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Origin of the Soft Component of Cosmic Rays

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The number of electrons arising from the decay of mesotrons has been calculated with the multiplication theory. A comparison with experimental data indicates the existence of an electron component having a sharper maximum in the vertical direction and increasing with altitude more rapidly than the decay electrons.

1. INTRODUCTION

THE question as to the origin of the soft component of cosmic rays in the atmosphere has been the subject of much discussion in the last years.[1–7] Until recently the general belief was that the soft component is produced partly by incoming primary electrons, partly by electrons arising from the decay of mesotrons and from collision processes of mesotrons in air. Lately Schein, Jesse, and Wollan[6] have suggested that practically all of the soft component may be a secondary product of mesotrons, the bulk of it being produced by the decay.

In the present paper the above hypothesis is developed quantitatively and the results are compared with experimental data. The main theoretical problem is to calculate the intensity of the soft component arising from the disintegration of mesotrons and the subsequent multiplication of the decay electrons in air. It has been shown by one of us[2] that this calculation can be carried out in a very elementary way, without making use of the multiplication theory of showers, if one assumes that the rate of production of the decay electrons does not change appreciably over a distance in which the showers generated by these electrons are completely absorbed. Bernardini and his collaborators[4] have found that the intensity of the soft component calculated by this method increases with height more slowly than the observed intensity. However, the assumption on which the above method of calculation is based represents only a very rough approximation. It is therefore desirable to treat the problem more accurately, and the use of the multiplication theory then becomes essential.

2. THEORETICAL PROBLEM

We want to determine the number of electrons at a certain point in the atmosphere arising from the decay of mesotrons in the upper layers. We shall consider mesotrons and electrons coming in a given direction and neglect scattering.

Let us denote by t the depth below the top of the atmosphere in radiation lengths, measured in the direction of the incoming particles, and by $f(t_1, p)dp$ the number of mesotrons with momentum between p and $p+dp$ present at the depth t_1. In general, only those mesotrons are recorded which go through a certain thickness x_0 of absorber and have, therefore, momenta larger than a certain value p_0. Hence the observed number of mesotrons at t_1 is given by

$$N(t_1)=\int_{p_0}^{\infty} f(t_1, p)dp. \tag{1}$$

For an absorber of 15 cm of lead, for instance, $p_0=3\times 10^8$ ev/c. Let us further indicate by N_1 the number of mesotrons with momentum smaller than p_0. N_1 is not measured directly, but can be evaluated approximately and is always small compared with N.

The number of mesotrons in the momentum interval (p, dp) which disintegrate in the air layer between t_1 and t_1+dt_1 is given by

$$\frac{\mu}{\tau_0}X_0\frac{f(t_1, p)dp}{p}\frac{dt_1}{\rho(t_1)},$$

where μ is the rest mass, τ_0 the proper lifetime of

[1] W. Heisenberg and H. Euler, Ergeb. d. exact. naturwiss. **17**, 1 (1938).
[2] B. Rossi, Phys. Rev. **57**, 469 (1940).
[3] E. Nelson, Phys. Rev. **58**, 771 (1940).
[4] G. Bernardini, B. N. Cacciapuoti, B. Ferretti, O. Piccioni, and G. C. Wick, Phys. Rev. **58**, 1017 (1940).
[5] L. W. Nordheim, Phys. Rev. **59**, 554 (1941).
[6] M. Schein, W. P. Jesse, and E. O. Wollan, Phys. Rev. **59**, 614 (1941).
[7] G. Cocconi, Phys. Rev. **60**, 532 (1941).

mesotrons, X_0 the radiation length in g/cm^2 of air, $\rho(t_1)$ the density of air at the depth t_1.[8] We shall assume that each mesotron disintegrates into an electron and a neutrino and that, in the frame of reference of the mesotron, the electron is emitted with equal probability in any direction. It follows that, in the frame of reference of the earth, the probability of an electron of energy (E, dE) being produced by the decay of a mesotron of momentum p is given by dE/p.[9]

The electrons produced in the layer (t_1, dt_1) multiply into a shower which can conveniently be described by a function $k(t_1, t)$ where $k(t_1, t)dt_1dt$ represents the energy dissipated by the shower electrons in the layer dt at a distance t from t_1. If we then indicate by $K(t_2)dt_2$ the energy dissipated by all shower electrons in the layer between t_2 and t_2+dt_2, $K(t_2)$ is given by

$$K(t_2)=\int_0^\infty k(t_2-t, t)dt. \tag{2}$$

The upper limit of the integral can be taken as infinity if the depth at which the measurements are made is larger than the maximum penetration of showers.

