This is in agreement with the findings of Altmann, Walker, and Hess[13] who have carried out an experiment similar to the one described here.

A final point may be brought out from the fact that the counting rate is practically the same at 12 as at 24 cm of lead. In consideration of the error in these readings, one can conclude that the maximum decrease in this counting rate could be only 3 to 4 percent, while the total decrease in the number of mesotrons brought about by 12 cm of lead is about 8 percent.[14] This difference may be explained by considering that only mesotrons of high energy are capable of producing knock-on showers. By considering the experimentally determined mesotron energy spectrum, Lovell[15] has calculated that the probability of observing knock-on showers is highest when the energy of the mesotron lies between 3 and 6×10^9 ev. If 5×10^9 ev is taken as the most probable mesotron energy for observing a knock-on shower with the present experimental arrangement, it can be shown that the energy spectrum above 5×10^9 ev is decreased by only 3 percent by 12 cm of lead. The slight decrease of this energy spectrum compared with the slight decrease in the counting rate indicates that only mesotrons above energies of the order of 5×10^9 ev produce knock-on showers.

In conclusion, the author wishes to thank especially Professor J. C. Stearns for his suggestion and direction of the experiment and also Professor D. K. Froman for helpful comments. To Professor Bruno Rossi and Mr. Kenneth Greisen of Cornell University the author is deeply grateful for valuable discussions on the interpretation of the results.

[14] B. Rossi and D. Hall, Phys. Rev. **59**, 223 (1941).
[15] A. C. B. Lovell, Proc. Roy. Soc. **A172**, 568 (1939).

FEBRUARY 1 AND 15, 1942 PHYSICAL REVIEW VOLUME 61

Frequency of Proton and Alpha-Tracks in Cosmic-Ray "Stars"

MAURICE M. SHAPIRO
Ryerson Physical Laboratory, University of Chicago, Chicago, Illinois
(Received December 4, 1941)

To obtain data concerning the multiple nuclear disintegrations produced by the cosmic radiation, special photographic plates were left on Mt. Evans for nearly eight months. From a statistical investigation of 365 tracks in 142 cosmic-ray "stars" found in one of these emulsions, it is estimated that more than 90 percent of the tracks were produced by protons. Most of the remaining star tracks are probably due to alpha-particles of energy less than 9 Mev.

INTRODUCTION

EVIDENCE of multiple nuclear disintegrations associated with the cosmic radiation has been provided by cloud chambers,[1] by photographic emulsions,[2] and indirectly by proportional counters.[3]

Of 20,500 counter-tripped cloud-chamber photographs of cosmic rays obtained by Brode and Starr[1a] at sea level, ten showed nuclear disintegrations produced by the cosmic radiation with the emission of heavy[4] particles. Even at an altitude of 4300 m[1b] only a few such disintegrations were observed in thousands of photographs. Because of the rare occurrence of these events, the continuously sensitive photographic emulsion has proved a useful tool in their investigation.

In the photographic emulsion a multiple nuclear disintegration produces a "fork," or group of microscopic tracks emanating from a common center. Each track consists of a row of discrete silver grains spaced several microns apart, the spacing being greater for protons than for alpha-

[1] (a) R. B. Brode and M. A. Starr, Phys. Rev. **53**, 3 (1938); (b) C. D. Anderson and S. H. Neddermeyer, Phys. Rev. **50**, 263 (1936); (c) J. Crussard and L. Leprince Ringuet, J. de phys. et rad. [7] **8**, 216 (1937).
[2] References 6–10 are to photographic emulsion studies of cosmic rays. Résumé and bibliography, M. M. Shapiro, Rev. Mod. Phys. **13**, 58 (1941).
[3] S. A. Korff, Phys. Rev. **59**, 949 (1941).
[4] The word "heavy" will be used here to describe particles at least as massive as protons.

particles of the same energy. Forks can also be produced by radioactive contamination of the plate, resulting in the step-wise disintegration of a single nucleus and its descendents, or in the emission of particles from several contiguous nuclei.[5] It is convenient to designate as "stars" those forks arising from multiple nuclear disintegrations associated with the cosmic radiation.

