(a-l\tan\varphi)\cos^4\varphi d\varphi},$$

where x_0 is the thickness perpendicular to the counter axes, a is the effective length of the counters, and l is the separation between the axes of the extreme counters. For $a=20$ cm, $l=36.4$ cm, we find that $\bar{x}=1.023x_0$. This correction is therefore negligible.

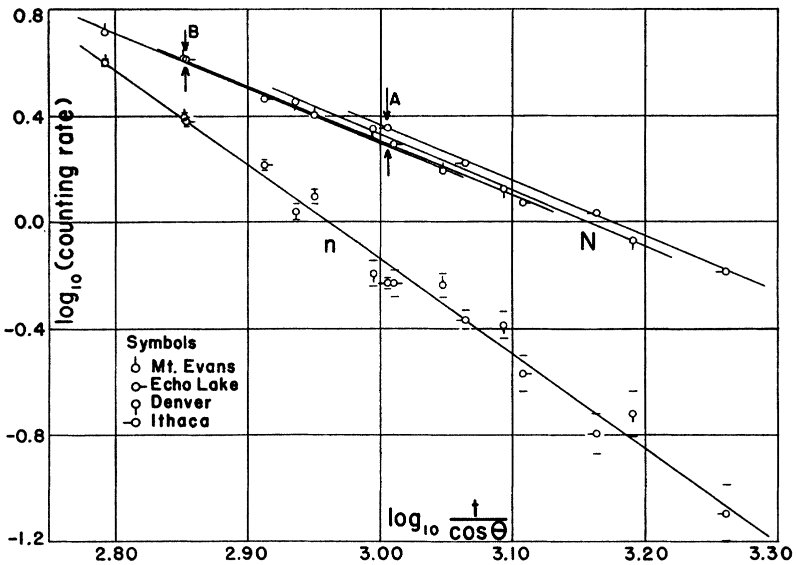

FIG. 2.

12 g/cm² of brass plus 1.7 g/cm² of wood. This number will be denoted by n', and may be considered as the intensity of "slow" electrons (energy roughly between 4×10^6 ev and 2×10^7 ev). We find at all elevations that n' is of approximately the same magnitude as n. The zenith angle variation of n' is not as significant as the zenith angle dependence of N or n, because of the large amount of scattering which the slow electrons undergo in the air.

In Table I we present, corresponding to the various altitudes and zenith angles at which they were measured, the rates $N+n$, N, and n', and the deduced values of n and $n+n'$, all expressed in coincidences per minute. The errors listed are the statistical (root mean square) errors. The column labeled "depth" indicates the atmospheric depth in g/cm², as calculated from the average barometric pressures.

The values for the intensities of the hard component in the vertical direction may be compared with similar measurements of Rossi, Hilberry, and Hoag,[11] with those of Rossi and Hall,[12] and with some recent measurements of Rossi and collaborators (unpublished). In the first of these experiments, triple coincidences were counted, while in the second and third the rates given are quadruple coincidence rates. The absorber thicknesses were 160, 196, and 196 g/cm² of lead, respectively. In all three experiments the number of side showers recorded was reduced by shielding the counters on the side with 10 or 11 cm of lead. In Table II we present the ratios of the intensities of the hard component at the other elevations to the intensity at Denver, as obtained in the present experiment and in the three experiments quoted.

The only values in this table which show a disagreement appreciably greater than the statistical errors are the ratios at Mt. Evans, and the discrepancy is in such a direction as to indicate that in the present experiment a few percent of the coincidences at Mt. Evans were due to air showers, which were not recorded in the experiment of Rossi, Hilberry and Hoag because of the shielding of the counters with lead.

TABLE II. Ratio of intensities of the hard component in the vertical direction to the intensity at Denver.

Location and elevation	Rossi, Hilberry, and Hoag	Rossi and Hall	Rossi and collaborators	Greisen
Mt. Evans 4300 meters	1.72 ±0.014			1.82 ±0.046
Echo Lake 3240	1.41 ±0.011	1.38±0.014	1.42±0.010	1.45 ±0.040
Denver 1616	1.00 ±0.00	1.00±0.00	1.00±0.00	1.00 ±0.00
Chicago 180	0.766±0.007			
Ithaca 259				0.795±0.021

[11] B. Rossi, N. Hilberry, and J. B. Hoag, Phys. Rev. **57**, 461 (1940).
[12] B. Rossi and D. B. Hall, Phys. Rev. **59**, 223 (1941).

