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On the Precision Measurement of Nuclear Magnetic Moments by the Molecular Beam Magnetic Resonance Method

The Moments of H^1, Li^7, F^{19}, and Na^{23} ‡

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A direct comparison of nuclear magnetic moments with the electronic moment is made. This is done by the measurement of the Larmor precession frequency of the nucleus and the frequencies of some of the lines of the radiofrequency spectra of the alkali atoms in the same magnetic field. In this way the magnetic moment of the proton is found to be 2.7896 ± 0.0008 nuclear magnetons. The precision of this value is very much greater than that which has heretofore been obtained by measurement of magnetic field by usual methods. The same methods are used in a redetermination of the magnetic moments of Li^7, F^{19} and Na^{23}. Since the nuclear gyromagnetic ratio of Li^7 has previously served as a standard in terms of which other g values were expressed, an improvement in the precision of this g value makes it possible to restate, with improved precision, the g values and magnetic moments of other nuclei.

INTRODUCTION

THE gyromagnetic ratios of a number of nuclei have been determined by the molecular beam magnetic resonance method.[1] The limitations on the precision of these measurements arose, for the most part, from uncertainties in the determination of magnetic fields by the usual methods, involving the use of flip coils and standards of mutual inductance. It was thus found difficult to determine, with the apparatus at hand, any nuclear g with a precision better than 0.5 percent. The relative values of many nuclear moments were much more precisely determined since such determinations could be made by measuring two frequencies at a fixed field.

Recent work[2, 3] on the radiofrequency spectra of the alkali atoms suggested the possibility of making a direct comparison between nuclear and electronic magnetic moments. In the experiments to be described, such a comparison was made by utilizing the field dependence of some of the lines in the radiofrequency spectra for the determination of the magnetic field in which the precession frequencies of nuclear magnetic moments were observed. This determination of magnetic field could be made with considerable precision and led to a greatly improved precision in the values of the nuclear magnetic moments of H^1, Li^7, F^{19} and Na^{23}. The improvement in precision of the proton moment is particularly desirable for the purpose of comparing the neutron moment obtained from the difference of the deuteron and proton moments with the value measured for free neutrons by the method of Alvarez and Bloch.[4] An improvement in the accuracy of the g value of Li^7 automatically improves the precision of the g values of many other nuclei, since all gyromagnetic ratios determined by the resonance method have heretofore been referred to the g value of Li^7 as a standard.

METHOD

The hyperfine separations of the ground states of the alkali atoms have been previously reported. These $\Delta\nu$'s were determined either by an observation of the Zeeman lines in very low magnetic field, or by the study of lines in the

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[1] I. I. Rabi, S. Millman, P. Kusch, and J. R. Zacharias, Phys. Rev. **55**, 526 (1939).

[2] P. Kusch, S. Millman, and I. I. Rabi, Phys. Rev. **57**, 765 (1940).

[3] S. Millman and P. Kusch, Phys. Rev. **58**, 438 (1940).

[4] L. W. Alvarez and F. Bloch, Phys. Rev. **57**, 111 (1940).

intermediate and Paschen-Back regions of field. In the cases of Li^7 and K^{39} both methods were used. In each of these experiments the choice of line or lines used in the determination of $\Delta\nu$ was such that the calculated values of $\Delta\nu$ did not depend on a precise knowledge of the magnetic fields at which the lines were observed. However, for purposes of calibrating a magnetic field, lines which have frequencies markedly dependent on magnetic field are used. If the frequency of such a line is measured and if the $\Delta\nu$ of the ground state is known, the magnetic field can at once be calculated by use of the quantum-mechanical expressions which give the energy of an alkali atom in a magnetic field. These expressions are given in Eqs. (9–12) of an earlier paper.[2] The lines chosen in our experiments are those represented by the transition $(2, -2)\leftrightarrow(1, -1)$ of Na^{23}, $(3, -3)\leftrightarrow(2, -2)$ of Rb^{85} and $(4, -2)\leftrightarrow(4, -1)$ of Cs^{133}. The notation is the same as that previously employed; the first symbol in the parenthesis indicates the F value of a state and the second symbol the m value.

In Figs. 1, 2 and 3 are plotted the frequencies of these lines as a function of field. The particular region of field employed in each of these cases covered the highest fields compatible with a marked dependence of the frequency of the line on field and with readily obtainable oscillator frequencies. The use of high magnetic fields is, of course, important since the error in locating the minimum of a nuclear resonance curve, obtained by varying frequency for a fixed field, decreases with increasing field. This arises from the fact that the width of the nuclear resonance curve is independent of the magnetic field. Li^7 and K^{39} are not suitable calibrating atoms for the present purposes, because no lines in the radiofrequency spectra of these two atoms show a marked dependence on field at values of the magnetic field above 1000 gauss and for frequencies not exceeding 1100 megacycles.

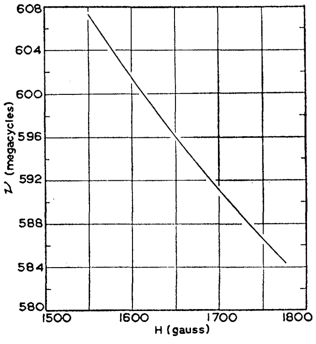

FIG. 1. The frequency of the line $(2, -2) \leftrightarrow (1, -1)$ of Na^{23} as a function of the magnetic field. $\Delta\nu(Na^{23}) = 1771.75\times10^6$ sec.$^{-1}$ has been assumed in the construction of the curve.

