

on to the data for angles greater than  $45^\circ$  where no recoil deuterons were present, and approach unity for the smallest angles. The low angle portion of the curve, due to the small amount of data taken and the method of applying the correction, is to be taken merely as indicative of the trend of  $R$ . More accurate data would have allowed an evaluation of the amount of contamination present if it is correct to assume that  $R$  should be unity at the smallest angles.

The curves show that the deviation from Rutherford scattering increases both with energy and angle, as might be expected. The angular

dependence is in qualitative agreement with calculations based on Primakoff's formula with the "one body" theory combined with a theoretical cross section for neutron-deuteron scattering given by Ochiai, but is smaller by almost a factor of two in magnitude. At the largest angles considerable corrections for reduced counter efficiency must be made so these values of  $R$  cannot be expected to be very good.

The writer wishes to express his thanks and appreciation to Professor G. Breit for much helpful discussion and correspondence concerning this problem.

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## Magnetic Scattering of Neutrons

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The problem of magnetic scattering of neutrons has been reconsidered. The possible sources of error in the previous estimate are discussed. The form factor of the  $3d$  shell of iron has been recalculated, with Hartree functions for Fe. This results in removal of the discrepancy between theory and experiment. Some predicted results of measurements with monoenergetic neutrons are given.

A RECENT theoretical evaluation<sup>1</sup> of neutron polarization experiments has indicated a marked discrepancy between theory and experiment. The calculated values are consistently too low by more than a factor two.

There appear to be three possible causes for this disagreement:

1. Since experimentally only the gyromagnetic ratio is measured, the possibility of neutron spin  $= \frac{3}{2}$  might be considered. This suggestion, which would result in an increase in the theoretical estimate, has been made by Halpern and Johnson.<sup>2</sup> However, it is well known to be irreconcilable with our present understanding of the structure of light nuclei.<sup>3</sup>

2. The velocity distribution of the neutrons is not very accurately known. H. H. J. (reference 1)

used the data of Dunning, etc.<sup>4</sup> obtained by use of a mechanical velocity selector. Though these results may contain considerable error, the theoretical estimate is not very sensitive to change in the distribution.

3. The calculated values of polarization effect are extremely sensitive to changes in the form-factor of the magnetically active  $3d$  shell. H. H. J.

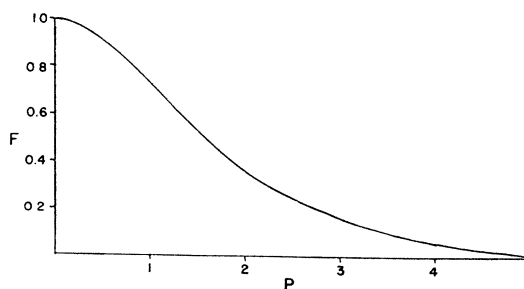


FIG. 1. Form factor for the  $3d$  shell as a function of  $P$ .

<sup>1</sup> O. Halpern, M. Hamermesh, and M. H. Johnson, *Phys. Rev.* **59**, 981 (1941). In what follows, this is referred to as H.H.J. and similar notation is used.

<sup>2</sup> O. Halpern and M. H. Johnson, *Phys. Rev.* **57**, 160 (1940).

<sup>3</sup> See also J. Schwinger, *Phys. Rev.* **52**, 1250 (1937).

<sup>4</sup> Dunning, Pegram, Fink, Mitchell, and Segrè, *Phys. Rev.* **48**, 704 (1935).

TABLE I. Values of  $w(x)$ .

