DECEMBER 1, 1941 PHYSICAL REVIEW VOLUME 60

Increase of Residual Magnetism Caused by a Current Flowing Through an Iron Bar

H. A. PERKINS AND H. D. DOOLITTLE
Jarvis Laboratory, Trinity College, Hartford, Connecticut

(Received June 14, 1941)

If a bar of high grade wrought iron or of permalloy is placed in a sufficiently strong magnetic field, and if this field is abruptly reduced to zero, a current set up in the iron causes an increase of the residual magnetism. This effect is not obtained when the field is gradually eliminated, nor is it obtained when the current flows in a wire along the axis of a bored out iron cylinder unless the cylinder is slotted longitudinally. Then the increase of magnetism is again observed. There seem to be two effects involved in this phenomenon. One of these results in decreasing the residual magnetism in accordance with Wiedeman's and Villari's observations, and is apparently associated with the circular flux set up in the bar by a current sent directly through it or in a separate wire along its axis. The other effect, if it could be completely isolated, would probably produce only increases in residual magnetism, and seems to depend on an unstable condition of the magnet.

INTRODUCTION

It has long been known that a current of electricity sent through an iron bar or wire previously magnetized tends to reduce the residual magnetism much like the effect of a mechanical jar. In 1862 Wiedeman[1] described this decrease of residual magnetism, and also a less vigorous increase when the current through the iron was broken. In 1865 Villari[2] published the results of extensive investigations in the same field, and he too observed the decrease in residual magnetism caused by a longitudinal current, but does not seem to have observed the increase when the circuit was broken. A study of his tabulations shows that after the first "make" the galvanometer deflections (by which alterations of flux were observed) caused by subsequent and repeated "makes" and "breaks," rapidly approach the same constant value though in opposite senses. This is obviously a purely inductive effect and has nothing to do with changes in the residual magnetism. So, although Villari reports galvanometer deflections suggesting increased magnetization, they are always subsequent to the initial "make," while the significant deflection produced when the current was first established always indicates the expected decrease.

The phenomena just cited seem to be closely associated with the effect of mechanical stresses of various kinds upon the magnetic properties of iron, but in general a jar or twist or pull tends to diminish residual magnetism. However in his collected *Mathematical and Physical Papers*, Vol. II, page 351, Sir William Thomson (Lord Kelvin) reports an increase in residual magnetism when tension was suddenly applied to a previously magnetized wire of soft iron, whereas a steel wire showed the usual decrease. In a footnote he says he later concluded that this could be explained by the influence of terrestrial magnetism, but as we found the same increase when a bar of wrought iron received an end on tap, it would seem as if the effect really existed in certain kinds of iron.

During the summer of 1939 we undertook a further study of the effect of a current on residual magnetism, using at first a bar of permalloy given us by the Bell Telephone Laboratories, and later bars of very pure Norway iron, cast iron and nickel, the latter being the gift of the International Nickel Company. In the course of these observations we were very much surprised to find that when a cylindrical bar either of permalloy or of Norway iron had been magnetized beyond a certain minimum value by a solenoid whose current was then *suddenly* broken, an increase instead of a decrease of the residual magnetism was produced by a current sent through the iron in either direction. Two effects seem to be present. One of them which we shall call "effect *A*" operates with striking rapidity, and tends to increase the residual magnetism. The other "effect *B*" results in decreasing the residual magnetism and is definitely slower in its action.

---

[1] G. Wiedeman, Poggendorff's Annalen **117**, 215 (1862).
[2] E. Villari, Poggendorff's Annalen **126**, No. 9, (1865).