The equations which describe the curves merely give the ratio of the frequency of the line ν to $\Delta\nu$, in terms of the parameter x, which is related to the field by the expression:

$$H=\frac{4\pi\Delta\nu}{(e/mc)(g_j-g_i)}x.$$

Clearly, the curves in Figs. 1, 2 and 3 are constructed for assumed values of $\Delta\nu$ and g_i. We have used for the values of $\Delta\nu$ the following: $\Delta\nu(Na^{23}) = 1771.75\times10^6$ sec.$^{-1}$; $\Delta\nu(Rb^{85}) = 3035.7\times10^6$ sec.$^{-1}$; $\Delta\nu(Cs^{133}) = 9192.6\times10^6$ sec.$^{-1}$. The $\Delta\nu$ of Rb^{85} has been redetermined for the purposes of this work and the value found to be as previously reported[3] but with the somewhat better precision of 0.005 percent. We have also redetermined the $\Delta\nu$ of Cs^{133} and find it to be as above instead of 9191.4×10^6 sec.$^{-1}$ as previously reported.[3] The present value is undoubtedly better and has a precision of about 0.005 percent.

Any uncertainty in the value of $\Delta\nu$ will, of course, be reflected in a corresponding uncertainty in the field calculated from the line frequency. The percent error of the value of the parameter x, determined from an observation of the frequency ν of the atomic line, is given by the percent error in $\Delta\nu$ multiplied by the percent change in x for unit percent change in the ratio $\nu/\Delta\nu$. It follows from Figs. 1, 2 and 3 that an increase of 0.1 percent in the assumed values of the respective $\Delta\nu$'s will result in an increase in the x value, calculated from a given frequency of the atomic line, of 0.38 percent in the case of Na^{23}, 0.2 percent in the case of Rb^{85} and a decrease of 0.22 percent in the x value associated with a line of given frequency of Cs^{133}. Since the value of $\Delta\nu$ also enters linearly into the expression which converts x into H, an increase in the assumed value of $\Delta\nu$ by 0.1 percent will result in an additional increase in H by 0.1 percent in all cases. It follows, therefore, that an error in $\Delta\nu$ is reflected by an error in H which is 4.8 times as large for the case of Na^{23}, 3.0 times as large for the case of Rb^{85} and only 1.2 times as large for the case of Cs^{133}. Since all the $\Delta\nu$'s are known to

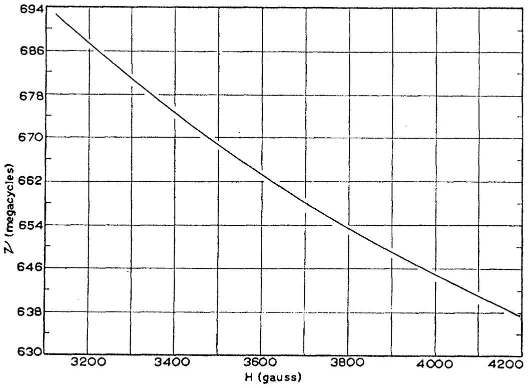

FIG. 2. The frequency of the line (3, −3) ⟷ (2, −2) of Rb^{85} as a function of the magnetic field. $\Delta\nu(Rb^{85}) = 3035.7 \times 10^6$ sec.$^{-1}$ has been assumed in the construction of the curve.

within 0.005 percent, the error in field due to uncertainty in $\Delta\nu$ alone will be about 0.024 percent in the Na^{23} calibration, 0.015 percent in the Rb^{85} calibration and 0.006 percent in the Cs^{133} calibration.

The values of g_i of the calibrating atoms enter both into the calculation of the ratio $\nu/\Delta\nu$ as a function of x and into the conversion from x to H. We have used for these g's the values listed in Table III. The contribution of terms involving g_i is so small that the indicated uncertainties in g_i introduce uncertainties in the field of only 0.001 percent in the case of Na^{23}, 0.002 percent in the case of Rb^{85} and 0.001 percent in the case of Cs^{133}. These uncertainties, therefore, do not affect the precision of our results.

In addition to the errors introduced by uncertainties in the values of $\Delta\nu$, the calculated values of the field will be subject to observational errors of the frequencies ν of the atomic lines. For any given atomic line the percent error in the field will be equal to the percent error in the frequency of the line multiplied by the percent change in x for unit percent change in $\nu/\Delta\nu$ as given earlier. Since the frequencies of lines may be determined to within 0.005 percent, the error arising from this factor should not exceed 0.02 percent for Na^{23}, 0.01 percent for Rb^{85} and 0.01 percent for Cs^{133}. The net precision within which the atomic lines may be used to determine the value of a magnetic field is about 0.035 percent for Na^{23}, 0.02 percent for Rb^{85} and 0.015 percent for Cs^{133}.

The determination of a field value from the observed frequency of an atomic line also depends on the value used for the constant e/mc. We have used for this quantity[5] the value 1.7591×10^7 e.m.u. per gram.