$ U ^*$	$A = 1 + \frac{ U ^2 \lambda^2}{4a^2}$	$F$	$f_0(v)$		$f_1(v)$		$f_2(v)$		$f_3(v)$		$f_4(v)$	
			$f( U )$	$fFA$	$f( U )$	$fFA$	$f( U )$	$fFA$	$f( U )$	$fFA$	$f( U )$	$fFA$
2	1.14	0.48	0.281	0.154	0.252	0.138	0.231	0.126	0.207	0.113	0.193	0.106
4	1.29	0.28	0.076	0.028	0.071	0.026	0.068	0.025	0.063	0.023	0.061	0.022
6	1.43	0.19	0.183	0.050	0.181	0.049	0.178	0.048	0.171	0.047	0.170	0.046
8	1.57	0.12	0.061	0.012	0.062	0.012	0.062	0.012	0.061	0.012	0.062	0.012
10	1.71	0.08	0.086	0.012	0.089	0.012	0.092	0.013	0.092	0.013	0.095	0.013
12	1.86	0.053	0.021	0.002	0.022	0.002	0.023	0.002	0.023	0.002	0.024	0.002
14	2.00	0.035	0.094	0.007	0.098	0.007	0.106	0.007	0.109	0.008	0.115	0.008
			Remainder	0.002		0.004		0.004		0.004		0.004
			Total	=0.267		0.250		0.237		0.222		0.213
			$w(x)$	=0.090		0.084		0.080		0.075		0.072

determined the form-factor by using Hartree functions for Cu, adjusting to fit the case of iron by expanding the radius of the shell in the inverse ratio of the effective charges. This procedure would be exact for hydrogenic functions. Since, however, the  $3d$  wave functions are not hydrogenic, and since the form factor is strongly dependent on the exact shape of the charge distribution, this procedure might lead to a considerable error.

Fortunately the Hartree functions for Fe have been computed by Manning and Goldberg.<sup>5</sup> The form factor for the  $3d$  shell has been calculated by numerical integration and is plotted in Fig. 1 as a function of  $p = 4\pi a_0 / \lambda \cdot \sin \theta / 2$  where  $a_0$  = Bohr radius,  $\lambda$  = neutron wave-length, and  $\theta$  is the angle of scattering. This form factor is throughout larger than that obtained by H. H. J.

By following the procedure of H. H. J., the quantity  $w(x)$  has been computed. Table I corresponds to their Table II.

From the values of  $w(x)$ , the polarization effect has been calculated, and is compared in Table II to various experimental results. (This replaces Table III of H. H. J.)

The new values for the form factor considerably improve the agreement. Whereas in the first three cases there still remains a discrepancy of unknown origin by about a factor three, the agreement in the last case<sup>6</sup> may be considered satisfactory.

<sup>5</sup> M. F. Manning and L. Goldberg, Phys. Rev. **53**, 662 (1938).

<sup>6</sup> L. W. Alvarez and F. Bloch, Phys. Rev. **57**, 111 (1940).

The lack of sensitivity of the theoretical estimate to changes in the velocity distribution

TABLE II.

Thickness	0.8	1.3	1.95	4.0
% Effect	0.23	0.63	1.38	5.1
Experimental	0.76	1.78	3.32	6.0

can be seen from Table I. The distribution in the initial beam leads to  $w=0.090$ , while that for a beam filtered through 1 cm gives  $w=0.084$ . The filtering can be described roughly as a decrease of the number of neutrons in the low velocity range (say up to 1.5 km/sec.) by 35 percent, as compared to the rest of the distribution. Yet this change in distribution would lead to only a 15 percent change in the final answer. Changes of this order would be sufficient to secure complete agreement with the Bloch-Alvarez case.

The change in form-factor also increases the calculated polarization for monochromatic beams. Some predicted results are given in Table III.

TABLE III.

$\lambda(A)$	$w$	Temp. ( $^{\circ}K$ )	% Effect for:	
			1 cm	4 cm
3	0.25	75	3.0	15.5
4	0.55	42	54.	367.

For  $\lambda > 4.04A$  the effect should disappear entirely, since the crystal becomes transparent. Experiments in this low velocity range should give a crucial test of the correctness of our theory.

I wish to thank Professor F. Bloch for discussion and suggestions concerning this problem.