values of $\Delta\nu/R$ compiled from experiments on the doublets from many different x-ray lines, and weighted according both to the number of lines entering into the determination of each value of $\Delta\nu/R$, and to the *a posteriori* consistency of each line used. Values of $\Delta\nu/R$, and of the corresponding weights, are compiled in Table III for the range of atomic number used (26 elements, running from $Z=60$ to $Z=92$). These values have been converted from the conventional Siegbahn scale, on which all experimental results are published, to the presumably true values of the so-called grating scale of wave-lengths, by the use of the factor $\lambda_g/\lambda_s=1.002034$.

Finally, the values of $\Delta(1/\alpha)$ and of B were found from these data by a least-square calculation, based directly on Eq. (8). The values obtained are $\Delta(1/\alpha)=-0.072$, $B=4.858\times 10^{-4}$. The statistical probable error in the determination of $\Delta(1/\alpha)$ is 0.034. These results give $1/\alpha=136.928\pm 0.034$.

The determination of the $f(\alpha Z)$, the correction of order $1/Z$ to the spin splitting, is incomplete due to our neglect of outer shells. We have estimated that this incompleteness might falsify the magnitude of f by not more than one percent, which would produce a change in $1/\alpha$ of 0.081. Adding three times the statistical probable error to this gives a figure of 0.18 for the limit of error on $1/\alpha$. Hence we conclude that the value of $1/\alpha$ is 136.93, correct to within 0.18.

We gratefully acknowledge our indebtedness to Professor J. R. Oppenheimer for suggesting this problem, and for pointing out the method of calculation. We also wish to express our gratitude to the National Youth Administration for furnishing assistance in the numerical calculations.

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The Mean Life of Neutrons in Water and the Hydrogen Capture Cross Section

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Direct observations of the time variation of slow neutron density in a large volume of water during and after irradiation by neutrons from the D−D reaction have been made. An exponential growth and decay is found. From an analysis of these data and data on the spatial density distribution which determines the effect of diffusion, the mean life of neutrons in water was found to be 205 ± 10 microseconds. This leads to a cross section of 0.33×10^{-24} cm^2 for the capture of slow neutrons by hydrogen.

Control of the ion beam responsible for neutron production from an artificial neutron source makes it possible to observe transient neutron phenomena. One such phenomenon is the time rate of change of neutron density in hydrogenous material. In water, for example, this depends on the probability of neutron escape from the surfaces, the loss by capture and the distribution of sources. For pure water, in which the capture is chiefly due to hydrogen nuclei, suitable measurements will lead to the capture probability and hence to the hydrogen capture cross section.

Baker and Bacher[1] have observed the mean life of neutrons emerging from a paraffin block after the source has been removed. The effect of diffusion, however, complicates considerably the proper interpretation of such data. The analysis is more straightforward if the time variation of neutron density is obtained in a region where the effect of diffusion can be evaluated.

Previous results on the mean life of neutrons in paraffin and water vary from 170 to 270 microseconds[2,3] although the hydrogen density, which is the important factor, varies by only about 30 percent. Such a fundamental quantity as the cross section for the $n+p\rightarrow d$ reaction

[1] C. P. Baker and R. F. Bacher, Phys. Rev. **59**, 332 (1941).

[2] E. Amaldi and E. Fermi, Phys. Rev. **50**, 899 (1936).

[3] O. R. Frisch, H. v. Halban, and J. Koch, Proc. Danske Vidensk. Selskab **15**, 10 (1938).

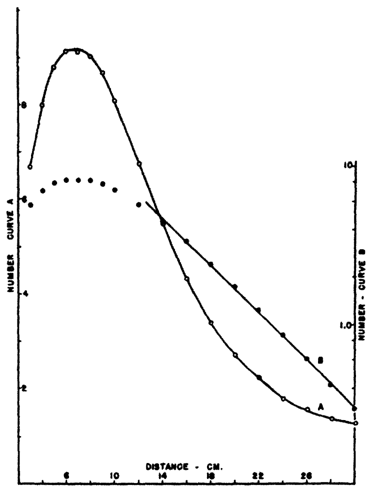

FIG. 1. Neutron density as a function of distance from target side of the tank. The center of the tank is at 22 cm. Curve A is an ordinary plot to show the point of inflection at 12 cm. Curve B is a logarithmic plot of the same data. The solid line is the function $n=n_0e^{-0.173x}$. Approximately 20,000 counts were taken for each point.

warrants more precise determination. An investigation with an intermittent neutron source was, therefore, undertaken.