The value of any nuclear g is found by the expression:

$$g = \frac{4\pi}{(e/Mc)} \times \frac{f}{H},$$

where f is the frequency at resonance of the nucleus in a magnetic field H. The best way of obtaining the constant e/Mc is by use of the faraday. For this quantity we have used the most recent value of Birge,[5] 9650.6 e.m.u. per gram equivalent, on the physical scale. On the same scale the atomic weight of H^1 is 1.00813. After correction for the mass of the electron, the quantity e/Mc becomes 9578.0 e.m.u. per gram. The above expression then becomes,

$$g = 1.31200 \times 10^{-3} f/H.$$

The nuclear g value is, of course, the ratio of the nuclear magnetic moment expressed in units of the nuclear magneton, $eh/(4\pi Mc)$, and the angular momentum in units of $h/2\pi$. If we use for h the most recent value given by Birge,[5] 6.6237×10^{-27} erg sec., the nuclear magneton, in terms of which our magnetic moments are given, is 5.049×10^{-24} e.m.u. It is to be noted that the constant h does not enter into our calculations as long as magnetic moments are expressed in terms

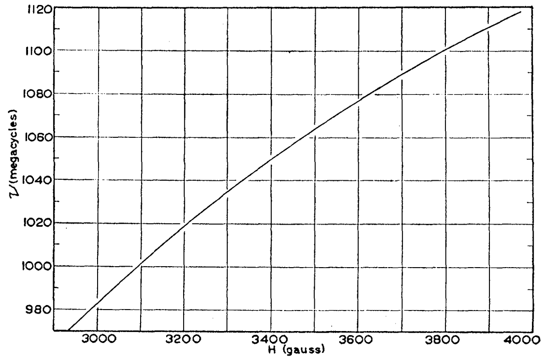

FIG. 3. The frequency of the line (4, −2) ⟷ (4, −1) of Cs^{133} as a function of the magnetic field. $\Delta\nu(Cs^{133}) = 9192.6 \times 10^6$ sec.$^{-1}$ has been assumed in the construction of the curve.

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[5] For the fundamental constants used in this paper we have used the values given by R. T. Birge in a privately circulated tabulation of August, 1939.

of the nuclear magneton; it is only when these moments are expressed in absolute units that the quantity *h* enters.

The nuclear *g* values which we find depend directly on the ratio of the quantities *e*/*mc* and *e*/*Mc* and therefore on *M*/*m*. For the values of *e*/*mc* and *e*/*Mc* that we have used, the ratio *M*/*m* is equal to 1836.6. Any increase in this quantity will result in a proportional increase in the *g* values reported in this paper.

Finally it is assumed in all our calculations that the magnitude of the electronic gyromagnetic ratio for an alkali atom in the ground state, g_J, is exactly 2. Dr. M. Phillips[6] discusses the validity of this assumption in a paper in this journal.

DETERMINATION OF THE GYROMAGNETIC RATIOS OF Li^7, F^{19} AND Na^{23}

Even though the value of a magnetic field may be determined within very narrow limits, as pointed out in the previous section, the use of such field values for the purpose of accurate determination of nuclear gyromagnetic ratios presents considerable difficulties. We have used two distinct methods of utilizing a field calibration in the determination of *g* values. The first method has been applied to the cases of Li^7, F^{19} and Na^{23} and the second method to the case of H^1.

For the purposes of determining the nuclear gyromagnetic ratios of Li^7 and F^{19}, runs were made by loading the oven with two substances, one of which produced the atomic portion of the beam and the other of which produced the molecular portion. Since a beam of the calibrating atom was produced at a lower temperature than the molecular beam, it was necessary to make the calibration of the field before the observation of any nuclear resonance curves. In the case of Na^{23}, a molecular beam of Na_2 accompanies a beam of atomic Na, so that calibration of field and observation of nuclear resonance curves may be accomplished at the same oven temperature.

In order that nuclear transitions may produce an appreciable drop in intensity at the detector, it is necessary that the inhomogeneous fields have large gradients. Such gradients will, however, serve to remove atoms from the beam, since the moment of an alkali atom is about 1000 times as great as that of a nucleus. It is, therefore, necessary that the deflecting fields have different values for the observation of nuclear and atomic transitions. However, because of the proximity of the deflecting magnets to the transition field, a change in the inhomogeneous field has a slight, but not negligible, effect on the value of the field in the transition magnet. This effect was found to be least when the transition field was demagnetized in the presence of the appropriate inhomogeneous field and was then raised to its final value.

The procedure which was followed in the determination of nuclear *g* values was to determine the calibration constant (field per unit current in the magnet coils) of the magnet in which the transitions occurred. This calibration constant was modified slightly for use in determining nuclear *g* values by an amount which depended on the difference between the value of the inhomogeneous field used for the observation of atomic and nuclear transitions. This correction was determined by flip coil measurements. In the region of 1600 gauss, where calibrations were made with the use of atomic transitions of Na^{23}, the correction was about 0.04 percent. In the region of 3200 gauss, where calibrations were made with the use of atomic transitions of Rb^{85}, the correction was about 0.10 percent. To average out effects arising from the non-reproducibility of the field as a function of the current in the field coils, all observations which were used in the determination of the calibration constant were repeated a large number of times, each for an independent demagnetization of the field.