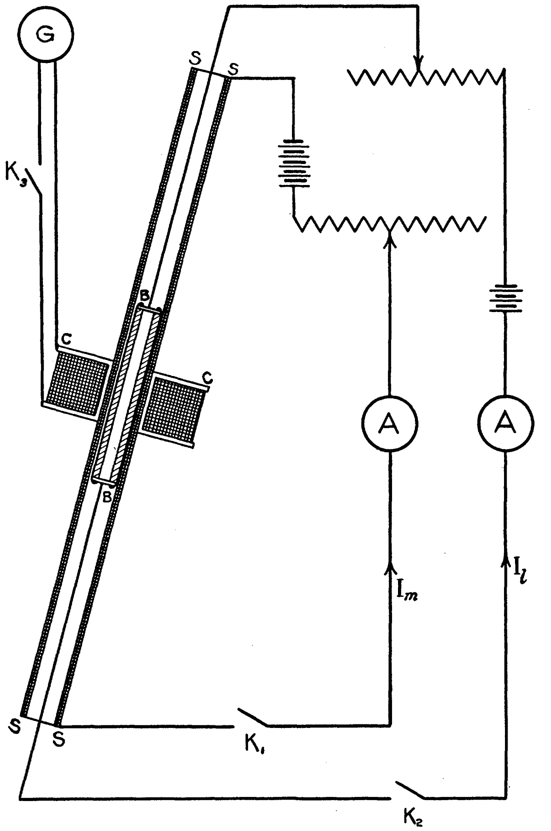

FIG. 1. *Diagram of apparatus. S......S*—magnetizing solenoid; *B......B*—bored out cylinder of Norway iron; *C......C*—galvanometer coil.

The difference in the speed with which these two effects take place appears in borderline cases where both are of nearly equal magnitude. The galvanometer starts vigorously in the direction indicating increasing magnetization, but is then deflected with less violence though often farther in the opposite direction. With still larger residual magnetization, the positive throw, indicating an increase, completely dominates the negative throw and increases with increased magnetization and also with an increase in the longitudinal current through the bar.

EXPERIMENTAL ARRANGEMENT

The apparatus used in these experiments has gone through a variety of modifications, but the following arrangement was finally adopted as the most satisfactory one. The bar under investigation is a hollow cylinder about 20 cm long and 2.20 cm diameter with an axial hole 1.2 cm in diameter. In a second exactly similar cylinder cut from the same bar of Norway iron a longitudinal slot about 2 mm wide was cut through the entire length of the cylinder, thus making its section like a nearly closed letter C. This slot was designed to minimize circular magnetization and it resulted in a very marked reduction in the "*B* effect." The ends of these hollow cylinders were covered with sheet copper caps tightly screwed down and the wires supplying the longitudinal current were soldered to the centers of these caps. In this way it was hoped that the current density throughout the bar's section would be made fairly uniform.

The bar is magnetized by a double layer solenoid of 15 turns per cm having a mean diameter of 3.6 cm and a length of 76 cm. This is set up in line with the earth's field, and Helmholz coils having a diameter of 60 cm are used to neutralize the rather slight influence of terrestrial magnetism on the phenomena under examination. The center of the bar coincides with the center of the solenoid, and a coil 5.5 cm long of 4455 turns of No. 24 wire surrounds the central portion of the solenoid. This coil is connected with a low resistance galvanometer having a sensitivity of 0.35 microvolt per millimeter, but it was used only as a ballistic in all our observations.

Figure 1 shows the essential parts of the apparatus without the Helmholz coils. The commutator switches for reversing the longitudinal current I_l, and the magnetizing current I_m are also omitted.

PROCEDURE

In order to obtain consistent results it was soon found necessary to establish a definite cycle of operations which, if rigidly followed gave galvanometer throws that could be repeated at any time with satisfactory regularity. The procedure adopted was as follows: The iron is first carefully demagnetized. Then I_m is brought up to the value desired by cutting out resistance in its circuit until some point as a in the $B-H$ curve, shown in Fig. 2, has been reached. If the circuit of the magnetizing current is then abruptly broken by opening K_1, the flux decreases from a to some point b on the B axis. Next the galvanometer circuit is closed by means of K_3, and any desired current I_l is sent through the bar by closing K_2. In those cases when the flux increased, the result is a throw of the galvanometer proportional to *bc*, or when the flux decreased, proportional to *be*. The next step is to open K_3 and close K_1 thus bringing the magnetism back to *d* or *f*. The cycle is then completed by closing K_3 and observing the galvanometer throw when I_l is broken by opening K_2. This throw is proportional to *da* showing a decrease of B, or to *af* showing an increase. These values were generally nearly equal to those corresponding to *cb* and *be* but of course in the opposite sense. This cycle may be repeated as many times as desired with only minor changes in the readings.