CALCULATION OF THE DIRECTIONAL INTENSITIES

In Fig. 2 we have plotted the values of log N and of log n against the corresponding values of log $(t/\cos\theta)$; i.e., against the logarithm of the distance in g/cm^2 to the top of the atmosphere. The statistical errors are indicated where possible, but for the values of N they are too small to appear outside the dots. The "semaphor" signs indicate the elevations at which the measurements were made. The "soft" electrons n' have not been included in the figure because the variation of n' with zenith angle is strongly affected by scattering of the low energy particles in air.

The graph will be discussed in greater detail below, but here we wish to stress the nature of the variation of the coincidence rates with zenith angle. It appears that at any elevation the intensity of the hard component and also (within the rather large errors) that of the soft component can be expressed by a power of $\cos\theta$,[13] and that the exponent does not change appreciably with elevation in the range from 259 m to 4300 m. It will be shown that under a single, mild, simplifying assumption, this experimental fact leads to the conclusion that the *directional intensities are strictly proportional to the counting rates*, although the proportionality constants may be slightly different for the soft and hard components.

Consider the counter telescope tipped so that the axes of the counters are horizontal and the line joining the centers of the counters (the "principal axis") makes an angle θ_0 with the vertical direction, as indicated in Fig. 3, where only the extreme counters are shown. Let θ be the angle between the vertical direction and any line passing through the axes of the counters, and φ the angle between this line and the principal axis. We wish to calculate the relation between the coincidence rate $N(\theta_0)$ and the cosmic-ray intensity along the principal axis, $I(\theta_0)$.

The single simplifying assumption is that over the small range of angle subtended by the diameters of the counters, $I(\theta)$ varies linearly with θ, so that integration over the diameter becomes unnecessary, and one may use as the average intensity over the diameter the intensity along a line passing through the axes of the counters.[14] Over the range of θ included by the variation of φ, the accurate expression

$$I(\theta)=I(\theta_0)(\cos^n\theta/\cos^n\theta_0)=I(\theta_0)\cos^n\varphi$$

will be used, where n is the exponent indicated by the slope of the lines in Fig. 2. The relation between $N(\theta_0)$ and $I(\theta_0)$ is (see Fig. 3):

$$N(\theta_0)=\int\int I(\theta)f(\varphi,x)d\varphi dx$$
$$=I(\theta_0)\int\int\cos^n\varphi f(\varphi,x)d\varphi dx,$$

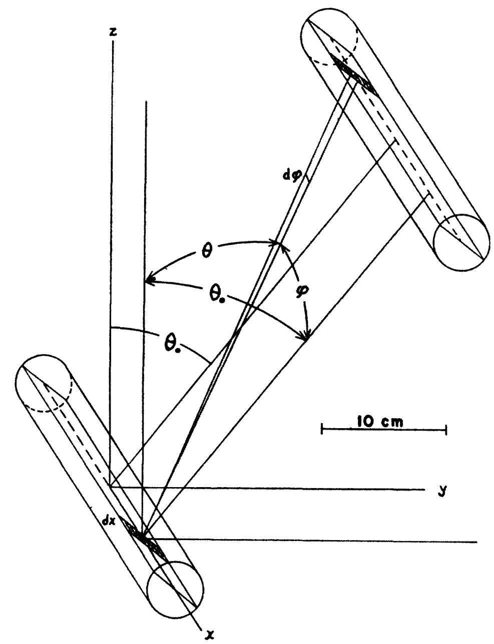

FIG. 3.

[13] The close resemblance of the zenith angle dependence of the total intensity to a $\cos^2\theta$ law has been pointed out by Johnson [T. H. Johnson, Phys. Rev. **43**, 307 (1933)], by Skobelzyn [D. Skobelzyn, Comptes rendus **194**, 118 (1932)], and by others.