The nuclear resonance curves were then observed at the same value of the field as that at which the calibration was made. Errors arising

TABLE I. *The nuclear g's of* Li^7, F^{19} *and* Na^{23} *as found by a direct determination. Each line in the table represents the results of a distinct run.*

CALIBRATING SUBSTANCE	MOLECULE	NUCLEUS	OBSERVED *g* VALUE
Na^{23}	Na_2	Na^{23}	1.4768
Na^{23}	Li_2	Li^7	2.1695
Na^{23}	NaF	F^{19}	5.2496
Na^{23}	Na_2	Na^{23}	1.4762
Rb^{85}	LiCl	Li^7	2.1691
Rb^{85}	LiCl	Li^7	2.1701
	Mean $g(Li^7)=2.1696$		
	$g(F^{19})=5.2496$		
	$g(Na^{23})=1.4765$		

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[6] M. Phillips, Phys. Rev. **60**, 100 (1941).

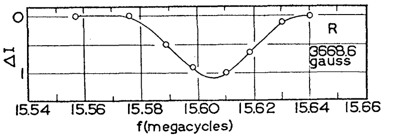

FIG. 4. Typical proton resonance curve in KOH observed for such values of the inhomogeneous fields that Rb^{85} atoms in the state $m = -2$ pass through each field without deflection.

from the non-reproducibility of the field as a function of the current in the coils were reduced, as in the calibration procedure, by observing resonance curves for many independent demagnetizations of the field. It was also necessary to observe the resonance minima for each of the two directions of the homogeneous field perpendicular to the beam direction. In this way asymmetries in the resonance curves arising from end effects in the wires which produce the oscillating field may be averaged out.[7]

In Table I are listed the observed nuclear g's. The error in any one of the tabulated g's does not exceed 0.1 percent. The consistency of the observed g values of Na^{23} obtained from two distinct runs may be noted. In the case of Li^7 such consistency is particularly significant since the individual results depend on different calibrating atoms, the $\Delta\nu$'s of which have been independently measured, on different molecules and on independently determined corrections to the transition field arising from the inhomogeneous field.

Any errors arising from uncertainties in fundamental constants have not been included in the above estimates of precision.

DETERMINATION OF THE PROTON MOMENT

The chief source of error in the experiments just described arose from the circumstance that the detection of atomic transitions requires very low gradients in the deflecting fields while nuclear reorientations produce appreciable changes in the path of the molecule only when large field gradients are used. Errors arising from the correction due to the presence of the deflecting fields, as well as those arising from the non-reproducibility of the transition field, would be eliminated if both nuclear and atomic transitions could be observed at the same setting of the homogeneous transition field as well as at the same setting of the deflecting fields. It was, therefore, thought worthwhile to investigate the possibility of observing nuclear transitions with low deflecting fields. Hydrogen is particularly favorable for such experiments since it has a large nuclear g value. Moreover, it was thought desirable to obtain the proton moment directly without any dependence on the moment of an intermediate nucleus.

In the attempt to obtain a proton resonance curve we have tried to obtain beams of a number of molecules containing an alkali atom as well as hydrogen. The alkali atom is essential for the detection of the beam by the surface ionization method. When LiH was heated in an iron or in a silver oven the beam which was detected was unsteady and practically 100 percent Li atoms. Similar results were obtained when $NaNH_2$ was heated in an iron or in a silver oven. In each case an atomic beam was produced with the evolution of large quantities of gas. NaOH and KOH were also completely dissociated when heated in iron or nickel ovens. However, when a silver oven was used, good molecular beams of NaOH and KOH were obtained. The dissociation in these cases was such that about one-third of the beam was molecular and about two-thirds atomic. This gave ample beam intensity of sufficient steadiness

TABLE II. *Results of the determination of the proton moment by direct measurement. Column 1. Calibrating atom and transition. Hydrogen-containing molecule used. Column 2. Field direction. Column 3. Observed atomic transition frequency.* $\nu \times 10^{-6}$ *(sec.*$^{-1}$*). Column 4.* H *(gauss) calculated from* $\nu/\Delta\nu$*. Column 5. Frequency of proton resonance minimum.* $f \times 10^{-6}$ *(sec.*$^{-1}$*). Column 6. Proton g in units of* $e/2Mc$*.* $g = 1.3120 \times 10^{-3} f/H$*. Column 7. Mean g values found for the field directions N and R separately. Column 8. Mean g value for each run.*

1	2	3	4	5	6	7	8
R^{85} $(3, -3)\leftrightarrow(2, -2)$ KOH	*N*	659.98	3665.1	15.583	5.5782		
		659.99	3664.9	15.585	5.5792		
		659.98	3665.1	15.581	5.5775	5.5783	
	R	659.76	3669.6	15.606	5.5796		
		659.81	3668.6	15.606	5.5811		
		659.81	3668.6	15.605	5.5808	5.5805	5.5794
Na^{23} $(2, -2)\leftrightarrow(1, -1)$ NaOH	*N*	588.97	1723.2	7.3251	5.5771		
		588.94	1723.5	7.3257	5.5766	5.5769	
	R	588.42	1729.1	7.3594	5.5841		
		588.38	1729.5	7.3600	5.5833	5.5837	5.5803
Cs^{133} $(4, -2)\leftrightarrow(4, -1)$ KOH	*N*	1056.04	3450.3	14.665	5.5764		
		1055.98	3449.9	14.665	5.5771		
		1056.49	3453.5	14.679	5.5766	5.5767	
	R	1055.46	3446.2	14.657	5.5800		
		1056.12	3450.9	14.681	5.5815		
		1056.41	3453.0	14.688	5.5808	5.5808	5.5788