3. METHOD OF CALCULATION FOR HIGH ENERGY MESOTRONS

We shall consider, for the moment, only the electrons arising from the decay of mesotrons with momentum larger than p_0, and we shall assume that p_0^2 is large compared with μ^2. We indicate by $z(t_1)dt_1$ the total amount of energy given to electrons by the mesotrons disintegrating in the layer (t_1, dt_1). In consequence of the formulae above, $z(t_1)$ is given by the following integral:

$$z(t_1)=\frac{\mu}{\tau_0}\frac{X_0}{\rho(t_1)}\int_{p_0}^\infty\frac{f(t_1, p)dp}{p^2}\int_{E_{\min}}^{E_{\max}}EdE, \tag{3}$$

where $E_{\min}$ and $E_{\max}$ are given by Eqs. (26) (see Appendix). If we neglect μ^2 with respect to p^2, the limits become practically 0 and p, and Eq. (3) yields

$$z(t_1)=\frac{1}{2}\frac{\mu}{\tau_0}X_0\frac{N(t_1)}{\rho(t_1)}. \tag{3a}$$

[8] See, for instance, B. Rossi, Rev. Mod. Phys. **11**, 296 (1939). For the system of units used in the present article, see B. Rossi, Phys. Rev. **57**, 660 (1940).

[9] See B. Ferretti, Nuovo Cimento **15**, 421 (1938).

$z(t_1)$ represents also the area under the "shower curve"; i.e., the curve giving $k(t_1, t)$ as a function of t:

$$z(t_1)=\int_0^\infty k(t_1, t)dt, \tag{4}$$

because the whole energy of the decay electrons is eventually dissipated by the particles of the shower to which they give rise.

Most of the electrons present at any particular depth t_2 arise from the decay of mesotrons not very far above t_2, say between $t_2-\Delta t$ and t_2. If one neglects the change in the shape of the energy spectrum of mesotrons over the distance Δt, one can put in Eq. (2)

$$\begin{aligned}k(t_2-t, t)&=\frac{z(t_2-t)}{z(t_2)}k(t_2, t)\\&=\frac{N(t_2-t)}{\rho(t_2-t)}\frac{\rho(t_2)}{N(t_2)}k(t_2, t).\end{aligned} \tag{5}$$

In fact, Eq. (5) gives a good approximation even if the energy spectrum of mesotrons, and consequently that of the decay electrons, undergoes an appreciable change. This is so because the *shape* of the shower curve depends only slightly on the energy of the particles which produce the shower. On the other hand, the *area* under the shower curve is correctly given by (5), according to Eqs. (3a) and (4).

The quantity $N(t_2-t)/\rho(t_2-t)$ can be developed in a Taylor series:

$$\frac{N(t_2-t)}{\rho(t_2-t)}=\left(\frac{N}{\rho}\right)_{t_2}-t\left[\frac{\partial(N/\rho)}{\partial t}\right]_{t_2}+\frac{1}{2}t^2\left[\frac{\partial^2(N/\rho)}{\partial t^2}\right]_{t_2}+\cdots \tag{6}$$

Substitution of (5) in (2) then gives

$$K(t_2)=\int_0^\infty k(t_2, t)dt-\frac{\rho(t_2)}{N(t_2)}\left[\frac{\partial(N/\rho)}{\partial t}\right]_{t_2}\int_0^\infty tk(t_2, t)dt+\frac{1}{2}\frac{\rho(t_2)}{N(t_2)}\left[\frac{\partial^2(N/\rho)}{\partial t^2}\right]_{t_2}\int_0^\infty t^2k(t_2, t)dt+\cdots \tag{7}$$

or, recalling (3a) and (4),

$$K(t_2)=\frac{1}{2}\frac{\mu}{\tau_0}X_0\left\{\left(\frac{N}{\rho}\right)_{t_2}-\bar{t}\left[\frac{\partial(N/\rho)}{\partial t}\right]_{t_2}+\frac{1}{2}\langle t^2\rangle_{\mathrm{Av}}\left[\frac{\partial^2(N/\rho)}{\partial t^2}\right]_{t_2}+\cdots\right\}, \quad (7a)$$

where $\bar{t}$, $\langle t^2\rangle_{\mathrm{Av}}$, $\cdots$, are the average values of t, t^2, etc.; i.e.,

$$\bar{t}=\frac{\int_0^\infty tk(t_2, t)dt}{\int_0^\infty k(t_2, t)dt},$$

$$\langle t^2\rangle_{\mathrm{Av}}=\frac{\int_0^\infty t^2k(t_2, t)dt}{\int_0^\infty k(t_2, t)dt}.$$