[14] The error thus introduced can easily be evaluated and shown to be small. If $I(\theta)$ varies as $\cos^2\theta$, the ratio between the correct average intensity over the diameter, and the assumed average intensity, is given by

$$1+\frac{1}{6}\left(\frac{b}{l}\right)^2(\tan^2\theta_0-1)$$

where b is the diameter of a counter, and l is the separation between extreme counters. Using the dimensions of our counter array, we find the above ratio is less than 1 by 0.23 percent at $\theta_0=0°$, increases to exactly 1 at $\theta_0=45°$, and is greater than 1 by 0.24 percent at $\theta_0=55°$. Thus over the range of angle we used, the error is not greater than $\frac{1}{4}$ percent. The percent error increases rapidly as one approaches 90°, and is 3 percent for $\theta_0=75°$; however, for such large zenith angles the percent errors involved in the measurements are so large that one is still justified in using the assumption.

where f is a function of the angle φ and of the dimensions of the apparatus, and the limits of integration are functions only of the dimensions of the apparatus. Therefore

$$N(\theta_0)=\text{const.}\times I(\theta_0),$$

where the constant depends only on the dimensions of the counter array and on the exponent n.[15]

Thus the graph in Fig. 2 may be used as a logarithmic graph of *intensity* of the hard and soft components against atmospheric depth in the direction of motion. The logarithms of the absolute intensities differ only by a constant from the logarithms of the counting rates, although the constant may have slightly different values for the hard and soft components.

In order to determine the proportionality constant, let us first consider the counters as *rectangular* sensitive areas, of length a and width b. b is equal to the inside diameter of the counters,[16] and a is equal to the length of the fine collecting wire (this is true for counters such as ours, in which contact is made by a thick rod to a fine collecting wire, and the outer cylinder extends several cm beyond the wire at both ends). If l represents the separation of the axes of the extreme counters, we then obtain (see Fig. 3):

$$N(\theta_0)=\int\int\frac{b^2}{l}I(\theta)\cos^2\varphi d\varphi dx$$

$$=I(\theta_0)\cdot\frac{2b^2}{l}\int_0^{\tan^{-1}a/l}(a-l\tan\varphi)\cos^{n+2}\varphi d\varphi.$$

TABLE III. Absolute intensity in the vertical direction $I(0)$; the number of particles per cm² per minute per unit solid angle.

Alt. (M)	Depth (g/cm²)	N	n	$n+n'$
259	1007	0.443±0.004	0.115±0.006	0.222±0.009
1616	857	0.557±0.013	0.213±0.014	0.41 ±0.03
3240	708	0.806±0.012	0.47 ±0.020	0.97 ±0.04
4300	616	1.014±0.014	0.78 ±0.022	1.83 ±0.05

[15] Similar calculations have been carried out by Johnson[13] and by Froman and Stearns [D. K. Froman and J. C. Stearns, Can. J. Research **A16**, 29 (1938)], under slightly different assumptions, so that the proportionality between $N(\theta_0)$ and $I(\theta_0)$ was not brought to light.

[16] J. C. Street and R. H. Woodward, Phys. Rev. **46**, 1029 (1934).

For the case $n=2(I(\theta)\propto\cos^2\theta)$, this becomes

$$\frac{N(\theta_0)}{I(\theta_0)}=\frac{b^2}{4}\left(3\frac{a}{l}\tan^{-1}\frac{a}{l}+\frac{a^2}{l^2+a^2}\right)=4.77$$

and for the case $n=3(I(\theta)\propto\cos^3\theta)$, we obtain

$$\frac{N(\theta_0)}{I(\theta_0)}=\frac{2b^2}{5}\left\{\frac{1}{l(a^2+l^2)^{\frac{1}{2}}}\times\left(l^2+3a^2-\frac{1}{3}\frac{a^4}{l^2+a^2}\right)-1\right\}=4.67.$$

The numbers 4.77 and 4.67 are calculated from

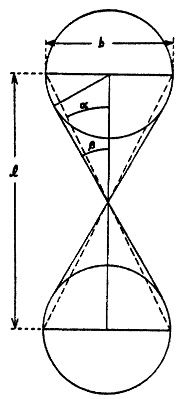

FIG. 4.

the following dimensions of our counter array: $a=20$ cm, $l=36.4$ cm, and $b=4.24$ cm.