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[7] S. Millman, Phys. Rev. 55, 628 (1939)

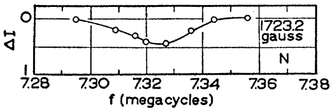

FIG. 5. Typical proton resonance curve in NaOH observed for such values of the inhomogeneous fields that Na^{23} atoms in the state $m=-1$ pass through each field without deflection.

for the observation of resonance curves of the proton. The partial dissociation of NaOH served a useful purpose in the determination of the proton moment with the use of atomic lines of Na^{23} for field calibration. In this case one oven charge gave the appropriate atomic and molecular beams in about the right proportion for the calibration of the magnet and the observation of proton resonance curves.

A preliminary study of the proton resonance curves obtained with KOH and NaOH showed that these could be observed with values of gradients in the deflecting fields as low as 3000 gauss/cm, compared with 100,000 gauss/cm normally employed for detecting nuclear transitions. The reduction of deflecting power of the magnets merely results in the lowering of the depth of the resonance curve but does not affect its width. In our apparatus such gradients are still too large for use with atoms having magnetic moments of the order of one Bohr magneton. However, if we take advantage of the fact that alkali atoms in some states have zero magnetic moment[8] for certain values of magnetic field, we can, with a suitable choice of atom and state, use deflecting fields that will not be excessive for observing certain atomic transitions and at the same time be sufficiently large for the purpose of detecting nuclear reorientations.

The first atom used for calibrating the field in this manner was Rb^{85}. In order to observe atomic and nuclear reorientations in rapid succession, so that errors due to field drift are minimized, it is important to arrange for simultaneous detection of the atomic Rb beam and the molecular KOH beam. It is, therefore, essential that the Rb atoms produce a beam at approximately the same temperature of the oven as the alkali hydroxide. It is clearly impossible to use metallic rubidium for this purpose, since the temperature at which the vapor pressure is of the order of 1 mm of Hg is about 600°C for KOH, 680°C for NaOH while only 200°C for metallic Rb. However, if instead of placing metallic Rb in the oven, we insert RbCl and Ca in a silver oven, we find that a rubidium beam is obtained at approximately the same temperature as a KOH beam. If RbCl and Ca are inserted in an iron oven, similar Rb vapor pressures are obtained at temperatures 50°C to 75°C higher than those required for the silver oven. We have, accordingly, constructed a silver oven containing two separate compartments, merging only at the slit. This prevents any possible reaction between KOH and the ingredients necessary for the production of the Rb beam. It was found that a beam of atomic Cs could be produced at the same time as a beam of molecular KOH by use of the same methods.

The atomic transition used for the calibration with Rb^{85} is the line $(3, -3)\leftrightarrow(2, -2)$. The magnetic moment of an atom in the state $m=-3$ is independent of the field and has a value of one Bohr magneton. Atoms in the state $m=-2$ have zero moment[9] at a field of about 725 gauss, which, in our apparatus, means a gradient of about 6000 gauss/cm. If, in the homogeneous field between the two deflecting magnets, atoms in the state $m=-2$ make a transition to the state $m=-3$, they will be deflected away from the original direction by the second deflecting field and will not reach the detector.

The procedure followed in these experiments was to observe the atomic transition at some fixed value of the homogeneous field, and then, without any changes except that involved in switching oscillators, to observe a proton resonance curve. We have obtained three such curves for one field direction and three curves for the reversed field direction. In each case a calibration was made between the successive curves. The procedure of using the transition field in each of two directions is necessary in order to eliminate errors in locating minima of resonance curves due to asymmetries introduced by end effects in the oscillating field. Figure 4 shows a typical proton resonance curve obtained in this way for the field direction which we designate as R. Evidently the value of the proton g as calculated from this curve will not coincide with the value finally arrived at. The drop in beam intensity ΔI is plotted in arbitrary

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[8] V. W. Cohen, Phys. Rev. **46**, 713 (1934).

[9] S. Millman and M. Fox, Phys. Rev. **50**, 220 (1936).

units. The results of the KOH-Rb run are tabulated in Table II.

The constancy of the g value for either field direction is a measure of the internal consistency of the results. Errors due to small changes in the homogeneous field between the calibration and the observation of the resonance curve, uncertainty in the location of the resonance minima, or in the readings of the wave meter, will cause variations in the observed g value. Errors in the assumed value of the $\Delta\nu$ of Rb^{85} and in the assumed value of M/m are, of course, common to all observations. The precision of the gyromagnetic ratio of the proton as determined from a calibration with Rb^{85} is about 0.04 percent. This includes all factors except the uncertainty in M/m.