(7a) can be written also as follows:

$$K(t_2)=\frac{1}{2}\frac{\mu}{\tau_0}X_0\left\{\left(\frac{N}{\rho}\right)_{t_2-\bar{t}}+\tfrac{1}{2}[\langle t^2\rangle_{\mathrm{Av}}-(\bar{t})^2]\left[\frac{\partial^2(N/\rho)}{\partial t^2}\right]_{t_2}+\cdots\right\}. \quad (8)$$

Thus the problem is reduced to the determination of the quantities $\bar{t}$, $\langle t^2\rangle_{\mathrm{Av}}$, $\langle t^3\rangle_{\mathrm{Av}}$, etc.

4. EVALUATION OF $\bar{t}$ AND $[\langle t^2\rangle_{\mathrm{Av}}-(\bar{t})^2]$

We indicate by $k_E(t)$ the function describing the shower produced by a single incident electron of energy E. An expression for $k_E(t)$ has been given by Serber in the form of a complex integral, and can be written as follows:

$$k_E(t)=\frac{\epsilon}{2\pi i}\int_{\delta-i\infty}^{\delta+i\infty}\frac{ds}{s}\frac{\sigma_0+\lambda_1(s)}{\lambda_1(s)-\lambda_2(s)}\times K_1(s, -s)\left(\frac{E}{\epsilon}\right)^s\exp[\lambda_1(s)t]-\frac{\epsilon}{2\pi i}\int_{\delta-i\infty}^{\delta+i\infty}\frac{ds}{s}\frac{\sigma_0+\lambda_2(s)}{\lambda_1(s)-\lambda_2(s)}\times K_2(s, -s)\left(\frac{E}{\epsilon}\right)^s\exp[\lambda_2(s)t]. \quad (9)$$

The function $k_E(t)$ is equal to ϵ times the function $\Pi(E_0, 0, t)$ defined by Eq. (2.103) in the article "Cosmic-ray theory" by Rossi and Greisen.[10] ϵ is the critical energy, σ_0 is the absorption coefficient of high energy photons, s is a complex variable and $\lambda_1(s)$, $\lambda_2(s)$, $K_1(s, -s)$ and $K_2(s, -s)$ are the functions of s defined in (CRT) by Eqs. (2.19) and (2.76).

We consider the Laplace integral of $k_E(t)$ with respect to t; i.e., the function

$$L_E(\lambda)=\int_0^\infty k_E(t)e^{-\lambda t}dt. \quad (10)$$

For $E\gg\epsilon$, $L_E(\lambda)$ can be evaluated by the method of residues, and the result is [see CRT, Eqs. (2.106)]

$$L_E(\lambda)=-\frac{\epsilon}{s}\frac{K_1(s, -s)}{\lambda_1'(s)}\frac{\sigma_0+\lambda_1(s)}{\lambda_1(s)-\lambda_2(s)}\left(\frac{E}{\epsilon}\right)^s, \quad (11)$$

where s is defined as a function of λ by the equation

$$\lambda_1(s)=\lambda. \quad (12)$$

It follows immediately from the definition of the Laplace integral that

$$\int_0^\infty k_E(t)dt=L_E(0), \quad (13a)$$

$$\int_0^\infty tk_E(t)dt=-(\partial L_E/\partial\lambda)_{\lambda=0}, \quad (13b)$$

$$\int_0^\infty t^2k_E(t)dt=(\partial^2L_E/\partial\lambda^2)_{\lambda=0}, \quad (13c)$$

etc.