In treating the counters as rectangular sensitive areas, we underestimate the angle subtended by the counters, both along the width and along the length of the counters, if we assume the counters are really sensitive *cylinders*. The error along the length of the counter arises because of the solid angle subtended by the circular ends of the cylinders, half of which angle extends beyond the solid angle subtended by the rectangular areas. If one regards the intensity as constant over the small additional solid angle, one obtains for the correction due to all four semicircular areas:

$$\Delta N(\theta_0)=I(\theta_0)\frac{\pi b^3}{2l}\int_0^{\tan^{-1}a/l}\cos^n\varphi\sin\varphi\cos\varphi d\varphi$$

$$=I(\theta_0)\frac{\pi b^3}{2l}\cdot\frac{1}{n+2}\left[1-\left(\frac{l^2}{l^2+a^2}\right)^{(n+2)/2}\right].$$

For the dimensions of our counter array, $\Delta N(\theta_0)=0.338I(\theta_0)$ if we take a $\cos^2\theta$ dependence, and $\Delta N(\theta_0)=0.323I(\theta_0)$ for the $\cos^3\theta$ dependence.

The error along the width of the counter is shown in Fig. 4, where the situation is exaggerated for the sake of clarity. The maximum angular divergence of the particles is really given by angle β rather than by α, where

$$\sin\beta=\tan\alpha=b/l.$$

Under our assumption that the intensity varies linearly along the diameters of the counters, the correction therefore to be applied to the factors calculated above is simply to multiply them by the ratio β/α, which in our case amounts to increasing the factors by 0.67 percent.

Finally, therefore, we have the results

$$N(\theta_0)=5.14I(\theta_0),\quad \text{if}\quad I(\theta)\propto\cos^2\theta,$$

$$N(\theta_0)=5.03I(\theta_0),\quad \text{if}\quad I(\theta)\propto\cos^3\theta.$$

TABLE IV. Integrated intensity J_1, the number of particles per minute which cross an area of 1 cm^2 placed horizontally. $J_1=2\pi\int_0^{\frac{1}{2}\pi}\sin\theta\cos\theta I(\theta)d\theta$.

Alt. (M)	Depth (g/cm^2)	N	$N+n$	$N+n+n'$	n	$n+n'$
259	1007	0.68	0.81	1.04	0.14	0.37
1616	857	0.86	1.13	1.46	0.27	0.60
3240	708	1.21	1.74	2.47	0.52	1.26
4300	616	1.62	2.53	4.00	0.91	2.38

TABLE V. Integrated intensity J_2, the number of particles per minute which cross a sphere of 1 cm^2 cross section. $J_2=2\pi\int_0^{\frac{1}{2}\pi}\sin\theta I(\theta)d\theta$.

Alt. (M)	Depth (g/cm^2)	N	$N+n$	$N+n+n'$	n	$n+n'$
259	1007	0.90	1.06	1.40	0.16	0.50
1616	857	1.14	1.47	1.92	0.33	0.78
3240	708	1.62	2.26	3.20	0.64	1.58
4300	616	2.17	3.28	5.13	1.11	2.96

The exponent n indicated by the lines in Fig. 2 is actually 2.1 for the hard component, and 3.6 for the soft component. If the "very soft" component n' were plotted, the slope for this component would be somewhat less than 3 (because of the scattering). However, the variation in the proportionality factor is only 2 percent as the exponent changes from 2 to 3. In the next section we have for the sake of ease in extrapolation and graphical integration always dealt with the total intensities N, $N+n$ and $N+n+n'$; for these, the indicated exponent is between 2 and 3, and we have taken the proportionality factor as 5.12 throughout. This is accurate for the hard component, and may be in error by about 1 percent for the intensities which include the soft component.

In Table III we present the absolute intensities (particles per cm^2 per minute, per unit solid angle) in the vertical direction, obtained simply by dividing the corresponding coincidence rates (see Table I) by 5.12. Since the statistical error for the soft component is in all cases more than 2 percent, the error made by using the factor 5.12 for the soft component as well as for the hard component is negligible.