The second calibrating atom used in these experiments was Na^{23}. We have used the transition $(2, -2)\leftrightarrow(1, -1)$, and therefore set the deflecting magnets at such a field value that the moment of atoms in the state $m=-1$ is zero. This field is about 320 gauss[10] and in our apparatus corresponds to a gradient of about 2600 gauss/cm. This gradient is less than half of that used for deflection in the KOH-Rb run and will, therefore, result in a very shallow proton resonance curve, thus introducing greater errors in the location of the minimum. Since the sensitivity of the frequency of the calibrating line to field demands that the calibration with Na^{23} be made at about half the value of the homogeneous field used for Rb^{85}, the ratio of the half-width of the resonance curve to the frequency at resonance is about twice as great for curves obtained in the NaOH-Na run as for curves obtained in the KOH-Rb run. This occurs because the half-width of the curve in frequency units is independent of the field. The net effect is that the g value of the proton as obtained by use of a calibration with the atomic line of Na^{23} is considerably less precise than that obtained by use of a calibration with the atomic line of Rb^{85}. Figure 5 shows a proton resonance curve, for the field direction which we designate as N, obtained in a molecular beam of NaOH under the conditions outlined above. The drop in beam intensity ΔI is plotted on the same scale as in the case of the KOH-Rb run. The generally low intensity drop is indicative of the difficulty of obtaining a precise value of g from these data. The precision of the g value of the proton as obtained from this calibration is probably not better than about 0.08 percent. The results obtained in this run are given in Table II.

Finally we have used the caesium atom for our field calibration. This atom is even more favorable for these experiments than Rb^{85}, since "zero moment peaks" of the former occur at considerably higher field values than do those of the latter. Since we are making use of the transition $(4, -2)\leftrightarrow(4, -1)$ we may set our deflecting fields so that the moment of atoms in either the state $m=-2$ or in the state $m=-1$ shall be equal to zero. However, since the zero moment peak for $m=-2$ occurs at twice the field as that for which the zero moment peak for $m=-1$ occurs, it is evidently desirable to use the former of the two peaks for our present purposes. This peak occurs at a field of about 1650 gauss.[8] An atom in this state suffers no deflection in the first of the two inhomogeneous fields. If a transition has taken place in the region between the two inhomogeneous fields, the moment of the atom will be considerably different from zero in the second deflecting field. The gradients are sufficiently strong to remove such an atom from the beam. The great advantage of this calibration lies in the fact that the relatively large gradients give rise to a fairly deep resonance minimum for the proton and a corresponding improvement in the precision of locating a minimum. A proton resonance curve for the field direction R taken in the KOH-Cs run is shown in Fig. 6. The results obtained in this run are tabulated in Table II. The g value of the proton as obtained from the Cs calibration can probably be expressed to a precision of 0.03 percent.

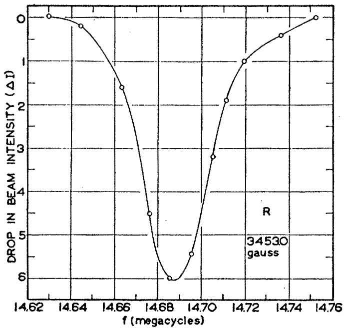

FIG. 6. Typical proton resonance curve in KOH observed for such values of the inhomogeneous fields that Cs^{133} atoms in the state $m=-2$ pass through each field undeflected.

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[10] M. Fox and I. I. Rabi, Phys. Rev. **48**, 746 (1935).

The difference between the apparent g values of the proton as obtained for the field directions R and N will depend on a number of factors.[7] For identical beams and for identical amplitudes of the oscillating fields the shift in the resonance minimum will be represented by a constant frequency difference. The apparent shift in the g value due to the end effects of the wires which produce the oscillating field will, therefore, be inversely proportional to the field at which the resonance curve is observed. This is apparent in Table II where the discrepancy between the two observed g values for the NaOH-Na run is about twice as great as in the case of the KOH-Cs run. The difference between the two observed values of the proton g for the KOH-Rb run is appreciably smaller than it is in the case of the KOH-Cs run. This may possibly result from lower oscillating currents in the first of the two cases. We believe that the mean of the apparent g's for the field direction R and N represents the true g value in all of the cases, since the magnitude of the oscillating currents used is of no importance if the same currents are used in the observation of resonance curves for each of the two field directions. We have fulfilled this requirement in all cases.

We also believe that the minima of the resonance curves represent the resonance of the Larmor precession frequency of the proton at the indicated values of the constant field with the frequency of the oscillating field. The small half-widths of the curves (32 kc for KOH-Rb and 36 kc for KOH-Cs) indicate that broadening arising from molecular interactions with the proton is very small indeed, and the generally symmetrical character of the curves indicates that such broadening is not likely to shift the minimum of the true proton resonance curve by any significant amount.

In arriving at the final result for the proton g value we average the results obtained with the three different calibrating atoms. We believe the proper weight factors to be about 6, 1 and 8 for the Rb^{85}, Na^{23} and Cs^{133} runs, respectively. The g value of the proton can then be given as: 5.5791±0.0016. The proton moment is then 2.7896±0.0008 nuclear magnetons. The precision measures do not include uncertainties in M/m.