From studies of these stars by Blau and Wambacher,[6] Schopper and Schopper,[7] Wambacher,[8] Filippov, Jdanov, and Gurevich,[9] and Stetter and Wambacher,[10] there emerges the following picture of nuclear "evaporations." A nucleus of light or medium weight excited by the cosmic rays to multiple disintegration usually emits between two and five heavy charged particles, and infrequently as many as six to fourteen. (While neutrons are probably released in similar numbers, they do not produce visible tracks; nor do any electrons which may be emitted possess a sufficiently high specific ionization to leave tracks in the emulsion.) A large sample of star tracks shows random distribution in direction.

The energy and frequency of stars increase rapidly, and the multiplicity (average number of tracks per star) slightly, with altitude. Star frequency has an altitude dependence similar to those of large cosmic-ray showers, bursts, and neutrons. The energy of a typical star registered at a depth of 6 to 8 m water equivalent below the top of the atmosphere is of the order of 70 Mev. As for the mechanism whereby a star is produced, the available data are not inconsistent with the Bohr model of a nuclear excitation, according to which several particles may be evaporated in succession from an activated compound nucleus.[10a]

Blau and Wambacher,[6] and Stetter and Wambacher,[10] indicated that most star tracks can probably be ascribed to protons by comparison with recoil-proton tracks produced in collisions with neutrons. For the purpose of calculation they assumed that all the tracks were made by protons, recognizing, however, the possibility that some of them might in fact be due to alpha-particles. On the other hand, Schopper and Schopper,[7] and Filippov, Jdanov, and Gurevich,[9] attributed certain of their star tracks to alpha-particles, basing this identification on measurements of grain spacing and track length.

The average grain spacing of alpha-ray and proton tracks serves as a means of distinguishing between them, provided that the alpha-particles in question have energies which do not exceed those in the natural radioactive region. This was demonstrated by Wilkins and St. Helens,[11] who registered alpha-particles from ThC and ThC′, and protons and deuterons from cyclotrons, on separate plates. They measured about 260 tracks of each type, and plotted the number of grains against the length of each track. Their distributions of alpha- and proton tracks are shown in Figs. 3 and 5 of reference 11. These show a fairly clear separation.

We have here applied the grain-spacing technique in an effort to compare statistically the occurrence of alpha-particles and protons in cosmic-ray stars.

MEASUREMENTS

To obtain sufficient star data, several kinds of plates were sent into the stratosphere in a number of unmanned balloon flights,[12] and others were exposed for many months at various mountain altitudes. Because of their relatively short duration, the stratosphere flights did not yield a sufficient concentration of data on any single plate to render its inspection worth while for our purpose. An Agfa "K-plate," which had been left vertically oriented for 239 days at an altitude of 4300 m,[13] was selected for this investigation. The emulsion, 43 microns thick, was developed for 5 minutes in a fine-grain

[5] For photomicrographs of such "forks," see references 7–9; or Shapiro, reference 2, p. 60.

[6] M. Blau and H. Wambacher, Akad. Wiss. Wien [IIa] **146**, 469 and 623 (1937).

[7] E. Schopper and E. M. Schopper, Physik. Zeits. **40**, 22 (1939).

[8] H. Wambacher, Physik. Zeits. **39**, 883 (1938).

[9] A. Filippov, A. Jdanov, and I. Gurevich, J. Phys. U.S.S.R. **1**, 51 (1939).

[10] G. Stetter and H. Wambacher, Physik. Zeits. **40**, 702 (1939).

[10a] N. Bohr and F. Kalckar, K. Danske Vid. Sels., Math.-fys. Med. **14**, No. 10 (1937).

[11] T. R. Wilkins and H. J. St. Helens, Phys. Rev. **54**, 783 (1938).

[12] These flights were conducted by Dr. W. P. Jesse, whose generous cooperation is acknowledged.

[13] Professor J. C. Stearns, of the University of Denver, very kindly arranged for storage of plates in the cosmic-ray station at Mt. Evans.