INTEGRATED INTENSITIES

The intensity J_1 may be defined as the number of particles per minute which cross, at any angle, a unit area placed horizontally. J_1 can be calculated as follows:

$$J_1=\int_0^{\frac{1}{2}\pi}2\pi\sin\theta\cos\theta I(\theta)d\theta$$

$$=\frac{\pi}{5.12}\int_{\cos^2\theta=0}^{\cos^2\theta=1}N(\theta)d(\cos^2\theta)$$

$$=(\pi/5.12)\times(\text{Area under graph of }N(\theta)\text{ against }\cos^2\theta).$$

The evaluation of the integral becomes particularly simple in the case of the hard component, because the intensity of this component is almost exactly proportional to $\cos^2\theta$. In the graphs of the intensities which include the soft component ($N+n$ or $N+n+n'$), the curvature is so small that the extrapolation to $\theta=\frac{1}{2}\pi$ is

TABLE VI. $I(0)$, J_1, and J_2, as obtained by other experimenters. The intensities include both hard and soft components.

Authors	Alt. (M)	$I(0)$	J_1	J_2
T. H. Johnson (reference 13)	189	0.70	0.96	
	1915	0.99	1.44	
J. C. Street and R. H. Woodward (reference 16)	0	0.80±0.028		1.48±0.055
D. K. Froman and J. C. Stearns (reference 15)	37	0.973±0.032	1.31±0.042	1.82±0.06
	4300	3.80±0.25	5.1±0.37	7.1±0.5

very easy in all cases. The intensities J_1 thus calculated are given in Table IV for the "fast" mesotrons N, for all particles capable of traversing the six counters $N+n$, and for all particles capable of traversing 2.3 g/cm² of brass (two counter walls) $N+n+n'$. The last columns give the differences n and $n+n'$.

The intensity J_2 may be defined as the number of particles per minute which cross a sphere of unit cross section. J_2 is directly related to the ionization per minute per unit volume observed in an ionization chamber, and is given by

$$J_2=\int_0^{\frac{1}{2}\pi} 2\pi \sin\theta I(\theta)d\theta$$

$$=\frac{2\pi}{5.12}\int_{\cos\theta=0}^{\cos\theta=1} N(\theta)d(\cos\theta)$$

$$=(2\pi/5.12)\times(\text{Area under graph of } N(\theta) \text{ against } \cos\theta).$$

The intensities J_2 thus calculated are listed in Table V.

In the evaluation of J_2, the contribution in the angular range between $\theta=55°$ and $\theta=90°$ is larger than in the evaluation of J_1; moreover, the slope of the graph of N against $\cos\theta$ changes more rapidly than does the slope of the graph of N against $\cos^2\theta$. Hence the results for J_2 cannot be considered as accurate as the results for J_1.

The numbers in Tables III, IV and V may be compared with the similar results of Johnson,[13] Street and Woodward,[16] and Froman and Stearns,[15] which are summarized in Table VI. The intensities listed in this table were obtained with no absorber except the walls of the counters and a thin roof, and hence include both soft and hard components. A comparison of the results in Tables III to V for $N+n$ and $N+n+n'$ demonstrates the critical dependence of the observed total intensities on the absorption which takes place in the counter walls. This alone may be sufficient to explain the differences in the results.[17]

[17] It seems likely that the high values obtained by Froman and Stearns at 4300-m elevation may be due to an over-correction for the inefficiency of their counters.

INTERPRETATION OF THE VARIATION WITH DEPTH

It is immediately obvious from the graphs in Fig. 2 that the decrease of n with depth is much more rapid than the decrease of N. In fact, the values of n are consistent with a straight line on the logarithmic graph, having a slope of -3.6. If one connects (as is done in the figure) the values of N obtained at *one elevation* and different zenith angles, one obtains for each elevation a straight line of slope -2.1; and if one connects the values of N obtained at *one zenith angle* and different elevations, one obtains curves showing a still less rapid decrease. The difference between the variation with depth of the soft and of the hard component has been observed by Rossi and Benedetti,[18] by Regener and Ehmert,[19] and by Bernardini and collaborators.[4] The implications, regarding the origin of the soft component, have been discussed by Rossi and Greisen.[9]

Let t represent the depth in g/cm² in the vertical direction and $y=t/\cos\theta$ represent the atmospheric depth in the direction of motion of the observed particles. The graph in Fig. 2 indicates definitely that N is not a function only of the depth y, since the values of N at two equal depths $y_1=t_1/\cos\theta_1$ and $y_2=t_2/\cos\theta_2$ are not equal unless $t_1=t_2$. This is especially true for the points obtained in Denver and in Ithaca, for which the possible effect of showers or other systematic errors is small and the differences are many times the statistical errors.