In our curves we have given changes in the residual flux, $\Delta\phi$, in terms of the galvanometer throws to which they are proportional. But in order to be able to interpret these throws in terms of maxwells we calculated the flux change for several deflections when there was no iron within the solenoid and found that one millimeter on the galvanometer scale corresponds to a change of 0.044 maxwell passing through the coil *CC*.

Before describing a typical series of observations we should point out two important considerations. In all cases in which the residual flux is increased by I_l, the magnetizing current was broken abruptly.[3] If instead I_m is reduced to a very small value by gradually introducing resistance in the circuit, the effect of I_l is invariably to produce a decrease in the residual magnetism. In cases where I_l after an abrupt break caused an increase, the gradual elimination of the magnetizing current was followed by a moderate decrease of residual flux. But in cases where I_l, after an abrupt break, caused a moderate decrease, a slow elimination of I_m was followed by a very large decrease of magnetization produced by I_l. This must mean that an abrupt break brings the point *b* (Fig. 2) to a lower point on the B axis than when the current is decreased gradually. To test this peculiarity still further we tried an intermediate procedure of pushing the slider along the drum rheostat as fast as possible from positions of low to high resistance. This gave intermediate results. That is in cases where the magnetism was strongly increased by I_l, after an abrupt break of I_m, the less abrupt decrease of the magnetizing field still resulted in an increase of magnetism, though less than before. In cases where I_l, after an abrupt break caused a decrease, the less abrupt elimination of I_m was followed by a decidedly larger decrease of flux, but not nearly as much as when the field was gradually eliminated.

The other matter to be noted is that there was always a small inductive effect between the I_l circuit and the coil *CC*. This resulted in a reversible throw of the galvanometer on make and break of I_l as has already been mentioned in connection with Villari's experiments. This disturbing throw varied with the orientation of the wires in the galvanometer and longitudinal current circuits, and by careful adjustment could be made as small as two or three millimeters. As this appeared as a constant increase or decrease in the throws caused by changes of flux, we have corrected those readings accordingly.

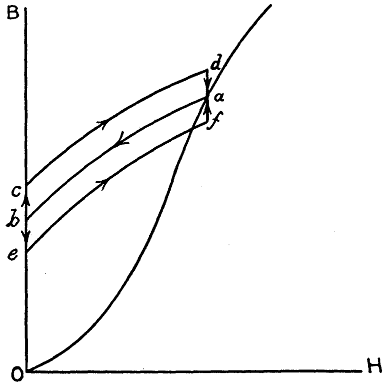

FIG. 2. Magnetic cycles.

RESULTS

Two typical curves obtained with increasing values of the magnetizing field (ignoring the demagnetization by the poles) from 0.0188 oersted created by a current of one milliampere, up to 56.4 oersteds, (3 amperes), and with I_l constant at some value as 0.5 ampere up to 4 amperes (though usually 2) are shown in Fig. 3. Here the axis of ordinates represents $\Delta\phi$, while the axis of abscissas measures the magnetizing field in oersteds. One curve gives the behavior

---

[3] We have found that in spite of the abrupt break of I_m, the condition of the iron is surprisingly stable. Even when a week elapsed between subjecting the bar to a field of about forty oersteds and the closing of the I_l circuit, the galvanometer indicated the same increase of flux as when there was no appreciable interval.

when K_2 was closed and corresponds to *da* or *fa* in Fig. 2. It is labeled "make." The other labeled "break" represents the behavior when K_2 was opened with I_m flowing, and corresponds to *da* or *fa*. These curves are almost identical in shape though of course reversed. However near the point of reversal, *a*, there is considerable uncertainty due to double throws already mentioned, and its exact position could not be determined with precision. But in general, with I_l at 2 amperes, it came when the field was in the neighborhood of 10 oersteds.