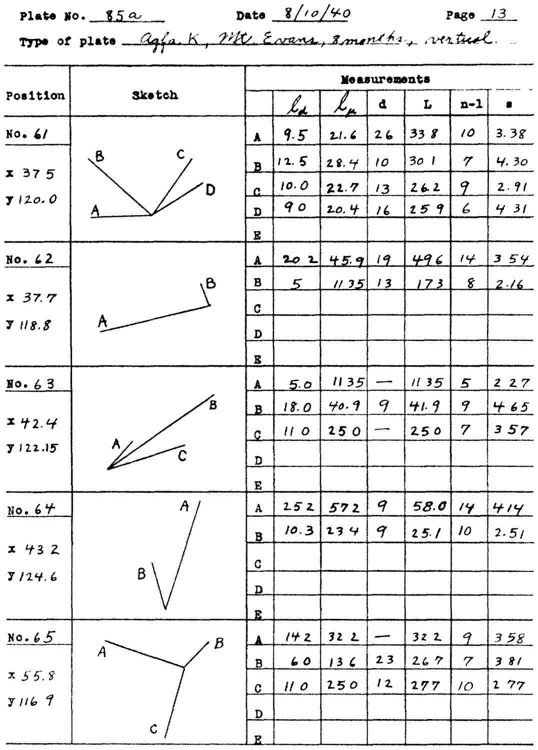

Plate No. 85a Date 8/10/40 Page 13

Type of plate Agfa K, Mt. Evans, 8 months, vertical.

Position	Sketch		Measurements					
			l_d	l_μ	d	L	n-1	s
No. 61		A	9.5	21.6	26	33 8	10	3.38
x 37 5		B	12.5	28.4	10	30 1	7	4.30
y 120.0		C	10.0	22.7	13	26.2	9	2.91
		D	9 0	20.4	16	25 9	6	4 31
		E						
No. 62		A	20 2	45.9	19	49 6	14	3 54
x 37.7		B	5	11 35	13	17 3	8	2.16
y 118.8		C						
		D						
		E						
No. 63		A	5.0	11 35	—	11 35	5	2 27
x 42.4		B	18.0	40.9	9	41.9	9	4 65
y 122.15		C	11 0	25 0	—	25 0	7	3 57
		D						
		E						
No. 64		A	25 2	57 2	9	58.0	14	4 14
x 43 2		B	10.3	23 4	9	25.1	10	2.51
y 124.6		C						
		D						
		E						
No. 65		A	14 2	32 2	—	32 2	9	3 58
x 55.8		B	6 0	13 6	23	26 7	7	3 81
y 116 9		C	11 0	25 0	12	27 7	10	2 77
		D						
		E						

FIG. 1. A page of representative data. l_d is the projection of the track-length on the surface of the emulsion, measured in divisions of the eyepiece scale. l_μ denotes the same quantity in microns. (1 division $=2.27\mu$.) d, L, and s denote, respectively, the depth, length, and mean grain spacing of the track in microns. n is the number of grains in the track.

developing solution; and examined under a binocular microscope[14] equipped with mechanical stage, eyepiece scale, objective with variable iris diaphragm, and dark-field condenser.

Many single tracks, as well as groups of tracks radiating from a common center were observed. In order to exclude from the data forks due to radioactive contamination, only those were studied which contained at least one track attributable to either an alpha of energy >9 Mev or to a proton. In practice this was accomplished by considering as cosmic-ray stars only those forks which have at least one track with a mean grain spacing ≥ 2.4 microns. (The choice of this minimum value is explained in footnote 18.) If only one track in a fork conformed to this description, it was further required that this track include at least five silver grains, so that the determination of grain spacing governing the identification of the fork as a star be based upon at least four spacings. A track with equivalent air range greater than 10 cm would have qualified its fork as a star even if it possessed alpha-spacing. No track with such length and spacing occurred, however. It should be noted that the application of these criteria restricts the generality of the conclusions to be drawn from this study; for it is quite possible that some of the forks containing *only* alpha-tracks of energies within the radioactive range are also of cosmic-ray origin.

Of 200 stars satisfying these requirements, 58 were rejected from this study because one or more of their tracks could not be measured accurately. In such cases either a part of the track was obscured by background silver grains, or else the track sloped too steeply into the emulsion to permit an accurate count of the number of grains in it. For the remaining stars, the following criterion was used to prevent the inclusion of fortuitous collineations of silver grains as tracks. A row of grains emanating from a star center was counted as a track only if it comprised at least four grains.

For each track of the 142 stars constituting the data, the following measurements were made at a magnification of 750 diameters. The projection l of the length upon the surface of the emulsion was measured with an eyepiece scale. The depth d was determined from scale readings on the fine-adjustment head of the microscope when the two ends of a track were successively brought into sharp focus. The refractive index (1.5) of the emulsion was taken into account. During such focusing the iris diaphragm of the objective was opened wide for minimum depth of field.