A reason for the differences is well known; they are the effect of spontaneous decay of the mesotrons, which causes the intensity to be a function not only of the depth in g/cm² but also of the geometrical distance from the place of production of the mesotrons. The differences could also be explained if the directional intensity of the primary rays were not isotropic, but since the zenith angle θ in our experiment was toward the south, and the latitudes were well above the knee of the latitude effect, there should be no differences due to magnetic effects. There is, in addition to the decay, another density effect,[20,21] which influences the energy

[18] B. Rossi and S. De Benedetti, Ricerca Scientifica **5** (2), 119 (1934).
[19] E. Regener and A. Ehmert, Zeits. f. Physik **111**, 501 (1938).
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TABLE VII. Predicted values of $\log_{10}(N_3/N_2)$, where N_3 and N_2 are observed at equal depths y but at different vertical depths t and different inclinations. t and y are expressed in g/cm². p_2 is the momentum at the depth y, in ev/c. The values are calculated on the assumption that $\tau_0/\mu=9\times10^{-4}$ and that the mesotrons are produced at a depth $y=103$ g/cm². The last row gives the observed values of $\log_{10}(N_3/N_2)$.

p_2(ev/c)	$y=t_3=1007$g/cm² $t_2=857$	$t_2=708$	$t_2=616$	$y=t_3=708$g/cm² $t_2=616$
3×10^8	0.129	0.31	0.47	0.128
8×10^8	0.085	0.20	0.31	0.078
2×10^9	0.050	0.12	0.18	0.042
Observed, for $p_2>3\times10^8$	0.04 ± 0.01	0.06 ± 0.01	0.07 ± 0.01	0.005 ± 0.01

loss by collision of the mesotrons; but in air this effect is negligible except for extremely high energies.

In order to compare the differences between values of N obtained at equal depths y with the predicted differences due to decay, one needs to know the place of production of the mesotrons, the angular distribution at the place of production, and the momentum spectrum. This information is not available. For a qualitative comparison, however, we may calculate the effect for mesotrons of certain momenta, taking a reasonable value for the depth of production and assuming that the mesotrons have the same direction as the primaries by which they are produced.

At two different vertical depths t_2 and t_3, let the zenith angles θ_2 and θ_3 be so chosen that $y=t_2/\cos\theta_2=t_3/\cos\theta_3$. Let N_2 represent the number of mesotrons of momentum p_2 observed at t_2 in the direction θ_2, and let N_3 represent the number of mesotrons of the same momentum observed at t_3 in the direction θ_3. Let us denote by L_2 and L_3 the corresponding distances from the place of production of the mesotrons (depth y_1) to the place of observation. If we assume that the momentum loss of the mesotrons is a constant given by a ev/c per g/cm² of air, we obtain the relation (see Euler and Heisenberg,[1] Rossi[22])

$$\log\frac{N_3}{N_2}=\frac{\mu}{\tau_0}(L_2-L_3)\cdot\frac{1}{P},$$

where

$$P=\frac{p_2+ay}{1+\log(p_1/p_2)/\log(y/y_1)}.$$

$p_1=p_2+a(y-y_1)$ is the momentum of the mesotrons at the place of production, τ_0 is the proper lifetime and μ is the rest-mass. We take for τ_0/μ the value 9×10^{-4} (cm-c/ev),[23] and we assume for y_1 the value 103 g/cm² (1/10 of the atmosphere). The lengths L may be evaluated with the aid of the relation $z=z_0\log(t_0/t)$ between altitude z and vertical depth t, in which (according to our barometer readings) $z_0=84\times10^4$ cm and $t_0=1036$ g/cm².

