effect, in that it tends to throw the elementary dipoles out of a longitudinal alignment. Thus the same two causes (jar and circular flux) seem to produce two effects, one tending to bring about a more stable alignment of elementary magnets that have overshot the mark, and the other tending to turn them out of that alignment. The fact that in the case of double throws (observed both with mechanical jar and abruptly created circular field) the trigger effect comes first shows that the decrease in longitudinal magnetization takes more time to be produced than the increase caused by a jar or by circular flux after the magnetizing field has been abruptly broken. This seems reasonable, because upsetting an unstable condition should take less time than a more or less permanent rearrangement of the dipoles associated with a decrease of magnetization.

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Internal Diamagnetic Fields*

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In the precise molecular beam experiments of Rabi and his collaborators, it is necessary to know the value at the nucleus of the magnetic field produced by the diamagnetism of the atomic electrons. This has been calculated on the basis of the Fermi-Thomas model, and checked for a number of atoms by use of the available Hartree calculations. The statistical treatment gives for ratio of induced to external field $0.319\times10^{-4}Z^{4/3}$, while the numerical coefficient on the basis of the Hartree model is lower by 19 percent at $Z=19$ and by 12 percent at $Z=80$. The effect is equivalent to a reduction of the nuclear g value by a factor of $(1-0.319\times10^{-4}Z^{4/3})$, and in this form, the correction may be applied in the calculation of hyperfine structure of heavy atoms. The influence of the diamagnetic fields on an orbital electron has also been considered, and it is shown that it is equivalent to a reduction in the g value of an outer s electron by an amount of just the same order of magnitude as the relativistic correction calculated by Margenau.

IN discussing the magnetic properties of solids, it is important to know the value of the magnetic field produced at one atom due to the action of all the *other* atoms. One uses for this some modification of the Lorentz formula.[1] We shall be concerned in this note with the field produced *within* an atom by its own diamagnetic moment when an external field is present. This problem has only arisen because of the very precise molecular beam measurements of Rabi and his collaborators.[2,3]

An external field H (taken along the z axis) may be described by the vector potential

$$\mathbf{A}=\tfrac{1}{2}[\mathbf{H}\times\mathbf{r}] \tag{1}$$

and this induces a diamagnetic current density in the atomic electrons of[4]

$$\mathbf{S}=e\mathbf{A}\rho(\mathbf{r})/mc, \tag{2}$$

where $-e$ is the charge on the electron and $\rho(\mathbf{r})$ is the charge density at $\mathbf{r}$. The induced field is then given by

$$\mathbf{A}'(\mathbf{r})=(e/2mc^2)\int d\mathbf{r}'\rho(\mathbf{r}')[\mathbf{H}\times\mathbf{r}']/|\mathbf{r}-\mathbf{r}'|. \tag{3}$$

When $\rho(\mathbf{r})$ is spherically symmetrical, this may

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[1] H. Casimir, *Magnetism and Very Low Temperatures* (Cambridge, 1940), p. 56.

[2] P. Kusch, S. Millman, I. I. Rabi, Phys. Rev. **55**, 1176 (1939); P. Kusch, S. Millman, Phys. Rev. **56**, 527 (1939); R. H. Hay, Phys. Rev. **60**, 75 (1941).

[3] P. Kusch, S. Millman, I. I. Rabi, Phys. Rev. **57**, 765 (1940); S. Millman, P. Kusch, Phys. Rev. **58**, 438 (1940); **60**, 91 (1941).

[4] This is most easily derived by considering the induced current in a conducting ring of radius r in the xy plane.

be written as

$$A'(\mathbf{r})=(e/6mc^2)[\mathbf{H}\times\mathbf{r}]\Big\{(1/r^3)\int_{r'<r}d\mathbf{r}'r'^2\rho(r')+\int_{r'>r}d\mathbf{r}'\rho(r')/r'\Big\}, \quad (4)$$

whence the induced magnetic field along the z direction is

$$H'_z=(eH/2mc^2r^5)(z^2-\tfrac{1}{3}r^2)\int_{r'<r}d\mathbf{r}'r'^2\rho(r')+(eH/3mc^2)\int_{r'>r}d\mathbf{r}'\rho(r')/r'. \quad (5)$$

The induced field at the center of the atom, i.e., that acting on the nuclear spin is just

$$H'(0)_z=(eH/3mc^2)\int_{r'>0}d\mathbf{r}'\rho(r')/r'=(eH/3mc^2)v(0) \quad (6)$$

and depends only on the electrostatic potential $v(0)$ produced at the nucleus by the electrons. One has $v(0)=-Ze(1/r)_{\text{Av}}$, and using an effective screening radius $r\sim a_0/Z^{\frac{1}{3}}$, one finds

$$H'(0)_z=-\gamma(e^2/mc^2a_0)Z^{4/3}H=-\gamma\alpha^2Z^{4/3}H, \quad (7)$$

where γ is a number of order unity, a_0 is the Bohr radius, and α is the fine structure constant. The value of γ may be estimated by use of the Fermi-Thomas atom model. According to this[5]

$$V(r)=(Ze/bx)\phi(x) \quad (8)$$

is the total electrostatic potential at a distance from the nucleus $r=bx$, where $b=0.885a_0/Z^{\frac{1}{3}}$, and

$$\phi(x)=1-1.588x+4/3x^{\frac{3}{2}}+\cdots \quad (9)$$

for small x. The electronic contribution to the potential at the nucleus is then

$$v(0)=-1.588Ze/b,$$

whence γ has the value $(1.588/0.885)/3=0.598$ and

$$H'(0)_z=-0.319\times10^{-4}Z^{4/3}H. \quad (10)$$

[5] E. Condon and G. Shortley, *Theory of Atomic Spectra* (Cambridge, 1935), p. 336; L. Brillouin, *L'Atome de Thomas-Fermi* (Hermann, Paris, 1934).

