rather close to the curve, the deviations for the smaller velocities are larger than the estimated experimental errors indicating some minor defects in the one or the other of the two methods of velocity determination. In the statistical method such a defect might perhaps be due to a failure to count all branches in the last, more branching part of the range; in the direct method a defect might be due to a possible inaccuracy in the assumed range-velocity relation of the α-particles in helium for the smallest velocities.

A comparison of the range-velocity relation in argon and in helium is attempted in Fig. 4. Here, curve I is an average range-velocity curve for the two groups of fission fragments in argon obtained in the previous paper. In curve II, the full-drawn part corresponds to the information contained in Fig. 2, while the dotted part is drawn in a way that the total average range is 4-mm normal air greater than in argon, in accordance with the earlier measurements. Although the information on which the upper parts of curves I and II are based is rather scanty, it seems that there can be no doubt about the general run of the curves. In particular, the point where the rate of velocity loss is a minimum is obviously much farther removed from the end of the range in helium than in argon. The shape of the curves seems altogether to fit the theoretical expectations, and the curves can, at least within the experimental errors, be drawn in such a way that the rate of velocity loss is alike in the two gases in the first part of the range where the stopping is mainly due to electron encounters, whereas the end part of the range-velocity curve in helium is considerably longer than in argon, corresponding to the effect of the smaller energy loss by nuclear collisions in helium than in argon.

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On the Pseudoscalar Mesotron Theory of β-Decay

E. C. NELSON
Department of Physics, University of California, Berkeley, California

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The calculations of Sakata which indicate that the pseudoscalar mesotron theory can successfully account for both mesotron decay and β-decay are shown to be in error. While the pseudoscalar mesotron theory gives the correct spin and parity selection rules for β-decay, it also gives β-decay lifetimes too long by a factor of 10^6 and having an inverse seventh power dependence on the upper limit of the β-spectrum. As these same conclusions can be made when the mesotron field is strongly coupled to nuclear particles, it would seem that they are sufficient to show that pseudoscalar mesotrons are not responsible for β-decay.

YUKAWA'S[1] suggestion that β-decay is to be interpreted as the emission of a mesotron by a neutron (or proton) and its subsequent decay into an electron (or positron) and a neutrino has received support from the observed instability[2] and apparent β-radioactivity of cosmic-ray mesotrons. However, before it can be regarded as a correct description of nuclear β-processes, it must give rise to a theory of β-decay possessing the following features: (1) Fermi energy distribution[3] with its consequent inverse fifth power dependence of the lifetime on the upper limit of the spectrum,[4] (2) Gamow-Teller selection rules,[4, 5]

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(3) calculated lifetimes of β-radioactive nuclei, with the mesotron-electron-neutrino coupling constant evaluated from the disintegration of the free mesotron and the mesotron-nuclear particle coupling constant evaluated from nuclear forces, in agreement with experiment. The scalar mesotron theory meets only the first point; the vector theory but the first two. Both these theories give β-decay that is too slow.[6-8] Its characteristic time, that factor in the lifetime independent of the energy and matrix element of the transition, is of the order of a hundred times greater than that evaluated from experiments on light nuclei.

Christy and Kusaka's[9] analysis of cosmic-ray bursts together with the spin dependence of nuclear forces indicates that mesotrons are best described by a pseudoscalar field.[10,11] Recently, the assertion has been made that pseudoscalar mesotrons provide a theory of β-decay in agreement with experiment. Calculations by Sakata[8] yielded the necessary "allowed" β-decay and slow disintegration of the free mesotron. However, his formula for β-decay turns out to be incorrect;[12] proper treatment of the problem shows that the lifetime discrepancy remains and is, in fact, increased from a factor of 10^2 to a factor of 10^6.