Thus we have calculated the differences $(\log_{10}N_3-\log_{10}N_2)$ which are given in Table VII. The calculations have been made for three different values of the momentum p_2: 3×10^8, 8×10^8, and 2×10^9 ev/c. In the first three columns of the table, N_3 represents the number of mesotrons in the vertical direction at Ithaca ($t_3=1007$ g/cm², $\theta_3=0$), and N_2 represents the number of mesotrons in an inclined direction at one of the other stations (Denver, Echo Lake, and Mt. Evans, respectively, for columns one, two, and three), the inclinations being so chosen that the depth in the direction of observation is the same at all four stations. If the mesotrons were monoenergetic, and all had at the depth of observation a momentum equal to one of the values chosen for p_2, the numbers in the row corresponding to that momentum should represent the separations in Fig. 2 between the topmost line and the three lower lines, respectively, at the depth indicated by arrow A. In the last column of the table, N_3 represents the number of mesotrons in the vertical direction at Echo Lake ($t_3=708$ g/cm², $\theta_3=0$), and N_2 represents the number of mesotrons in an inclined direction at Mt. Evans, so chosen that the depth in the direction of observation is the same at both stations. If the mesotrons were monoenergetic, and had one of the momenta for which the effect has been calculated, the number in the last column corresponding to that momentum should represent the separation

[22] B. Rossi, Rev. Mod. Phys. **11**, 296 (1939).

[23] See Rossi and Hall, reference 12. This result has been confirmed recently in another experiment by Rossi and collaborators, an account of which is to be published shortly.

between the two lines in Fig. 2 at the depth indicated by arrow B.

In the last row of Table VII, we have given the observed differences ($\log_{10} N_3 - \log_{10} N_2$), which are taken directly from Fig. 2 at the depths indicated by arrows A and B. It may be noted that the observed difference between Denver and Ithaca (0.04 ± 0.01) is in agreement with the predicted separation for $p_2 = 2 \times 10^9$ ev/c (0.050), which is a reasonable estimate of the effective momentum at sea level. All of the other observed differences are much lower than those predicted. In particular, the predicted effect between the higher elevations (which is given at depth B by the last column in the table, and at depth A by the difference between the figures in the second and third columns) is of the same order of magnitude as the predicted effect between the lower elevations (given by the first column); whereas the observed effect between the higher elevations is practically zero. This discrepancy remains, even if the calculations are repeated using different values for the lifetime or for the depth of production of the mesotrons.

The discrepancy might be accounted for if many side showers were recorded at the high elevations, since these might reduce the apparent decrease in intensity with increasing zenith angle, and since the number of showers increases with altitude faster than the number of mesotrons. However, auxiliary experiments described below show that the number of such showers recorded with six counters in coincidence is at most a very small fraction of the number of mesotrons, while the discrepancy is very large.

The discrepancy may be satisfactorily explained, however, if one takes into account that mesotron production takes place at *all* depths below the top of the atmosphere, and if one makes two assumptions: (1) that mesotron production is still an important process at depths corresponding to the elevation of Mt. Evans (4300 meters), and (2) that mesotrons produced with small momentum have a large angular divergence from the direction of the primaries by which they are produced. Under these two assumptions, many of the mesotrons which are observed at the higher elevations in an inclined direction have been produced *near the place of observation* by primaries which have traversed the atmosphere in nearly vertical directions. These mesotrons have not been appreciably reduced in number by decay. Looking at the phenomenon from another point of view, we may say that the effective depth of observation for these mesotrons lies between t and $t/\cos\theta$. Near sea level, however, mesotron production is rare, so that most of the observed mesotrons are produced far from the place of observation. The mesotrons observed at sea level must therefore have been created with such large momentum that their divergence from the direction of the primaries is small. For *these* mesotrons, the effective depth of observation is very nearly $t/\cos\theta$, as is assumed in the calculations presented above.

This explanation might account for the difference ($\log N_3 - \log N_2$) being small when one compares intensities observed at the higher altitudes, and for the differences being roughly as predicted by the theory when one compares intensities observed near sea level. We consider, therefore, that the results of this experiment on the intensity of the hard component as a function of altitude and of zenith angle are an indication of the production of mesotrons at fairly low altitudes.