The function L_E and its derivatives can be calculated with Eq. (11). The result is [see CRT, Eqs. (2.106)]

$$\int_0^\infty k_E(t)dt=E, \quad (14a)$$

$$\int_0^\infty tk_E(t)dt=E(b\log E/\epsilon+a), \quad (14b)$$

$$\int_0^\infty t^2k_E(t)dt=E(b^2\log^2 E/\epsilon+d\log E/\epsilon+c), \quad (14c)$$

[10] B. Rossi and K. Greisen, Rev. Mod. Phys. **13**, 233 (1941). In what follows we shall refer to this article as (CRT).

where $b=-1/\lambda_1'(1)=1.01$, $a=0.40$, $d=2ab-\lambda_1''(1)/[\lambda_1'(1)]^3=2.42$, and $c=-0.1$. The primes denote differentiation with respect to s.

The approximations made in the deduction of the above formulae are only valid when E is large compared with ϵ; moreover the mathematical procedure becomes meaningless for $E<\epsilon$. Nevertheless, Eq. (14a) happens to be rigorously correct for every value of E because it expresses the obvious fact that the energy dissipated by all shower particles equals the energy of the electron which has produced the shower.

Let $k_p(t_1, t)dpdt_1$ be the function representing the shower produced by the decay of mesotrons with momentum (p, dp) in the layer (t_1, dt_1). According to Section 2, k_p is given by

$$k_p(t_1, t)=\frac{\mu}{\tau_0}\frac{X_0}{\rho(t_1)}\frac{f(t_1, p)}{p^2}\int_{E_{\min}}^{E_{\max}} k_E(t)dE, \quad (15)$$

Consequently,

$$\int_0^\infty k_p(t_1, t)dt=\frac{\mu}{\tau_0}\frac{X_0}{\rho(t_1)}\frac{f(t_1, p)}{p^2}\times\int_{E_{\min}}^{E_{\max}} dE\int_0^\infty k_E(t)dt, \quad (16a)$$

$$\int_0^\infty tk_p(t_1, t)dt=\frac{\mu}{\tau_0}\frac{X_0}{\rho(t_1)}\frac{f(t_1, p)}{p^2}\times\int_{E_{\min}}^{E_{\max}} dE\int_0^\infty tk_E(t)dt, \quad (16b)$$

$$\int_0^\infty t^2k_p(t_1, t)dt=\frac{\mu}{\tau_0}\frac{X_0}{\rho(t_1)}\frac{f(t_1, p)}{p^2}\times\int_{E_{\min}}^{E_{\max}} dE\int_0^\infty t^2k_E(t)dt. \quad (16c)$$

The expression (16a) can be evaluated from (14a) and yields, if one neglects terms of the order of $(\mu/p)^2$,

$$\int_0^\infty k_p(t_1, t)dt=\frac{1}{2}\frac{\mu}{\tau_0}\frac{X_0}{\rho(t_1)}f(t_1, p). \quad (17a)$$

In the expression (16b) one can use (14b) for $E>\epsilon$. On the other hand, since $\epsilon\ll p$ the integral with respect to E from $E_{\min}$ to ϵ is certainly negligible with respect to the integral from ϵ to $E_{\max}$, both because comparatively few electrons are emitted with energy smaller than ϵ and because these electrons have a very small range. Hence one can extend the integral from ϵ to $E_{\max}$ rather than from $E_{\min}$ to $E_{\max}$. The same conclusion applies to (16c). Neglecting terms of the order of $(\epsilon/p)^2$, one obtains

$$\int_0^\infty tk_p(t_1, t)dt=\frac{1}{2}\frac{\mu}{\tau_0}\frac{X_0}{\rho(t_1)}f(t_1, p)\times\left(B\log\frac{p}{\epsilon}+A\right), \quad (17b)$$

$$\int_0^\infty t^2k_p(t_1, t)dt=\frac{1}{2}\frac{\mu}{\tau_0}\frac{X_0}{\rho(t_1)}f(t_1, p)\times\left(B^2\log^2\frac{p}{\epsilon}+D\log\frac{p}{\epsilon}+C\right), \quad (17c)$$

where, if a, b, c and d are as given above, $A=a-\frac{1}{2}b=-0.10$, $B=b=1.01$, $C=c-\frac{1}{2}d+\frac{1}{2}b^2=-0.8$, and $D=2AB-\lambda_1''(1)/[\lambda_1'(1)]^3=1.40$.