Yukawa treats the connection between nuclear forces and β-decay with an interaction of the mesotron field with nuclear particles and with electrons and neutrinos symmetrical in all respects except in strength. In the pseudoscalar theory this is expressed in the following Lorentz-invariant terms in the Lagrangian:

$$L=-(8\pi)^{\frac{1}{2}}\int dV_4\left\{\frac{2M}{\mu}g\psi^\dagger\gamma^5\frac{(\tau_x+i\tau_y)}{2}\psi U+\frac{2M}{\mu}g\psi^\dagger\gamma^5\frac{(\tau_x-i\tau_y)}{2}\psi U^*+g'\varphi^\dagger\gamma^5\frac{(\tau_x+i\tau_y)}{2}\varphi U+g'\varphi^\dagger\gamma^5\frac{(\tau_x-i\tau_y)}{2}\varphi U^*\right\}. \quad (1)$$

U is the pseudoscalar field and U^* its complex conjugate. ψ and φ are the neutron-proton and the electron-neutrino wave fields, $\psi^\dagger=i\psi^*\beta$ and $\varphi\dagger=i\varphi^*\beta$ their respective adjoints. γ^5 and β are Dirac matrices, τx and τy are isotopic spin matrices, $M=$proton mass, and $\mu=$ the mesotron mass. The mesotron-nuclear particle coupling constant g is normalized so that the potential in the singlet state of the deuteron is $g(e^{-\kappa r}/r)$, $\kappa=\mu c/\hbar$.

It might be expected that interaction terms containing the first derivatives of U as well as ones containing U should occur in the Lagrangian. However, such terms do not really represent a different type of interaction, for in virtue of the equations of motion of ψ, they may be derived from the terms in (1) by partial integration.

$$\psi^\dagger\gamma^5\frac{(\tau_x+i\tau_y)}{2}\psi \cong -\frac{\hbar\partial_\nu}{2Mc}\left\{\psi^\dagger\gamma^5\gamma^\nu\frac{(\tau_x+i\tau_y)}{2}\psi\right\}.$$

The neglected terms are of the order g^2, are bilinear in the U fields, and give no contribution to β-decay. They also give no contribution to nuclear forces in the usual approximations. Thus there is no need to introduce two independent constants, f and g, in the pseudoscalar theory as there is in the vector theory.

Since (1) does not contain the time derivative of U, i.e., the canonically conjugate field momentum, the interaction terms in the Hamiltonian are just the negative of the ones in L.

$$H_{\text{int}}=-L. \quad (2)$$

In this formulation there are no terms directly coupling neutrons and protons to electrons and neutrinos in either the Hamiltonian or the Lagrangian. Such terms do appear in the

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[11] J. R. Oppenheimer and J. S. Schwinger, Phys. Rev. **60**, 150 (1941).

[12] Sakata has apparently neglected the singularity arising from the derivatives of the Yukawa potential in his Eq. (79d). As has been pointed out by Serber, reference 7, $\boldsymbol{\sigma}_1\cdot\nabla_1\boldsymbol{\sigma}_2\cdot\nabla_2$ (exp $(-\kappa|\mathbf{r}_1-\mathbf{r}_2|)/|\mathbf{r}_1-\mathbf{r}_2|$), which also occurs in the Bethe-Nordheim, reference 7, treatment of the vector theory, contains $(4\pi/3)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\delta(\mathbf{r}_1-\mathbf{r}_2)$. Upon integration over the electron-neutrino coordinates in the matrix element for β-decay, this delta function term cancels in the usual approximation for "allowed" transitions, wave functions of electron and neutrino treated as constant over nuclear dimensions, the $-(4\pi/3)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$ from the other terms.

Hamiltonian in the equivalent formulation in which derivatives of U occur in the interaction Lagrangian. There the inclusion of such terms in the Hamiltonian is essential to preserve the covariance of the theory and to give the same matrix elements as (2). Any attempt to eliminate them by adding invariant direct interaction terms to the Lagrangian succeeds only in replacing them by larger ones.

The matrix element, H_{if}, for β-decay follows from (2) by a straightforward calculation using second-order perturbation theory, and plane Dirac waves for the electron and neutrino.

$$H_{if}=\frac{gg'(\hbar c)^2}{W^2-E_k^2}BB', \tag{3}$$

$$B=\frac{2Mi}{\mu}\sum_s\int dV\Psi_f{}^*\beta_s\gamma_s{}^5\frac{(\tau_x{}^s+i\tau_y{}^s)}{2}\times\Psi_i\exp[-i\mathbf{k}\cdot\mathbf{r}_s], \tag{3'}$$

$$B'=iu_e{}^*\beta\gamma^5u_\nu.$$

Ψ_i and Ψ_f are the initial and final wave functions of the nucleus. The summation s is carried out over all the particles in the nucleus. u_e and u_ν are the spin functions of the electron and the neutrino.