The correction in magnetic field acting on the nucleus must also be applied to the field produced by an orbital electron when one calculates hyperfine structure. It is easy to show that this case may be combined with that of an external field by keeping the applied field constant, and changing the nuclear g factor to $g(1-0.319\times10^{-4}Z^{4/3})$, except for some negligible errors due to the field produced by the outermost electrons.

These simple formulae can be improved in any particular case by use of the Hartree model. For a number of atoms, values of $v(0)$ have been given explicitly by Hartree and others. Thus for[6] ${}_{19}$K, ${}_{20}$Ca, ${}_{26}$Fe, ${}_{29}$Cu, ${}_{37}$Rb, ${}_{55}$Cs, ${}_{74}$W, ${}_{80}$Hg, one finds that the number 0.319 in Eq. (10) should be replaced by the numbers 0.259, 0.259, 0.263, 0.268, 0.270, 0.274, 0.277, 0.280, respectively. Thus it is seen that for these elements, the Fermi-Thomas $Z^{4/3}$ law is checked very well, although the numerical coefficient is too high by an amount varying smoothly from 19 to 12 percent.

In the radiofrequency experiments with atoms,[3] one has beside the nuclear spin, an orbital electron usually in a ${}^2S_{\frac{1}{2}}$ state. We shall now discuss the effect of the induced field on this outer electron. It is seen from Eq. (5) that the induced field H'_z at any distance r may be divided into two parts. One is due to electrons inside the sphere of radius r, the other due to electrons outside of the sphere. The first contribution varies like the second Legendre polynomial $P_2(\cos\theta)$ with angle, and averages to zero in its effect on an s electron. The second contribution does not vary with angle, but it is small for an outer electron which does not penetrate much into the core. We can estimate the effective induced field for such an electron as follows. We must average

$$H''(r)_z=(eH/3mc^2)\int_{r'>r}d\mathbf{r}'\rho(r')/r' \quad (11)$$

over the orbit of the outer electron. With Poisson's equation

$$\frac{d^2V}{dr^2}+\frac{2}{r}\frac{dV}{dr}=-4\pi\rho(r), \quad (12)$$

[6] The references used are given in F. Seitz, *Modern Theory of Solids* (McGraw-Hill, 1940), p. 251, except for the case of Fe which was obtained from M. Manning and L. Goldberg, Phys. Rev. **53**, 662 (1938).

the integral, which is just the potential due to the electrons of the core beyond r, can be expressed as

$$\int = V(r)+r(dV/dr), \tag{13}$$

where $V(r)$ is the potential acting on the orbital electron. If we write $V(r)=eZ(r)/r$, this becomes $edZ(r)/dr$, so that the effective magnetic field is

$$(H''_z)_{\text{Av}}=(e^2/3mc^2)H(dZ(r)/dr)_{\text{Av}}. \tag{14}$$

By applying the virial theorem to a potential of the above form, one finds

$$e^2(dZ(r)/dr)_{\text{Av}}=W-T, \tag{15}$$

where W is the binding energy of the orbital electron and T is its average kinetic energy. The two quantities are of course equal for a pure coulomb field, but the average kinetic energy is larger than the binding energy for an atomic field because of the penetration of the orbital electron into regions of the core where the effective Z is larger than unity. A very simple, although rough way to estimate the difference $T-W$ is the following: For orbits with small penetration, one can expand $Z(r)$ as a power series in inverse powers of r. For large r only, one could write

$$Z(r)=1+\zeta(a_0/r), \text{ where } \zeta \text{ is a constant.}$$

Then

$$T-W=\zeta a_0 e^2(1/r^2)_{\text{Av}}. \tag{16}$$

Now the binding energy of the electron is equal to the hydrogenic value $W_0=e^2/(2n^3a_0)$ increased by

$$\zeta a_0 e^2(1/r^2)_{\text{Av}}.$$

Hence $T-W\backsimeq W-W_0$, and we can express our results simply in terms of the excess binding energy of the electron over the hydrogenic value, so that

$$(H''_z)_{\text{Av}}\backsimeq-(H/3mc^2)(W-W_0). \tag{17}$$

This induced field is always of the order 10^{-5} of the applied field, and hence does not affect the discussion of the radiofrequency experiments in their present state of precision, although this can probably be increased.

It will be noticed that the atomic electron behaves as if its g value were reduced from the value 2 to the value

$$g=2(1-(T-W)/3mc^2). \tag{18}$$

This is of the same order, but distinct from the relativistic change in the g factor discussed by Margenau,[7] who found

$$g=2[1-\tfrac{2}{3}(T/mc^2)]. \tag{19}$$

His effect arises because of the change in the magnetic moment of an electron due to its motion in a fixed potential field, while we have considered the fact that the potential field itself is changed by the presence of an external magnetic field, at least in the present case where this field is produced by electronic charges. Both corrections must be considered in order to obtain the effective g value of an s electron.

These calculations were suggested to me by members of the molecular beam laboratory of Columbia University. I am indebted to them for helpful discussions.

[7] H. Margenau, Phys. Rev. **57**, 383 (1940). It has been shown by M. Phillips, Phys. Rev. **60**, 100 (1941) that interconfiguration interactions do not affect the g value of an alkali atom in the ground state.