From l and d the actual length L of the track was deduced; and by counting the number n of silver grains in it, the average spacing $s=L/(n-1)$ between grains was determined. Many of the L values corresponded to incomplete ranges, the tracks in such instances running out of the emulsion. Along with these measurements, the coordinates (x, y) of each star, given by the mechanical stage of the microscope, were

[14] For the loan of this microscope, the writer is indebted to Professor W. H. Taliaferro, Dean of the Division of Biological Sciences, University of Chicago.

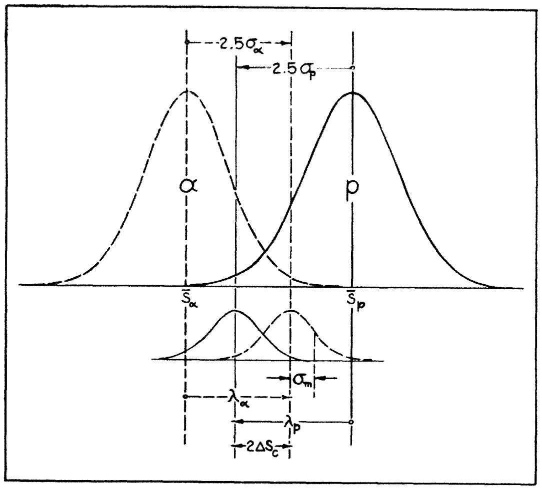

FIG. 2. Diagram illustrating choice of Δs_c. Track frequency is plotted against average grain-spacing. In the comparison data the alpha-group extends to an upper limit $\bar{s}_\alpha+2.5\sigma_\alpha$, and the proton group to a lower limit $\bar{s}_p-2.5\sigma_p$. Because of our error in measurement, σ_m, the upper limit is increased to $\bar{s}_\alpha+\lambda_\alpha$, and the lower one decreased to $\bar{s}_p-\lambda_p$, where $\lambda_\alpha=[(2.5\sigma_\alpha)^2+\sigma_m^2]^{\frac{1}{2}}$ and $\lambda_p=[(2.5\sigma_p)^2+\sigma_m^2]^{\frac{1}{2}}$. The extent of overlapping of λ_α and λ_p determines the interval of uncertainty $2\Delta s_c$.

recorded together with a sketch of the star to insure that it would not be counted twice. Figure 1 shows a representative page of data.

RESULTS

The 142 stars studied, comprising between two and five tracks each, contained a total of 365 tracks, with s values ranging from 1.36 to 5.52 microns. An estimate of the relative numbers of proton and alpha-tracks occurring in the stars can be obtained by deducing a critical value s_c of s such that tracks with $s>s_c$ may be attributed to protons, and those with $s<s_c$ to alpha-particles. In a tabular summary, Wilkins and St. Helens[11] give an average spacing 1.74μ for the group of alphas with greatest s values, and an average 2.68μ for the group of protons with shortest spacing. It would appear easy, then—if we are dealing with alphas which are not more energetic than those from ThC′—to separate alphas from protons. It must be remembered, however, that these s values are *averages* for many tracks. Actually, the mean spacings for proton and alpha-tracks overlap because of inhomogeneities in the distribution of AgBr grains in the emulsion and fluctuations due to errors in measurement.[15]

This overlapping gives a band of s values lying within an interval $s_c\pm\Delta s_c$ such that tracks with s values in this interval cannot be identified on the basis of spacing alone. The half-width Δs_c of this interval determines the error in the estimate of the proportion of alpha-particles and protons. To evaluate Δs_c for the population of cosmic-ray tracks, it is necessary to consider (a) the "spread" of s values in the most energetic alpha-group and that in the least energetic proton group of the comparison data (of Wilkins and St. Helens[16]), and (b) the additional spread arising from experimental error in our measurements of the cosmic-ray tracks. (See Fig. 2.)

(a) Let D_α be the total spread (difference between the largest and smallest s values) for the most energetic alpha-group, and D_p the spread for the least energetic proton group. Denote the standard deviations of these groups by σ_α and σ_p; and the mean s values by $\bar{s}_\alpha$ and $\bar{s}_p$, respectively. To a sufficiently close approximation, the standard deviations are $\sigma_\alpha=D_\alpha/6$ and $\sigma_p=D_p/6$. From the data in Figs. 3 and 5 of reference 11,

$$\bar{s}_\alpha=1.70\mu \qquad \bar{s}_p=2.40\mu$$
$$\sigma_\alpha=0.17\mu \qquad \sigma_p=0.20\mu.$$

Now, for the alpha-group, $\bar{s}_\alpha+2.5\sigma_\alpha$ may be taken[17] as an upper limit for the range of s values; similarly, $\bar{s}_p-2.5\sigma_p$ is a reasonable lower limit for the proton group in the comparison data.