W = upper limit of the β-spectrum,
E_k = energy of the mesotron,
$\hbar\mathbf{k}$ = its momentum.

Since by the equations of motion of ψ in non-relativistic approximation

$$\psi^\dagger\gamma^5\frac{(\tau_x+i\tau_y)}{2}\psi\cong\frac{\hbar}{2iMc}\nabla\cdot\left(\psi^*\boldsymbol{\sigma}\frac{(\tau_x+i\tau_y)}{2}\psi\right), \tag{4}$$

$$B=\frac{\hbar\mathbf{k}}{c\mu}\cdot\sum_s\int dV\Psi_f{}^*\boldsymbol{\sigma}_s\frac{(\tau_x{}^s+i\tau_y{}^s)}{2}\times\Psi_i\exp[-i\mathbf{k}\cdot\mathbf{r}_s]=\frac{\hbar\mathbf{k}}{c\mu}\cdot\mathbf{B}_1 \tag{4'}$$

$\boldsymbol{\sigma}$ represents the Pauli spin matrices. The transition probability for β-decay is:

$$\frac{1}{\tau}=\frac{2\pi}{\hbar}\int d\Omega_e\int d\Omega_\nu\int_{mc^2}^{W}dE_e|H_{if}|^2\rho_e\rho_\nu,$$

$$\rho_e=\frac{P_e{}^2dP_e}{(2\pi\hbar)^3dW},\quad \rho_\nu=\frac{P_\nu{}^2}{(2\pi\hbar)^3}\frac{dP_\nu}{dW}.$$

E_e = energy of the electron,
$\mathbf{P}_e$ = momentum of the electron,
E_ν = energy of the neutrino,
$\mathbf{P}_\nu$ = momentum of the neutrino,

$$\mathbf{P}_e+\mathbf{P}_\nu=\hbar\mathbf{k},$$

$$\sum_{\text{spins of } e \text{ and } \nu}|B'|^2=1-\frac{\mathbf{P}_e\cdot\mathbf{P}_\nu c^2}{E_eE_\nu}, \tag{5}$$

$$\frac{1}{\tau}=\frac{1}{\tau_\beta}\frac{1}{3}|\mathbf{B}_1|^2I(\omega_0), \tag{6}$$

$$\frac{1}{\tau_\beta}=\frac{32}{\pi}\frac{\mu c^2}{\hbar}\left(\frac{gg'}{\hbar c}\right)^2\left(\frac{m}{\mu}\right)^7, \tag{7}$$

m = mass of the electron,

$$I(\omega_0)=\int_1^{\omega_0}d\omega\left\{\frac{8\omega(\omega-\omega_0)}{3}+\omega_0{}^2+\frac{2\omega_0}{\omega}-\frac{5}{3}\right\}\times\omega(\omega^2-1)^{\frac{1}{2}}(\omega_0-\omega)^2$$
$$=(\omega_0{}^2-1)^{\frac{1}{2}}\left\{\frac{\omega_0{}^6}{70}-\frac{13\omega_0{}^4}{140}-\frac{4\omega_0{}^2}{21}+\frac{2}{105}\right\}+\frac{\omega_0{}^3}{4}\ln[\omega_0+(\omega_0{}^2-1)^{\frac{1}{2}}]\sim\omega_0{}^7/70, \tag{8}$$

$$\omega=Ee/mc^2,\quad \omega_0=W/mc^2.$$

It is instructive to compare B, the nuclear matrix element, and B', the electron-neutrino matrix element, (3) in the special case of the decay of the neutron. Here all the particles may be appropriately described by using plane waves. The contribution of B' is expressed in (5). Since both the neutron and proton have non-vanishing rest mass, the contribution of B differs from that of B' by mass terms.

$$\sum_{\text{spins of } P \text{ and } N}|B|^2=\left(\frac{2M}{\mu}\right)^2\left(1-\frac{\mathbf{P}_N\cdot\mathbf{P}_Pc^2+M_NM_Pc^4}{E_NE_P}\right)$$
$$=\left(\frac{2M}{\mu}\right)^2\frac{(\mathbf{P}_P)^2c^2}{2(M_Pc^2)^2}$$
$$=\left(\frac{2M}{\mu}\right)^2\frac{(\mathbf{P}_e+\mathbf{P}_\nu)^2}{2(M_Pc)^2}\sim\left(\frac{m}{\mu}\right)^2, \tag{5'}$$

while (5) is of the order unity.