The neutron source, method of control, and recording circuits have been described previously.[4,5] Bursts of fast neutrons from the D–D reaction were used to irradiate a cylindrical tank of water 44 cm in diameter by 54 cm high. The tank was located so that the center of its vertical axis was approximately 60 cm from the target of the accelerating tube and in the same horizontal plane. A cylindrical BF_3 proportional counter (sensitive volume 2 cm in diameter by 5 cm long) was supported with its axis vertical by a small copper tube so that it could be moved in the median plane of the tank. The total absorption of the counter itself was measured and found to be equivalent to 90 cc of water, only slightly more than the displacement of the counter. The amplified pulses from this counter, appearing on an oscilloscope screen[5] with a suitable time scale, were photographed in order to record their time distribution. The tank, which was paraffin-lined to prevent solution of metallic ions, contained 85.6 liters of distilled water.

[4] J. H. Manley, L. J. Haworth, and E. A. Luebke, Rev. Sci. Inst. **12**, 587 (1941).
[5] L. J. Haworth, J. H. Manley, and E. A. Luebke, Rev. Sci. Inst. **12**, 591 (1941).

In order to obtain the effect of diffusion, neutron densities at different points in the tank must be known. Measurements of this distribution were made for the steady state case. The neutron density along a line from the target to the center of the tank is shown by curve A of Fig. 1. Distances are measured from the side of the tank nearest the target. Curve B is a logarithmic plot of the same data to indicate that near the center of the tank, $x=22$ cm, the experimental points follow the curve $N=N_0e^{-0.173x}$. Measurements in planes perpendicular to this line reveal that over several diffusion lengths from it the density is essentially constant. It will, therefore, be possible to apply the one-dimensional diffusion equation to the data of Fig. 1.

It is possible to obtain the desired information on the effect of capture and diffusion by observation at a single space point of either the growth of neutron density during irradiation or the decay after irradiation. The size of the oscilloscope screen and considerations of ease of counting the recorded neutron pulses made it more feasible to observe the growth and decay in separate experiments with an overlap for correlation. Since this was done at three space points, $x=12$, 14, and 22 cm, there are six independent observations. Irradiation occurred cyclically with a time of irradiation sufficient to reach saturation and a lapse long enough to eliminate any measurable carry-over of neutrons to the next cycle. The number of counts in each 50-microsecond time interval was summed over a large number of cycles. A typical composite growth and decay curve is shown in Fig. 2.

Determination of the saturation value neces-

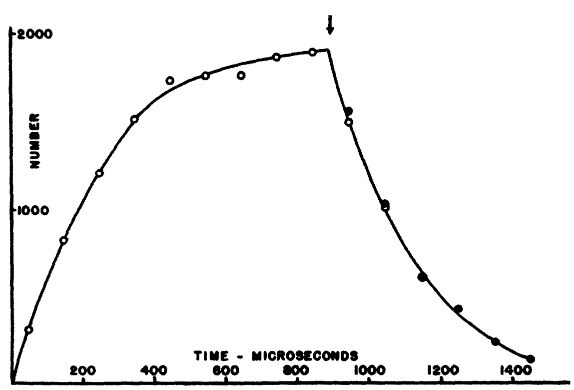

FIG. 2. A typical growth-decay curve. The arrow indicates the end of irradiation.

sary for the analysis of the growth curve introduced additional error which does not enter the analysis of the decay curve. Even though the statistical accuracy was good on the points near saturation the high density of pulses on the recording film increased the probability of missing them in the counting process. The decay data were, therefore, considered more reliable even though growth data are in satisfactory agreement. Exponential plots[6] of the decay data at three different space points are shown in Fig. 3. For the reason mentioned above the highest points were not given as much weight in determining the best straight line representing the data. As is to be expected, the observed mean life as given by the curves of Fig. 3 depends on the position at which the measurements were made. If the effect of diffusion at these positions can be evaluated, the mean life due to capture alone can be obtained from each set of data.