The function $k(t_1, t)$ describing the shower produced by the decay of mesotrons of all momenta larger than p_0 in the layer (t_1, dt_1) is given by

$$k(t_1, t)=\int_{p_0}^\infty k_p(t_1, t)dp. \quad (18)$$

It follows that

$$\int_0^\infty k(t_1, t)dt=\frac{1}{2}\frac{\mu}{\tau_0}\frac{X_0}{\rho(t_1)}\int_{p_0}^\infty f(t_1, p)dp=\frac{1}{2}\frac{\mu}{\tau_0}\frac{X_0}{\rho(t_1)}N(t_1), \quad (19a)$$

$$\int_0^\infty tk(t_1, t)dt=\frac{1}{2}\frac{\mu}{\tau_0}\frac{X_0}{\rho(t_1)}\times\left[B\int_{p_0}^\infty f(t_1, p)\log\frac{p}{\epsilon}dp+AN(t_1)\right], \quad (19b)$$

$$\int_0^\infty t^2k(t_1, t)dt=\frac{1}{2}\frac{\mu}{\tau_0}\frac{X_0}{\rho(t_1)}\times\left[B^2\int_{p_0}^\infty f(t_1, p)\log^2\frac{p}{\epsilon}dp+D\int_{p_0}^\infty f(t_1, p)\log\frac{p}{\epsilon}dp+CN(t_1)\right]. \quad (19c)$$

Equation (19a) is identical with Eq. (3a). Equations (19b) and (19c) give, from the definitions of $\bar{t}$ and $\langle t^2\rangle_{\mathrm{Av}}$, and by inserting the values of the constants, the following results:

$$\bar{t}=1.01(\log p/\epsilon)_{\mathrm{Av}}-0.10, \tag{20}$$

$$\langle t^2\rangle_{\mathrm{Av}}-(\bar{t})^2=1.02\left[\left(\log^2\frac{p}{\epsilon}\right)_{\mathrm{Av}}-\left(\log\frac{p}{\epsilon}\right)^2_{\mathrm{Av}}\right]+1.61\left(\log\frac{p}{\epsilon}\right)_{\mathrm{Av}}-0.8, \tag{21}$$

where

$$(\log p/\epsilon)_{\mathrm{Av}}=\frac{\int_{p_0}^{\infty}f(t_1,p)\log\frac{p}{\epsilon}dp}{\int_{p_0}^{\infty}f(t_1,p)dp}$$

and

$$(\log^2 p/\epsilon)_{\mathrm{Av}}=\frac{\int_{p_0}^{\infty}f(t_1,p)\log^2\frac{p}{\epsilon}dp}{\int_{p_0}^{\infty}f(t_1,p)dp}$$

represent the average values of $\log p/\epsilon$ and $\log^2 p/\epsilon$ over the momentum spectrum of mesotrons from p_0 to infinity.

5. LOW ENERGY MESOTRONS

We have considered so far only mesotrons with momentum larger than p_0. The average energy of the electrons arising from the decay of mesotrons with momentum less than p_0 is small and we may assume that it is completely dissipated in a short distance from the place of production. Therefore the function $K_1(t_2)dt_2$ representing the energy dissipated in the layer (t_2, dt_2) by the shower arising from the decay of mesotrons with momentum smaller than p_0 (but different from zero) can be regarded as equal to the energy of the decay electrons produced in the same layer. This energy has already been calculated by one of us,[2] assuming that the absorption curve for mesotrons has a constant slope for thicknesses less than x_0, the range of mesotrons of momentum p_0. Since the whole term is very small compared with the contribution of the mesotrons with momentum above p_0, the error introduced by this assumption is negligible. The result is

$$K_1(t_2)=\frac{1}{2}\frac{\mu}{\tau_0}\frac{X_0}{\rho(t_2)}\frac{N_1(t_2)}{ax_0}\times[p_0-\mu\tan^{-1}(p_0/\mu)], \tag{22}$$

where a is the momentum loss per g/cm² of the absorber used for mesotrons of momentum p_0, and N_1 is the number of mesotrons stopped in the absorber; i.e., the number of mesotrons with momentum smaller than p_0.

The contribution of mesotrons which disintegrate after having been stopped by collision losses is small and moreover uncertain, because part of the mesotrons may be captured by the nuclei before disintegrating. Therefore this contribution will be neglected.

Should the mesotrons disintegrate each into an electron and a photon, rather than an electron and a neutrino, the intensity of the soft component would be approximately twice as large as that given by these calculations.