The presence of $\boldsymbol{\sigma}$ in B (4') ensures Gamow-Teller selection rules. Although $\beta\gamma^5$ is a pseudoscalar, there is no parity change in the nuclear state in the most "allowed" transition. This is so because B vanishes in view of (4) when $\exp[-i\mathbf{k}\cdot\mathbf{r}]$ is replaced by the first term, 1, in its expansion and the most "allowed" transition comes from the second term in the expansion, $-i\mathbf{k}\cdot\mathbf{r}$, which when multiplied by $\beta\gamma^5$ does not change sign under reflection of the space coordinates. Since B is proportional to the momentum of the mesotron, the lifetime τ has an inverse seventh power dependence on the upper limit of the β-spectrum in disagreement with the inverse fifth power dependence shown by the series of light positron emitters.[4]

Only slow mesotrons are involved in β-decay, $\hbar k \ll \mu c$. Thus the $\hbar\mathbf{k}/\mu c$ in B contributes, upon averaging over the angles of the electron and neutrino, effectively $(m/\mu)^2$ to the transition probability, slowing up β-decay in that proportion.

The probability per unit time that a free pseudoscalar mesotron decay, obtained after a similar but simpler calculation, is

$$1/\tau_\mu = (\mu c^2/\hbar)(g'^2/\hbar c) \qquad (9)$$

from (7) and (9)

$$\tau_\beta = \tau_\mu \frac{\pi}{32} \frac{\hbar c}{g^2} \left(\frac{\mu}{m}\right)^7$$

with $\tau_\mu = 2\times 10^{-6}$ sec., $g^2/\hbar c = 0.1$, $\mu = 200$ m, $\tau_\beta^{\mathrm{calc}} = 2.4(10)^{10}$ sec. while from (6) and experiments on Ne[19],[4] $\tau_\beta^{\mathrm{exp}} = 6.7(10)^3$ sec. $\tau_\beta^{\mathrm{calc}}/\tau_\beta^{\mathrm{exp}} \sim 10^6$, a discrepancy greater by $(\mu/m)^2$ than that in the scalar or in the vector theory.

In deriving these results perturbation-theoretic methods have been used. The large mesotron-nuclear particle coupling renders these methods unsuited to many problems. However, recent investigation of this interaction in the limit of strong coupling, with extended sources used to ensure convergence, indicates that the normal state of the field of pseudoscalar mesotrons "bound" to nuclear particles differs from the classical static field only by a factor of $1/\sqrt{6}$.[13] Since the classical field gives for β-decay the same results as perturbation theory, no new feature, except the $\frac{1}{6}$ which only increases the lifetime discrepancy, is introduced into this problem by strong coupling. For allowed transitions β-decay does not depend on the details of the source. As preliminary results of strong coupling calculations indicate that $g^2/\hbar c$ is not a large number,[11] it is unlikely that a detailed treatment of the problem would produce results altered enough to give a correct description of β-decay. Therefore, it would seem that the magnitude of the lifetime discrepancy and the seventh power energy dependence are sufficient to show that pseudoscalar mesotrons are not responsible for β-decay.

Two alternative interpretations of β-decay have been suggested: (1) There are two kinds of mesotrons—long-lived pseudoscalar mesotrons observed near sea level in cosmic rays and fast decaying mesotrons of another type to give β-decay.[14] Møller and Rosenfeld[15] suggested, for other reasons, a theory giving a combination of pseudoscalar and vector mesotron fields in nuclei. Rozental has shown that the constants in this theory can be chosen to give correctly the observed mesotron decay and β-decay.[16] (2) β-decay occurs as in the original Fermi theory by means of a direct interaction between the nuclear particles and electrons and neutrinos. Since the constant characterizing this interaction is dimensionally different from that which couples mesotrons to electrons and neutrinos, it does not seem possible to relate them in any unique way and establish thereby a simple connection between β-decay and the decay of a free mesotron. The suggestion[17] that mesotron decay occurs through the virtual emission of a neutron-proton pair is dependent on a highly speculative application of a not yet substanstiated relativistic theory of neutrons and protons.

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[13] The author is indebted to Professor J. R. Oppenheimer for information on this point.

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