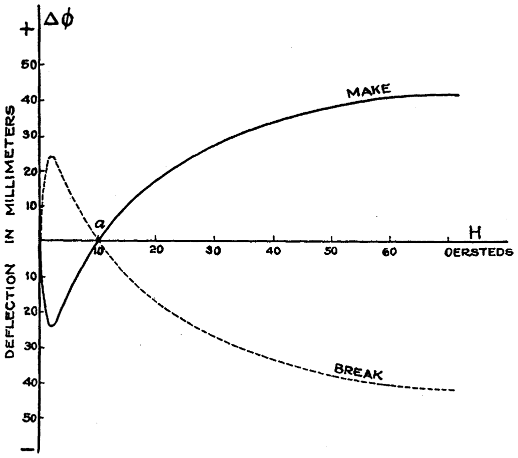

FIG. 3. Typical make and break curves.

The shape of these curves strongly suggests the existence of the two effects we have postulated. For instance the "make" curve, *M*, as shown in Fig. 4, may be decomposed into two other curves *A* and *B* indicating the increasing and decreasing flux caused by the *A* and *B* effects, respectively. This hypothesis was strongly substantiated by observing the effect of a current I_a flowing in a wire placed in the axis of the hollow cylinder. Such an axial current sets up an even stronger circular flux than one flowing in the iron itself, as is found from the following calculation: If we assume the permeability of the iron to be the same when the same current is sent along the axis as when it flows through the iron, and if we suppose the current to be of uniform sectional density, then in the case of the axial current, a familiar calculation gives

$$\phi_a = 2i\mu \log_e \frac{R}{r}, \tag{1}$$

where *R* and *r* are the outer and inner radii of the bored-out cylinder and ϕ_a is the total flux per unit length. In the case of the longitudinal current I_l, the total flux per unit length is given by

$$\phi_l = i\mu\left(1 - \frac{2r^2}{R^2-r^2} \times \log_e \frac{R}{r}\right). \tag{2}$$

In our cylinders $r=6$ mm, and $R=11$ mm. Substituting these values and dividing (1) by (2) we obtain $\phi_a/\phi_l = 2.5$ very nearly, Now when *H* is small (before the *A* effect has really developed) the galvanometer throws indicate roughly twice as great a decrease of magnetization when the current is axial as that produced by a current through the iron. This is near enough the calculated ratio of 2.5 to suggest that the *B* effect tends only to demagnetize the iron.

This demagnetizing effect can be discounted in two ways thus exposing, so to speak, the *A* effect more fully. One way was to send suitably adjusted currents of different values and in opposite directions through the iron and along its axis. Under these circumstances the *B* effect was very much reduced, though we were unable to neutralize it altogether; while the *A* effect gave us larger deflections than when a longitudinal current of the same value acted alone. Apparently the opposite direction of the currents did not alter the *A* effect but by reducing the *B* effect made it appear larger.

The other and even more effective way of reducing the circular flux was to cut a slot in the cylinder as already explained. This greatly increased the reluctance of the bar to circular flux and greatly reduced the *B* effect.

Our latest series of observations was made with the two similar hollow cylinders described above, one slotted, the other not. We made a series of readings for each of four cases beginning with a magnetizing field of less than 0.02 oersted, and ending with about 60 oersteds. The longitudinal and axial currents were both two amperes. These cases are:

No. 1, current through iron, no slot,
No. 2, current through axis, no slot,
No. 3, current through iron, slotted bar,
No. 4, current through axis, slotted bar.