Reduction of the diffusion problem to one dimension has already been experimentally justified. For the general case of a neutron density $N(x, t)$ resulting from a distribution of sources producing $\rho(x, t)$ neutrons per second

$$dN/dt - Dd^2N/dx^2 + N/\tau = \rho(x, t). \tag{1}$$

τ is the mean life for capture and D the diffusion coefficient. A solution of this equation appropriate to the case in which the source is removed at $t=0$ leaving a spatial distribution $N(x)$ is

$$N(x, t) = \frac{e^{-t/\tau}}{2(\pi Dt)^{\frac{1}{2}}} \int_{-\infty}^{\infty} dx' N(x') \times \exp\left[-\frac{(x'-x)^2}{4Dt}\right]. \tag{2}$$

In order to investigate the behavior of $N(x, t)$ for a general $N(x')$ we note that the integrand has a saddle point for a particular x'

$$x'_0 = 2Dtf' + x \quad \text{where} \quad f = \log N.$$

Then

$$N(x, t) = Ne^{-t/\tau - Dtf'^2}/(1-2Dtf'')^{\frac{1}{2}} \tag{3}$$

in which N and its derivatives are to be evaluated at x'_0. The result is valid if

$$f'''(2Dt)^{\frac{3}{2}}/(1-2Dtf'')^{\frac{3}{2}} \ll 1$$

and if higher derivatives of f are negligible at x'_0.

[6] Similar plots were obtained for the growth.

It is clear that for sufficiently long times the error function in (2) will be of so great a width as to make higher derivatives of f important; under these conditions (3) will be only approximately valid. The observations actually included points for which (3) could not be used; for these cases numerical integration of (2) gave results which differed appreciably from the approximate ones obtained by the saddle point method.

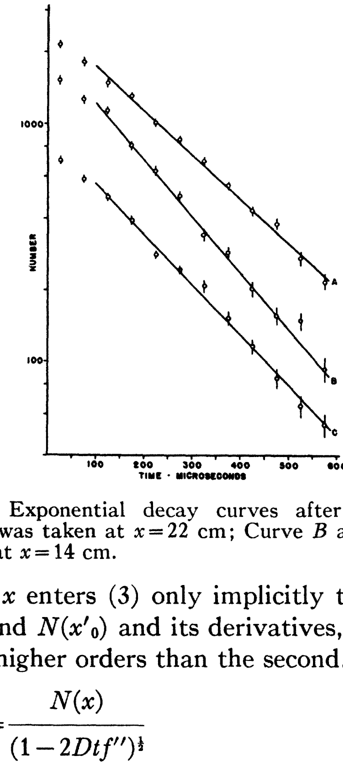

FIG. 3. Exponential decay curves after irradiation. Curve A was taken at $x=22$ cm; Curve B at $x=12$ cm; Curve C at $x=14$ cm.

Since x enters (3) only implicitly through x'_0 we expand $N(x'_0)$ and its derivatives, and again neglect higher orders than the second. Then

$$\begin{aligned} N(x, t) &= \frac{N(x)}{(1-2Dtf'')^{\frac{1}{2}}} \times \exp\left\{-t/\tau - Dt\left[f'^2 - \frac{2f'^2}{1-2Dtf''}\right]\right\} \\ &\backsimeq N(x)\exp\{-[1/\tau - D(f'^2+f'')]t\} \\ &= N(x)\exp\left\{-\left[1/\tau - D\frac{N''}{N}\right]t\right\} \end{aligned} \tag{4}$$

if $2Dtf'' \ll 1$. $N(x)$ is the observed spatial distribution at $t=0$. As would be expected, diffusion effects would vanish if the time dependence of $N(x, t)$ were observed at a point for which N'' and all higher derivatives vanish.

This analysis applies equally well to the rise of neutron density during irradiation. Let us consider a steady-state spatial distribution resulting from distributed sources which are constant in time. Then it can be shown in a very general way, including the effects of diffusion and capture, that the rise to this steady state as a function of space and time is the same as the fall from the steady state when the sources are removed.