SYSTEMATIC ERRORS IN THE MEASUREMENTS

(1) Inefficiency

Auxiliary experiments performed with the counters used in this experiment[24] indicate that the inefficiency of the counters due to the "dead-time" following each pulse is 0.2 percent or less at 259-m elevation. This implies an error in the measurements with six counters which is about 1 percent in Ithaca and increases to about 2.5 percent at the highest elevation.

Other types of inefficiency, i.e., that due to time lags and that due to occasional failure of the particles to produce ions in a counter, have been shown by our auxiliary experiments to be smaller than 0.5 percent per counter, and are a constant fraction of the counting rate. Therefore they have no influence on the relative rates, and introduce an error less than 3 percent in the absolute rates.

[24] We expect to publish a detailed account of these experiments in the near future.

(2) Chance Coincidences

An experiment performed with the same apparatus in Ithaca, in which the counting rates were increased by a radium source, indicates that the resolving time of the circuit is about 6 or 7 microseconds, and that the number of sixfold chance coincidences is completely negligible.

(3) Scattering

The decrease in the number of coincidences, due to scattering of particles out of the beam, has been neglected.

(4) Showers

Occasionally two or more electrons which are members of the same shower may simultaneously traverse all of the counters and be recorded as only one particle. This effect tends to reduce slightly the number of electrons recorded. No effect at all of this type is expected for mesotrons.

A more serious error, which affects the counting rates with lead as well as those without lead between the counters, is caused by *side showers* which discharge all the counters simultaneously. This effect tends to increase all of the counting rates.

The error caused by showers is the most difficult to eliminate or evaluate. If one wishes to record only mesotrons of high energy, one may reduce the number of side showers recorded by shielding the counters on the sides (as well as above the counters) with a large thickness of lead. If one wishes to record electrons, however, the presence of lead alongside and near the counters distorts the results because of the showers which originate in the lead itself. Likewise one cannot use the device of placing alongside the coincidence counters additional counters, which are so connected that a coincidence is recorded only when the side counters are not simultaneously discharged. Such a device would discriminate against true coincidences caused by electrons, since all observed electrons are members of more or less dense air showers. This device also reduces the coincidences recorded for mesotrons by several percent,[25] because the side counters are sometimes discharged by knock-on electrons accompanying the mesotrons.

It has been suggested that one may correct for the showers recorded, by measuring the coincidence rate with one of the counters slightly out of line, and subtracting this rate from the coincidence rates with all the counters in line. The correction thus obtained, however, is much too large, both because of the knock-on electrons which discharge the counter out of line when a particle goes through the other counters, and because of the shower particles accompanying the electrons which traverse the counter array.

In our experiment, the number of side showers recorded was reduced by using a large number (six) of counters in coincidence. Auxiliary experiments were performed in which one of the middle counters (in line) was so connected that a coincidence between the other counters was only recorded when this counter was not simultaneously discharged. Such measurements were taken with varying numbers of the other counters in coincidence, from the two extreme counters alone to all of the five remaining counters. The results are not conclusive enough to be entirely satisfactory, but indicate that the number of showers recorded is quite large when only two counters are in coincidence, and decreases rapidly as the number of counters in coincidence is increased. With six counters in coincidence, the number of showers recorded seems to be not more than a few percent of the coincidence rates at the highest elevation, and negligible at the lowest elevation.

The error in the measurements due to the recording of side showers is in a direction opposite to that of the other errors, which also increase somewhat with altitude and partially compensate for the error due to showers. The exact corrections for all the errors are not known, but we know that the corrections are small, of the order of magnitude of the statistical errors. Hence we have not applied to the data any corrections for the systematic errors discussed above.

The writer wishes to express his thanks to Professor Bruno Rossi for suggesting this experiment, and for assistance in carrying out the measurements and evaluating the results.

[25] See Rossi, Hilberry and Hoag (reference 11), Schwegler [A. Schwegler, Zeits. f. Physik **96**, 62 (1935)]. This result is substantiated by similar experiments now in progress.