The curves obtained are very illuminating, and are shown in Fig. 5 with case No. 2 plotted on a scale one-tenth that of the others with respect to the axis of abscissas which is laid off in galvanometer deflections for convenience, although it measures $\Delta\phi$, the change of flux.

If we contrast curves 2 and 4, it becomes evident that the slot has so greatly reduced the very large negative deflections of 2 that the *A* effect is again exhibited. A comparison of curves 1 and 3 also illustrates the effect of the slot though in a lesser degree than when 2 and 4 are compared. The positive galvanometer throws begin earlier and reach higher values in curve 3 than in the other three curves. It should further be noted that in this case the current flows through the iron of a slotted bar, so the circular flux is weaker than in the other cases. At any rate there is no evidence that the *A* effect is stronger in one case than in any other.

We have not yet studied permalloy beyond obtaining a qualitative agreement with the Norway iron. That is, when sufficiently magnetized, there is an increase in the residual flux caused by a current sent through a solid cylinder of the alloy. In the case of a cast iron cylinder bored out and slotted like our other cylinders, we find only a decrease in residual flux as far as we have been able to carry the magnetizing field. However it is significant that whereas an axial current causes a progressively larger decrease in the residual magnetism, as that is raised to higher and higher values, the effect of a current sent through the iron begins to diminish with large values of residual magnetism, and the curve, though still negative, slants upward toward the *H* axis which it seems to be approaching. This suggests the possibility that with sufficiently large magnetizing currents (larger than our coil would carry) the curve might cross the *H* axis, indicating that the *A* effect exists even in cast iron. The nickel bar gave results similar to cast iron, but as yet we have not examined its behavior very thoroughly.

A final experiment was a repetition of what we had roughly measured early in our work. This was a study of the effect of a mechanical jar on residual magnetism. The jar was produced by giving a light tap with a brass rod to the upper end of the bar which had previously been demagnetized both circularly and longitudinally, and then brought up to the desired magnetic intensity by first closing and then opening K_1. The results were surprisingly similar to curve 1, showing first a decrease of residual flux, then, for higher intensities, showing a reversal associated with double throws, and finally quite large deflections indicating the usual increase of residual magnetism. If however the magnetizing current was not broken, when *H* was small, the tap produced a rather large increase in magnetization, while with large values of *H*, beyond about twenty oersteds, the tap caused a decrease. Thus we found more or less complimentary curves like those of Fig. 3.

Discussion of Results

In explaining our observations we have to account for a number of effects which are not easily harmonized. It is necessary to find one or more causes which will simultaneously account for the facts: (a) that a longitudinal current through an iron bar tends to decrease small residual fluxes and increase large ones; (b) that a mechanical jar does the same thing; (c) that a current through the axis causes only a decrease in the residual magnetism, unless the bar is slotted, when results similar to (a) are obtained; (d) that these phenomena depend upon a sudden break in the magnetizing current, and that a more or less gradual decrease in that current tends more or less to result in decreases of the residual flux; (e) that in borderline cases there are

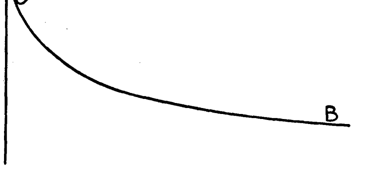

FIG. 4. Proposed decomposition of experimental curve.