TABLE III. *The values of nuclear gyromagnetic ratios observed by the molecular beam magnetic resonance method, the values of the nuclear spin and the resulting values of nuclear magnetic moments.*

NUCLEUS	OBSERVED g VALUE	SPIN i	MOMENT μ	DIAMAGNETIC CORRECTION (%)	REF.
$_1H^1$	5.5791±0.0016	1/2	2.7896	0	15
$_1H^2$	0.8565±0.0004	1	0.8565	0	15
$_3Li^6$	0.8213±0.0005	1	0.8213	0.01	1
$_3Li^7$	2.1688±0.0010	3/2	3.2532	0.01	1
$_4Be^9$	0.784 ±0.003	3/2	−1.176	0.02	a
$_5B^{10}$	0.598 ±0.003	1	0.598	0.03	b
$_5B^{11}$	1.791 ±0.005	3/2	2.686	0.03	b
$_6C^{13}$	1.402 ±0.004	1/2	0.701	0.03	c
$_7N^{14}$	0.403 ±0.002	1	0.403	0.04	14
$_7N^{15}$	0.560 ±0.006	1/2	0.280	0.04	13
$_9F^{19}$	5.250 ±0.005	1/2	2.625	0.06	1
$_{11}Na^{23}$	1.4765±0.0015	3/2	2.215	0.08	14
$_{13}Al^{27}$	1.452 ±0.004	5/2	3.630	0.10	d
$_{17}Cl^{35}$	0.547 ±0.002	5/2	1.368	0.14	e, f
$_{17}Cl^{37}$	0.454 ±0.002	5/2	1.136	0.14	e, f
$_{19}K^{39}$	0.260 ±0.001	3/2	0.391	0.16	14
$_{19}K^{41}$	0.143 ±0.001	3/2	0.215	0.16	2, 16, 17
$_{37}Rb^{85}$	0.536 ±0.003	5/2	1.340	0.39	e
$_{37}Rb^{87}$	1.822 ±0.006	3/2	2.733	0.39	e
$_{55}Cs^{133}$	0.731 ±0.002	7/2	2.558	0.67	14
$_{56}Ba^{135}$	0.558 ±0.002	3/2	0.837	0.68	g
$_{56}Ba^{137}$	0.624 ±0.002	3/2	0.936	0.68	g

[a] P. Kusch, S. Millman, and I. I. Rabi, Phys. Rev. **55**, 666 (1939).
[b] S. Millman, P. Kusch, and I. I. Rabi, Phys. Rev. **56**, 165 (1939).
[c] R. Hay, Phys. Rev. **58**, 180 (1940).
[d] S. Millman and P. Kusch, Phys. Rev. **56**, 303 (1939).
[e] P. Kusch and S. Millman, Phys. Rev. **56**, 527 (1939).
[f] E. F. Shrader, S. Millman, and P. Kusch, Phys. Rev. **58**, 925 (1940).
[g] R. Hay, Phys. Rev. **59**, 686 (1941).

NUCLEAR g VALUES AND MAGNETIC MOMENTS REFERRED TO THE NEW SCALE

All the values of nuclear g's previously published are subject to a rather large uncertainty arising from the difficulty of determining the absolute value of a field at resonance. Since the ratio of two nuclear g values may, in general, be determined to a higher order of precision than the g value itself, it has been the custom to report all g values relative to that of Li^7. The g value of Li^7 was chosen as a standard because the resonance curve associated with the nuclear g of Li^7 may readily be observed in a great variety of molecular beams, and because the resonance curve is generally deep and narrow. In view of the fact that the absolute value of the hydrogen moment has been established to a high order of precision, it seems desirable to tabulate all the moments, obtained by the molecular beam magnetic resonance method, referred to our new standard. For this purpose the g value of Li^7 will serve as a secondary or intermediate standard. Since its value is very well known, no appreciable error is introduced by use of this value. In the first column of Table III are listed the nuclei for which data are available. In the second column are listed the observed g values corrected to the scale for which the g value of the proton is 5.5791. The corresponding values of the spin are listed in the third column. Most of these have been determined by reliable experimental procedures. Exceptions occur in the cases of Be^9, B^{10}, B^{11} and C^{13} where the tabulated values of the spin are the result of theoretical calculations, and in the cases of Cl^{35} and Cl^{37} where the tabulated spins are those obtained from an observation[11, 12] of the alternating intensities of lines in the band spectra of the homonuclear diatomic molecules of chlorine. The last of these procedures may not be conclusive for spins as large as 5/2. The fourth column lists the magnetic moments obtained from the observed g value and the spin. The signs of the moments have been experimentally determined[7] for all nuclei except N^{15}.[13] They are all positive except that of Be^9. A diamagnetic correction, which serves to increase the value of the moment, must be applied, as discussed in an earlier paper.[14] This correction has not been applied to the values of the moments listed in Table III. The magnitude of the correction is indicated in each case in the fifth column of the table. A brief discussion of some of the g values follows.

${}_1H^2$. The value of the deuteron moment has been determined from our value of the proton moment and from the value $\mu_P/\mu_D = 3.257 \pm 0.001$ gives by Kellogg, Rabi, Ramsey, and Zacharias.[15]

${}_3Li^7$. We have made a careful determination of the ratio $g(H^1)/g(Li^7)$ which we find to be 2.5729 ± 0.0008. If this ratio is applied to the known g of the proton we obtain for the g of Li^7 the value 2.1684. The directly determined value of the g of Li^7 is 2.1696. The discrepancy lies well within the experimental error of the determinations. It is to be noted that the two determinations are entirely independent and involve only the common values of $\Delta\nu$. The first of the two indicated values is somewhat more precise, and we have given this value a weight of two as compared to a weight of one assigned to the second of the two values. The g of Li^7 then becomes 2.1688 ± 0.0010. The discrepancy between this value and that previously reported is 0.10 percent.