(b) If we take into account the standard error $\sigma_m=0.1\mu$,[18] in our measurement of s for the

[15] The overlapping is much more serious if the population of tracks in question includes alphas with energies exceeding 9 Mev. This case will be discussed separately below.

[16] These authors do not give explicitly the standard deviations in the spacings within these groups of tracks. This information was obtained from the frequency distributions shown in their graphs (cf. Figs. 3 and 5 of their paper).

[17] For a normal frequency distribution with standard deviation σ, approximately 99 percent of the total area is included between the ordinates at $\pm2.5\sigma$.

[18] Taken as one-third the estimated maximum error in measurement of s. σ_m is combined with $2.5\sigma_\alpha$ as follows: $\lambda_\alpha\equiv[(2.5\sigma_\alpha)^2+\sigma_m^2]^{\frac{1}{2}}=0.45\mu$ (see Fig. 2).

We may now explain the choice of $s=2.4\mu$ for at least one track in a fork as a minimum value for the latter's admission to the star data: $\bar{s}_\alpha+2.5\sigma_\alpha+2.5\sigma_m=2.4\mu$. Thus the probability that a track with $s\geq2.4\mu$ was produced by an alpha of radioactive origin is vanishingly small. The application of this cautious criterion virtually eliminates error due to radioactive contamination.

cosmic-ray tracks, we arrive at the following limits:

Upper limit for the α-group: $\bar{s}_\alpha + 0.45\mu = 2.15\mu$
Lower limit for the p group: $\bar{s}_p - 0.50\mu = 1.90\mu$

i.e., the "interval of uncertainty" within which tracks cannot be identified with confidence is given by $1.9 < s < 2.15\mu$. We may put $s_c = 2.02\mu$ and $\Delta s_c = 0.12\mu$.

The star tracks may now be classified as in Table I.

One-half of the number of tracks in this borderline group lies within the half-interval Δs_c. Thus in our sample population of star tracks, the percentage of protons in stars containing at least one proton is 93.7 ± 3.2 percent. The standard error 3.2 percent in the relative frequency is increased only to 3.4 percent if we take into account the error due to sampling, which is estimated as 1.3 percent.[19]

In Table II the stars are divided into two groups—those having at least one alpha-track, and those containing only protons—and these in turn are classified according to track multiplicity.

DISCUSSION

Our statistical separation of protons from alphas based on the criterion of grain spacing is valid for a population of tracks in which the alpha-energies do not exceed 9 Mev. It was shown by Schopper and Schopper[7] that to alpha-energies greater than this there correspond s values higher than those in the less energetic group of alphas from natural radioactive substances. These higher values are in fact as large as those of less energetic protons. Thus to each s value of an energetic[20] alpha there corresponds a proton with the same grain spacing. If, then, an unknown population of tracks contain an appreciable number of energetic alphas, it becomes necessary to know the complete range r of a particle, as well as its grain spacing. From these data and Schopper's curves plotting grain spacing against length for protons and alphas, one can identify the track.

Many of the tracks in our experiment, however, are incomplete, either because the emulsion was not sufficiently thick to record their entire length, or because they originated too close to one of its surfaces. It might seem arbitrary to classify such tracks as protons. It can be shown, however, that in the plate investigated the number of energetic alpha-tracks, if any, was negligible. In the first place, no clear evidence for such alphas was found on the plate. Very few tracks with air-equivalent lengths greater than 10 cm were observed, and of these, those which lay completely in the emulsion could be identified as protons by reference to Schopper's curves. There was not a single complete track with $l > 7$ cm air equivalent which fitted on Schopper's alpha-curve. It remains to be shown, however, that only a negligible number of the incomplete tracks with $s > 2.15\mu$ can be due to alphas.

Consider the probability $P(l)$ that a track with range r which originates in an emulsion of thickness k has a length l in the emulsion between l_1 and l_2. From geometrical considerations it can be shown that when $r > k$, and l ranges over the intervals indicated below, then for a population of tracks with isotropic[20a] distribution, $P(l)$ has the following values:

$$k \leq l_1 \leq l \leq r \qquad P(l_1 \leq l \leq r) = k/2l_1$$

$$l < r \qquad P(l_1 \leq l \leq l_2) = 1 - \frac{k}{2l_2} - \frac{l_1}{2k}.$$

TABLE I.