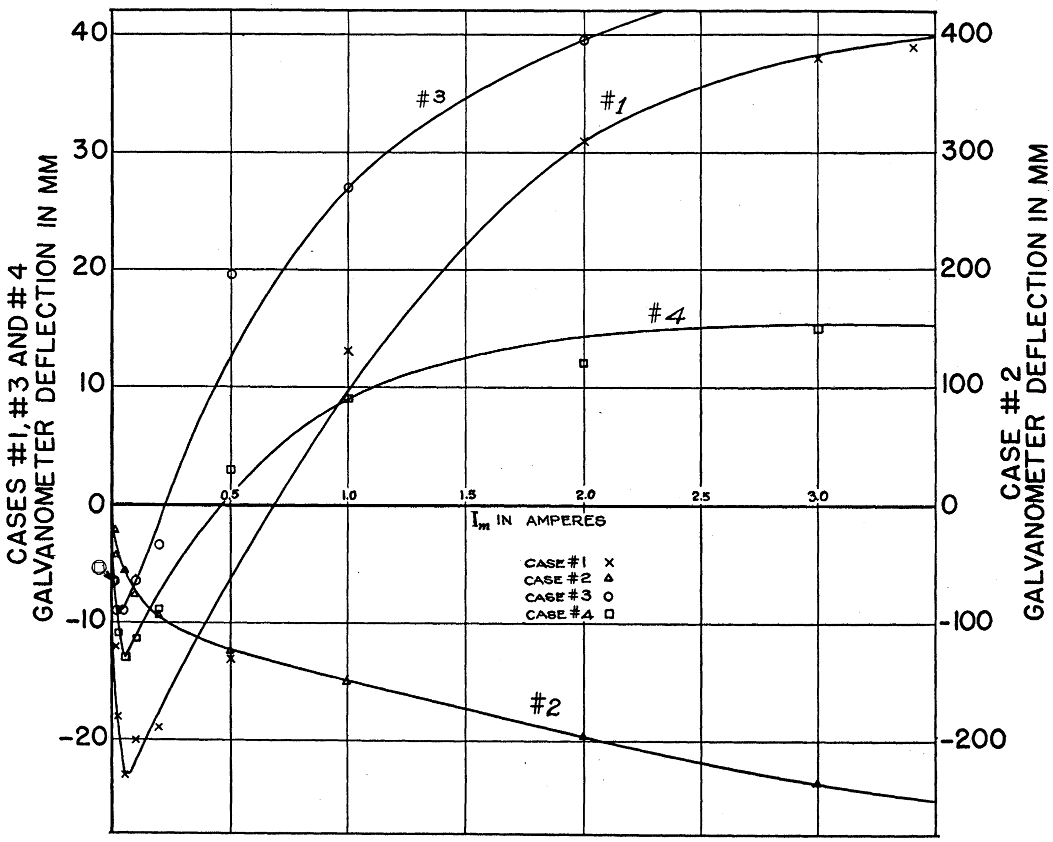

FIG. 5. Four experimental curves, with 2 plotted on a reduced scale of ordinates.

two galvanometer throws, the first being invariably the one indicating an increasing flux.

It might be argued that a circular field applied to a bar which has a residual longitudinal flux would produce a twist in the bar, and that such a twist combined with the circular flux would result in an increase in the residual flux provided the circular flux was small. This effect, based on Wiedeman's experiments, was observed by Nagaoka and Honda[4] in 1902. It amounts to a "Lenz law" reaction to the current which originally created the longitudinal magnetization. But we do not feel that this explanation is satisfactory, because it does not explain the increased flux caused by a mechanical jar after the bar had been carefully deprived of any residual circular flux. Nor does it take into account the fact that slow demagnetization destroys the phenomenon. And further it does not account for the remarkable result of slotting the bar which, if anything, should result in a reduced twist and therefore less of the proposed "Lenz law" reaction, as a result of weakening the circular flux, whereas we found just the opposite to be true.