To correct the observations for the effect of diffusion, the coefficient D must be known. This may be computed from the kinetic theory value, $D=\frac{1}{3}\lambda\bar{v}$. A value of 2.2×10^5 cm/sec. for $\bar{v}$ has been chosen although there is some indication that this may be too low. Different experiments on the mean free path, λ, are not in complete agreement. Goldhaber and Briggs[7] give $\lambda=2.5$ mm in paraffin and find the scattering per hydrogen atom to be the same in water within 2 percent. This would lead to $\lambda=3.0$ mm for H_2O. Brickwedde, Dunning, Hoge, and Manley[8] have measured the scattering of H_2O directly with a geometrical arrangement very similar to that of Goldhaber and Briggs. Their results give $\lambda=3.35$ mm. With 3.2 mm as an average of these two results, $D=2.4\times10^4$ cm^2/sec. The effect of diffusion on the observed time dependence of the neutron density at various positions may be found by application of Eq. (2) or (4).

At the 12-cm position for which $N''=0$, Eq. (4) predicts no diffusion correction. However, since $f''\simeq-0.01$ cm^{-2}, (4) is not valid for times after irradiation greater than 200 μsec. Numerical integration of (2) for different values of t reveals that, in the range of measurement, $e^{-t/\tau}$ is multiplied by a slowly varying linear function of time. This function may accordingly be represented by e^{at}. For this position, $a=-200$ sec.$^{-1}$ rather than zero as predicted by (4). A similar situation exists at $x=14$ cm except that $a=50$ sec.$^{-1}$.

Since the spatial distribution on either side of $x=22$ cm is rather accurately given by $N=N_0e^{-0.173x}$ for which $f''=0$, Eq. (4) would yield $a=DN''/N=720$ sec.$^{-1}$ for this position. A check numerical integration of (2), however, gives $a=600$ sec.$^{-1}$. This integration reveals that for $t=500$ μsec. the width of the error function of (2) is such that values of $N(x)$ outside the region of validity of the exponential contribute to the integral. The experimental points in this region lie below the exponential and, therefore, a lower value of a is obtained. This means that this deviation which starts at $x=14$ cm affects the observations at $x=22$ cm through higher derivatives at this point which are not negligible. The saddle point integration used in obtaining (4) is, therefore, a poor approximation for such times. The complete results are summarized in Table I.

TABLE I. Data on mean life of neutrons.

Position (x) cm	12	14	22
Observed mean life (growth), μsec.	180	195	214
Observed mean life (decay), μsec.	182 ± 9	202 ± 12	240 ± 9
Reciprocal mean life, observed, sec.$^{-1}$	5500	4950	4180
Diffusion correction a, sec.$^{-1}$	-200	50	600
Corrected mean life, μsec.	189 ± 10	200 ± 12	209 ± 9

The maximum diffusion correction is 15 percent, and errors in the coefficient are, therefore, relatively unimportant. They show that for $x=14$ cm there is almost no net diffusion while at 12 cm diffusion removes neutrons from this position and at 22 cm there is an inflow. From the geometrical arrangement the small, neglected curvature in other directions would be negative, corresponding to a removal of neutrons, and more important at smaller x. This may account for the trend in the results for the mean life, in which case the 22-cm value of $\tau=209$ μsec. would be more reliable. However, to within 5 percent we may take $\tau=205$ μsec. as a weighted mean.

The capture cross section is related to the mean life or the reciprocal capture probability by

$$1/\tau=P=n\sigma\bar{v}.$$

It may be assumed that σ is proportional to $1/v$ so the capture probability becomes independent of velocity. The average cross section, when we take $\bar{v}=2.2\times10^5$ cm/sec. and the above mean life, is 0.33×10^{-24} cm^2, the largest uncertainty being in $\bar{v}$. The capture cross section of oxygen which is less than 3 percent of this value has been neglected.

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[8] F. G. Brickwedde, J. R. Dunning, H. J. Hoge, and J. H. Manley, Phys. Rev. **54**, 266 (1938).