	Number of tracks	Percentage
$s > 2.02\mu$	342	93.7
$s \leq 2.02\mu$	23	6.3
Total	365	100.0
$1.90 < s < 2.15$	23	6.3

TABLE II. Numbers of "alpha-stars" (those containing at least one alpha) and "proton stars" (those with protons only).

Multiplicity	p stars	α-stars	Totals
2-fold	63	10	73
3-fold	50	9	59
4-fold	7	1	8
5-fold	1	1	2
Totals	121	21	142

TABLE III. Expected number of long tracks for several values of f.

f	1	0.5	0.1	0.05
$0.455fN$	86	43	9	4

[19] This is combined with the error 3.2 percent by computing the square root of the sum of their squares.

[20] We shall use "energetic" to describe alphas with energy exceeding 9 Mev.

[20a] The directional distribution of star tracks is very nearly isotropic, according to Stetter and Wambacher, reference 10.

Suppose that in the stars investigated all tracks whose spacing satisfies the relation $2.15 < s < 4.0\mu$ were alphas. From Schopper's alpha-curve it is found that for all such tracks $r \gtrsim 140\mu$. Compare the probability P_a that such a track have an emulsion length $\gtrsim 140\mu$ with the probability P_b that such a track have a length between 20μ and 50μ. For the emulsion used here ($k = 43\mu$),

$$\frac{P_a(140 \leq l \leq r)\mu}{P_b(20 \leq l \leq 50)\mu} = 0.455.$$

Thus, if in a given plate area we find a group G of N tracks with l satisfying $20 \leq l \leq 50\mu$ and s in the interval $2.15 < s < 4.0\mu$, then we should expect to find $0.455N$ tracks in the same spacing interval with $l \gtrsim 140\mu$, provided that all the N tracks are alphas. If only a fraction f of the tracks are alphas, we should expect $0.455fN$ tracks with $2.15 < s < 4.0\mu$ and $l \gtrsim 140\mu$. The experimental data give $N = 190$ tracks between 20 and 50μ. Table III shows the expected number of long tracks for several values of f. Actually, *no* tracks were observed satisfying $2.15 < s < 4.0\mu$ and $l \gtrsim 140\mu$. It is thus improbable that an appreciable number of the tracks in group G are due to alphas.

If we assume, on the other hand, that group G consists of protons, then from Schopper's proton curve we find that the maximum range for such tracks is 140μ, and we should expect to find *no* tracks within the s interval of G longer than 140μ. This agrees with the observations. A similar investigation for the tracks with $s > 4.0\mu$ leads to the same conclusion: there is very little evidence for the presence of energetic alphas in the plate studied.

We should expect—and this has been confirmed[11] in the case of deuterons—that the grain spacings of H^2, H^3, or He^3 particles will have values lying between those of alphas and protons. It seems possible, therefore, that a few of our star tracks were produced by particles of mass intermediate between those of proton and alpha-particle. However, the emission of an H^2, H^3, or He^3 is improbable compared with the competitive processes of proton or alpha-emission, since the binding energies of the former—especially in nuclei of medium weight—are large compared with those of the latter. We cannot exclude the possibility that protons of double charge[21] are responsible for some of the tracks with alpha-spacing. Taylor, Fraser, and Dabholkar[22] have concluded that the detection of doubly charged protons with the photographic method is not possible.

Finally, we might inquire whether any of the dense tracks identified as alphas are really due to heavier nuclei. This is possible; but even the smallest average grain spacings in our group of alphas lie within the maximum spread of s values in the comparison group of alphas. Hence it is not necessary to ascribe any of our star tracks to recoil nuclei.

An investigation of the nature of the star-producing radiation is now in progress, and will be reported in another paper.

ACKNOWLEDGMENTS

The author is grateful to Dr. M. Schein, Dr. H. W. Norton, Mr. G. W. Brier, and other members of the staff at Ryerson Laboratory for helpful suggestions. He is particularly indebted to Professors Arthur H. Compton and George S. Monk for their inspiring counsel and generous interest.

---

[21] H. J. Bhabha, Phys. Rev. **59**, 100 (1941).
[22] H. J. Taylor, D. Fraser, and V. D. Dabholkar, Nature **146**, 777 (1940).