It seems to us that a possible explanation is that on an abrupt break, the elementary dipoles in swinging away from the position of maximum magnetic intensity overshoot the mark, so to speak, by a kind of momentum, and are then brought back to a more stable position either by a trigger-like action of the mechanical jar, or by the abrupt setting up of a circular flux. This might account for our *A* effect. Our *B* effect is also brought about either by a jar or by abruptly setting up a circular field, but it invariably involves a decrease of magnetic intensity. This would be the expected result of a jar, and therefore it seems necessary to suppose that the abrupt creation of a circular field has much the same

---

[4] H. Nagaoka and K. Honda, Phil. Mag. **4**, 45–72 (1902).

effect, in that it tends to throw the elementary dipoles out of a longitudinal alignment. Thus the same two causes (jar and circular flux) seem to produce two effects, one tending to bring about a more stable alignment of elementary magnets that have overshot the mark, and the other tending to turn them out of that alignment. The fact that in the case of double throws (observed both with mechanical jar and abruptly created circular field) the trigger effect comes first shows that the decrease in longitudinal magnetization takes more time to be produced than the increase caused by a jar or by circular flux after the magnetizing field has been abruptly broken. This seems reasonable, because upsetting an unstable condition should take less time than a more or less permanent rearrangement of the dipoles associated with a decrease of magnetization.

DECEMBER 1, 1941 PHYSICAL REVIEW VOLUME 60

Internal Diamagnetic Fields*

W. E. Lamb, Jr.
Columbia University, New York, New York

(Received October 6, 1941)

In the precise molecular beam experiments of Rabi and his collaborators, it is necessary to know the value at the nucleus of the magnetic field produced by the diamagnetism of the atomic electrons. This has been calculated on the basis of the Fermi-Thomas model, and checked for a number of atoms by use of the available Hartree calculations. The statistical treatment gives for ratio of induced to external field $0.319\times 10^{-4}Z^{4/3}$, while the numerical coefficient on the basis of the Hartree model is lower by 19 percent at $Z=19$ and by 12 percent at $Z=80$. The effect is equivalent to a reduction of the nuclear g value by a factor of $(1-0.319\times 10^{-4}Z^{4/3})$, and in this form, the correction may be applied in the calculation of hyperfine structure of heavy atoms. The influence of the diamagnetic fields on an orbital electron has also been considered, and it is shown that it is equivalent to a reduction in the g value of an outer s electron by an amount of just the same order of magnitude as the relativistic correction calculated by Margenau.

In discussing the magnetic properties of solids, it is important to know the value of the magnetic field produced at one atom due to the action of all the *other* atoms. One uses for this some modification of the Lorentz formula.[1] We shall be concerned in this note with the field produced *within* an atom by its own diamagnetic moment when an external field is present. This problem has only arisen because of the very precise molecular beam measurements of Rabi and his collaborators.[2,3]

An external field H (taken along the z axis) may be described by the vector potential

$$\mathbf{A}=\tfrac{1}{2}[\mathbf{H}\times\mathbf{r}] \tag{1}$$

and this induces a diamagnetic current density in the atomic electrons of[4]

$$\mathbf{S}=e\mathbf{A}\rho(\mathbf{r})/mc, \tag{2}$$

where $-e$ is the charge on the electron and $\rho(\mathbf{r})$ is the charge density at $\mathbf{r}$. The induced field is then given by

$$\mathbf{A}'(\mathbf{r})=(e/2mc^2)\int d\mathbf{r}'\rho(\mathbf{r}')[\mathbf{H}\times\mathbf{r}']/|\mathbf{r}-\mathbf{r}'|. \tag{3}$$

When $\rho(\mathbf{r})$ is spherically symmetrical, this may

---

* Publication assisted by the Ernest Kempton Adams Fund for Physical Research of Columbia University.

[1] H. Casimir, *Magnetism and Very Low Temperatures* (Cambridge, 1940), p. 56.

[2] P. Kusch, S. Millman, I. I. Rabi, Phys. Rev. **55**, 1176 (1939); P. Kusch, S. Millman, Phys. Rev. **56**, 527 (1939); R. H. Hay, Phys. Rev. **60**, 75 (1941).

[3] P. Kusch, S. Millman, I. I. Rabi, Phys. Rev. **57**, 765 (1940); S. Millman, P. Kusch, Phys. Rev. **58**, 438 (1940); **60**, 91 (1941).

[4] This is most easily derived by considering the induced current in a conducting ring of radius r in the xy plane.