6. COMPARISON WITH EXPERIMENTS

We shall compare our theoretical results with some recent measurements by one of us[11] on the intensity of the hard and of the soft component as a function of altitude and of zenith angle. The measurements were performed by counting coincidences between Geiger-Müller tubes arranged in a straight line. For the measurements on the hard component the tubes were shielded by a total absorber thickness equivalent to 167 g/cm² of lead. In this condition the counting rate is a measure of the number N of mesotrons with momentum larger than about 3.0×10^8 ev/c ("fast" mesotrons). Measurements were taken at altitudes of 259, 1616, 3240 and 4300 m above sea level. The values of N in the vertical direction are plotted in Fig. 1 as solid dots against atmospheric depth in radiation lengths. The curve connecting the dots represents therefore the number of "fast" mesotrons as a function of depth. This curve has been extrapolated to small depths by means of the results of Schein, Jesse and Wollan.[12]

[11] K. Greisen, to be published shortly.
[12] M. Schein, W. P. Jesse and E. O. Wollan, Phys. Rev. **57**, 847 (1940).

The difference n between the counting rates with and without the absorber is taken as a measure of the intensity of the "soft" component. The values of n in the vertical direction are plotted as open dots in Fig. 1. The soft component includes mesotrons below 3.0×10^8 ev/c and electrons of sufficient energy to penetrate the counter walls; i.e., in the present experiments, 2.3 g/cm^2 of brass. The number of "slow" mesotrons has been indicated by N_1. Let us denote by D, D_1 and C the numbers of electrons arising from the decay of "fast" mesotrons, from the decay of "slow" mesotrons and from collision processes of mesotrons in air, respectively. If no electrons of different origin are present,

$$n=N_1+D+D_1+C. \qquad (23)$$

The various terms in Eq. (23) are evaluated as follows:

(*a*) *"Slow" mesotrons*, N_1. The values of N_1/N at 1616 m and at 3240 m have been obtained by extrapolating to zero thickness differential absorption curves measured recently with an anticoincidence method. The value of the same ratio at 259 m can easily be calculated from the absorption curve at Echo Lake or at Denver, by taking into account the decay, and the result obtained is in agreement with direct measurements. The value taken for N_1/N at 4300 m is a liberal estimate. The results are listed in Table I. No claim of accuracy is laid on the figures for N_1/N. However, it may be noted that an error in the evaluation of N_1 does not affect the final results seriously because N_1 itself is a small fraction of the total number of soft particles.

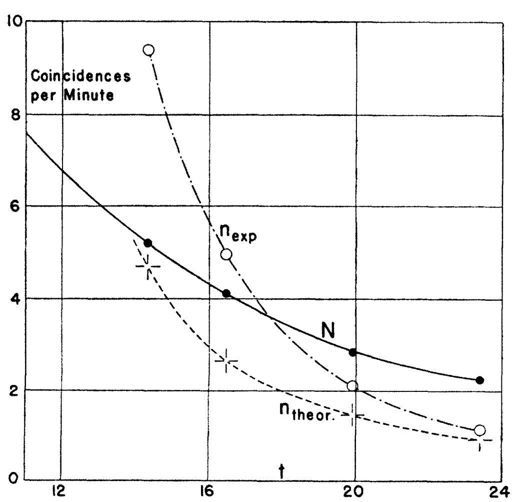

FIG. 1. The number of "fast" mesotrons N, the calculated number of "soft" particles n_{theor}, and the observed number of "soft" particles n_{exp}, as functions of depth in radiation lengths.

(*b*) *Electrons arising from the decay of "fast" mesotrons*, D. This number, which is the dominant term in Eq. (23), may be regarded as proportional to the quantity K given by Eq. (8). In the evaluation of K we may take $X_0=43$ g/cm^2, $\epsilon=10^8$ ev, $\tau_0/\mu=9\times10^{-4}$.[13] $N(t)$ is given by Fig. 1 and $\rho(t)=t\rho_0/t_0$, where ρ_0 and t_0 are the values of ρ and t at sea level. The quantities $\bar{t}$ and $\langle t^2\rangle_{\mathrm{Av}}-(\bar{t})^2$ can be calculated at sea level, where direct measurements of the momentum spectrum of mesotrons are available. Using Blackett's results[14] one obtains for $p>3\times10^8$ ev/c [see Eqs. (20) and (21)]: $(\log p/\epsilon)_{\mathrm{Av}}=3.11$, and $(\log p/\epsilon)_{\mathrm{Av}}=10.78$, hence $\bar{t}=3.04$ rad. lengths, $\langle t^2\rangle_{\mathrm{Av}}-(\bar{t})^2=5.3$ (rad. lengths)^2. No direct measurements of the momentum spectrum of mesotrons at high elevation have been reported so far. Probably the values of $\bar{t}$ and $\langle t^2\rangle_{\mathrm{Av}}-(\bar{t})^2$ decrease slightly with increasing altitude. If we neglect this decrease we slightly overestimate the increase of K with altitude, although the error is probably small. Numerical calculations show that already the second term in Eq. (8) is negligible with respect to the first one. Hence, in the vertical direction the expression for K reduces to