${}_3Li^6$. For the determination of the nuclear g value of Li^6 we have used the previously published value[2] of the ratio $\mu(Li^7)/\mu(Li^6) = 3.9601 \pm 0.0015$. This gives $g(Li^6) = 0.8213 \pm 0.0005$.

${}_9F^{19}$. The previously published value[1] for the g of F^{19}, when corrected to the present scale, becomes 5.247. The new and independently determined value is 5.250. We believe this agreement to be fortuitous and adopt the new value as the better one.

${}_{11}Na^{23}$. The previously published value[14] for the g of Na^{23}, when corrected to the present scale, becomes 1.4771. This value is to be compared to the present directly determined value of 1.4765. The agreement is better than is to be expected and may be regarded as fortuitous. We believe the value determined by methods described in this paper to be the better one and shall adopt it.

${}_7N^{14}$. The present value of the nuclear g of N^{14} depends on a new determination of the ratio, $g(Li^7)/g(N^{14}) = 5.37$. This leads to the value $g(N^{14}) = 0.403$, which is in good agreement with that previously published.[14] We believe the present value to be the better one.

${}_{19}K^{41}$. The nuclear g of K^{41} has not been directly determined. However, since the spin[16] of K^{41} is the same as that of K^{39},[17] the g value of the former can be obtained from the measured g value of the latter by multiplying by the ratio of the observed[2] $\Delta\nu$'s of the ground state of these atoms.

All other nuclear g values listed in Table III are simply the previously published g values corrected to the new scale.

The Magnetic Moment of the Neutron

A simple subtraction of the proton moment from the deuteron moment gives the value -1.933 ± 0.001 for the moment of the neutron.

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[12] E. F. Shrader, Phys. Rev. **58**, 475 (1940).

[13] J. R. Zacharias and J. M. B. Kellogg, Phys. Rev. **57**, 570 (1940).

[14] P. Kusch, S. Millman, and I. I. Rabi, Phys. Rev. **55**, 1176 (1939).

[15] J. M. B. Kellogg, I. I. Rabi, N. F. Ramsey and J. R. Zacharias, Phys. Rev. **56**, 728 (1939).

[16] J. H. Manley, Phys. Rev. **49**, 921 (1936).

[17] S. Millman, Phys. Rev. **47**, 739 (1935).

If we take the expression given by Rarita and Schwinger[18] for the calculation of the neutron moment from the observed values of μ_P and μ_D,

$$\mu_n=\frac{\mu_D-\frac{3}{4}\times 0.039}{1-\frac{3}{2}\times 0.039}-\mu_P,$$

we obtain the value -1.911 ± 0.001 for μ_n, wherein the precision measure includes only experimental errors in the determination of μ_P and μ_D. These results are to be compared with the value of 1.935 ± 0.02 obtained by Alvarez and Bloch[4] for the magnetic moment of the free neutron. The uncertainty in the latter experiment is thus too great to yield an experimental check on the theoretical predictions. It is highly desirable that the moment of the free neutron be measured with a precision comparable to that obtained for the difference $\mu_D-\mu_P$.

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On Electronic *g* Factors for Alkali Atoms

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Possible theoretical deviations from $g_J=2$ for the ground state of the alkalis are investigated, and it is concluded that none is large enough to affect measurably the experimental results.

It is assumed by Millman and Kusch[1] that for the ground level of an alkali atom $g_J=2$ is good to the accuracy of the experiments. This value of the gyromagnetic ratio for the electronic system follows at once from the description of an alkali atom in terms of a single electron moving in the central static field of the nucleus and core electrons. The energy level scheme predicted on this model is generally satisfactory, but inverted or anomalously narrow doublets occur frequently in alkali spectra,[2] indicating that the simple picture is not always justified. In view of this the question of possible perturbations of the magnetic moment must be considered seriously when high experimental accuracy is involved.

The anomalous doublet separations are adequately explained by taking into account the excitation of core electrons.[3] Possible dependence of the magnetic moment of the ground state on core excitation is here investigated. Two other aspects of the atom which are neglected in first approximation, the motion of the nucleus and relativistic effects, are also considered.

It can be easily shown that departures of the core from rigidity do not affect the mean value of the magnetic moment operator $(e/2mc)(L_z+2S_z)$ in this case. Such an effect would arise only if the ground state were not pure ${}^2S_{\frac{1}{2}}$ but contained some admixture of other $J=\frac{1}{2}$ states with different L or S. Perturbations due to excited levels may be found by writing the normal state wave function as $\Psi=\psi_0+\sum_i[V_{0i}/(E_0-E_i)]\psi_i$. Here ψ_0 is the approximate function for the lowest configuration, ψ_i that for some excited state i, and V_{0i} is the interconfiguration matrix element of the perturbing energy. The possible interacting configurations must be even, and the lowest of these would differ from the ground state only in that one of the core electrons has its total quantum number increased by unity. (No even states characterized by one-half unit of total angular momentum arise from excitation of the valence electron except for higher ${}^2S_{\frac{1}{2}}$

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[2] Bowen and Millikan, Phys. Rev. 25, 301 (1925).

[3] M. Phillips, Phys. Rev. 44, 644 (1933).