$$K(t)=4.44\times10^8 N(t-3.04)/(t-3.04). \qquad (24)$$

If the walls of the counters were infinitely thin, one would have $D=K/\epsilon$, since ϵ represents the energy loss of electrons per radiation length. Because of the finite thickness of the counter walls, electrons with energy below a certain limit are not recorded and D has a smaller value. We may write $D=\beta K/\epsilon$, with $\beta<1$. In the present case, the value of β probably lies between 0.8 and 0.9. Some values of D/N, calculated with $\beta=0.85$, are listed in Table I.[15]

[13] B. Rossi and D. B. Hall, Phys. Rev. **59**, 223 (1941). This value of τ_0/μ has been confirmed by recent experiments to be published shortly.

[14] P. M. S. Blackett, Proc. Roy. Soc. **159**, 1 (1937).

[15] It may be noted that the values of D thus calculated are considerably larger in absolute value and increase with height at a faster rate than the values calculated by previous, less accurate methods (see references 2 and 4).

TABLE I. Calculated and observed numbers of "soft" particles per "fast" mesotron at various altitudes.

Altitude (meters)	Depth (rad. lengths)	N_1/N	D/N	D_1/N	C/N	$\frac{N_1+D+D_1+C}{N}$	n/N observed
259	23.41	0.07	0.22	0.010	0.10	0.40	0.50±0.020
1616	19.93	0.09	0.31	0.014	0.10	0.51	0.74±0.053
3240	16.47	0.13	0.39	0.025	0.10	0.65	1.20±0.050
4300	14.33	0.27	0.47	0.060	0.10	0.90	1.81±0.058

(*c*) *Electrons arising from the decay of "slow" mesotrons*, D_1. We may take $D_1=\beta_1 K_1/\epsilon$, where K_1 is given by Eq. (22) and β_1 is a constant dependent on the thickness of the walls of the counters. β_1 is somewhat smaller than β because the average energy of electrons arising from the decay of "slow" mesotrons is smaller than that of electrons arising from the decay of "fast" mesotrons. Using $\beta_1=0.8$ we obtain for D_1/N the values listed in Table I.

(*d*) *Electrons produced by collision processes*, C. The number C can be regarded as proportional to the number N of fast mesotrons because low energy mesotrons cannot transfer any large fraction of their energy to electrons. The actual value of C depends again on the thickness of the counter walls. A rough estimate gives, for the present case, $C=0.1N$ (see, for instance, reference 5).

The above results yield for the total numbers of soft particles the values plotted in Fig. 1 as crosses. The ratios of these numbers to N are listed in Table I. In particular, at 4300 m the ratio between soft and hard particles is

$$(n/N)_{\text{theor}}=0.90\pm0.30,$$

where the error corresponds to a generous estimate of the maximum uncertainty of the various quantities which enter in the calculation. On the other hand, the observed value for n/N at the same altitude is

$$(n/N)_{\text{exp}}=1.81\pm0.06$$

and is therefore much larger than the calculated one. We conclude that *not all the electrons observed at 4300* m *can be accounted for by disintegration or collision processes of mesotrons.* It may be added that if the scattering of the electrons had been taken into account, the disagreement between the experimental and theoretical values of n/N would have been even more pronounced.

Inspection of Fig. 1 shows that not only the absolute values of the observed number n, but also the rate of variation of n with altitude differs strongly from what one would expect under the assumption that all electrons are secondaries of mesotrons. This conclusion is independent of any particular assumption about the values of the constants which enter into the calculations. It would also be valid if each mesotron decayed into an electron and a photon, instead of into an electron and a neutrino. In fact, the terms N_1, D_1 and C cannot account for more than a small fraction of the number n of soft particles observed at high altitude, for any reasonable assumption about the value of N_1. On the other hand, the rate of increase of the dominant term D with altitude depends only on $\bar{t}$ and on $N(t)$, since

$$\frac{D(t_1)}{D(t_2)}=\frac{N(t_1-3.04)}{N(t_2-3.04)}\times\frac{t_2-3.04}{t_1-3.04}.$$

In Table II we list the ratios of the values of N, n and D at various altitudes to the values at 259 m. It is seen that the calculated number of decay electrons increases with altitude more rapidly than the number N of fast mesotrons, but much less rapidly than the observed number n of soft particles.

The conclusion that not all electrons are secondaries of mesotrons is further strengthened by a consideration of the variation of the intensities with zenith angle. In Table III we list

TABLE II. Variation with height of the numbers of "fast" mesotrons (N), "soft" particles (n), and electrons arising from the decay of "fast" mesotrons (D).

Altitude (meters)	N/N_{259}	n/n_{259}	D/D_{259}
259	1.00	1.00	1.00
1616	1.26±0.026	1.86±0.15	1.73
3240	1.82±0.032	4.38±0.24	3.17
4300	2.29±0.037	8.27±0.41	4.83

TABLE III. Variation with zenith angle of the numbers of "fast" mesotrons (N), "soft" particles (n), and electrons arising from the decay of "fast" mesotrons (D).

Altitude (meters)	N°/N^{46}	n°/n^{46}	D°/D^{46}
259	2.10 ± 0.043	3.7 ± 0.67	2.4
1616	2.15 ± 0.062	2.7 ± 0.36	2.5
3240	2.10 ± 0.055	4.0 ± 0.47	2.4
4300	2.05 ± 0.048	3.2 ± 0.22	2.5

the ratios between the observed intensities at 0° and at 46° for the hard and soft components at various altitudes, along with the theoretical ratios for electrons arising from the decay of "fast" mesotrons. Here the soft component was measured with a permanent thickness of 15 g/cm² between the Geiger-Müller tubes so as to eliminate electrons of very small energy, which undergo considerable scattering. It is seen that the ratios for D are only slightly larger than the corresponding ratios for N and are considerably smaller than those for n.

CONCLUSION

The comparison of the theoretical results with experimental data, summarized in Fig. 1 and Tables I, II and III, indicates the existence of an electron component which increases more rapidly with altitude and has a sharper maximum in the vertical direction than the electrons arising from the decay or from collision processes of mesotrons. These electrons are probably the product of cascade processes of primary electrons. At a depth of about two-thirds of an atmosphere, they form a large fraction of the total electron intensity, while near sea level they seem to be not very abundant in comparison with the electrons produced by mesotrons (see Fig. 1). It is possible that at very high altitudes the decay electrons may again be responsible for most of the total electron intensity because the maximum in the number of decay electrons should occur at a higher altitude than the maximum in the number of electrons from primary cascade showers.

APPENDIX

The following relation between the energy of the decay electron and the angle at which it is emitted can be obtained from the conservation of energy and momentum, if one neglects the rest energy of the particles arising from the decay:

$$E=\tfrac{1}{2}\mu^2/[(p^2+\mu^2)^{\frac{1}{2}}-p\cos\theta] \tag{25}$$

where p is the momentum, μ the rest mass of the mesotron, E the energy of the decay electron and θ its angle of emission with respect to the path of the mesotron. It follows from (25) that the maximum and minimum energies of the decay electron are, respectively,

$$E_{\max}=\tfrac{1}{2}[(p^2+\mu^2)^{\frac{1}{2}}+p]$$
$$E_{\min}=\tfrac{1}{2}[(p^2+\mu^2)^{\frac{1}{2}}-p], \tag{26}$$

or, if $p^2\gg\mu^2$,

$$E_{\max}=p+\mu^2/4p$$
$$E_{\min}=\mu^2/4p. \tag{26'}$$

Equation (25) shows that no appreciable error is made by neglecting the finite angle of emission of the decay electrons, in the calculation of the soft component arising from the decay. In fact, the contribution to the soft component of decay electrons with energy smaller than μ is negligible. The angle of emission of electrons of energy μ, if $p\gg\mu$, is given by

$$1-\cos\theta=\mu/2p$$

and is therefore small.