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The Values of R and of e/m, from the Spectra of H, D and He^+

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The 1940 data of Drinkwater, Richardson and Williams (DRW) on the H_α and D_α lines have been completely recalculated, to get values of the Rydberg constant for hydrogen and deuterium, and from these, for helium and for infinite mass. Explicit equations, some of which have not previously been published or used, are derived for all the necessary calculations. The value of R_∞ now obtained is appreciably higher than that published by DRW, but it is surprisingly low, compared to the value commonly used for the past decade, which is based on the 1927 work of Houston on H and He^+. His work, as well as later work by Chu, is also completely recalculated, with the use of the latest values of all auxiliary constants. About one-half of the proposed change in the value of R_∞ is due to new values of the index of refraction of air. Recent work on this subject is summarized and put in a form suitable for use by spectroscopists. The data of DRW, as well as that of Houston and Chu, lead also to values of E (atomic weight of the electron) and e/m. The explicit equations needed for such calculations are derived. All recent precision work on e/m is also recalculated. Thus one gets finally twelve different precision values of e/m, obtained by seven distinctly different experimental methods. Although the data are not as consistent as could be desired, there are no really serious discrepancies. If the adopted value of the Faraday were raised by about 30 coulombs, almost complete consistency of the e/m values would be obtained. There is, however, no direct experimental evidence to justify such a change. The recommended values of R, e/m and E are listed at the end of the paper. That for e/m is $(1.7592 \pm 0.0005) \times 10^7$ abs. e.m.u., as contrasted with the value 1.7591 ± 0.0003 recommended three years ago.

INTRODUCTION

THE simplest known type of atom is one that consists of a nucleus and a single external electron. Examples of this type are neutral hydrogen, neutral deuterium, and singly ionized helium. Since such an atom represents a two-body problem, it is possible to obtain a complete solution for the frequencies of the resulting spectrum. The solution involves the important constants R_∞ (the Rydberg constant for infinite mass), α (the fine-structure constant) and E (the atomic weight of an electron). Hence, from sufficiently precise and detailed frequency measurements in the spectrum of H, D and He^+, one can evaluate these three constants.

To obtain a value of α, certain measurements in *any one* of the three spectra are sufficient, whereas to obtain R_∞ and E, measurements in any *two* spectra are required. In this latter case the only auxiliary constants needed are the two relevant isotopic masses (in atomic weight units). If, in addition, one assumes a value of the Faraday F, the value of e/m for an electron may be calculated. Furthermore, in the case of the spectra of H and D, only certain small frequency *differences* are needed to evaluate E or e/m; but in order to obtain R_∞, the actual frequencies must be measured. On the other hand, when He^+ is one of the two spectra used, the actual frequencies must be known to obtain E, (and e/m), as well as to obtain R_∞. Finally, it should be noted that it is a wave-length *in air* that is ordinarily measured, and hence the value of the corresponding index of refraction of air is another necessary auxiliary constant. In fact the results of recent work on the index of refraction of air are chiefly responsible for the appreciable change now recommended for the hitherto accepted value of the Rydberg constant.

The primary purpose of the present paper is to consider in some detail the numerical values of R_∞ and e/m given by recent experimental work in this field, as recalculated with a consistent set of auxiliary constants, and as directly as possible from the original data. The calculation of α involves so many additional considerations, and so much additional experimental data, that a separate paper is needed for any adequate presentation. Such discussion of α as occurs in the present paper is therefore purely incidental to the calculation of R and of e/m.

In the course of these calculations I have come across several small but not negligible errors, some in the theory and others purely arithmetic. Furthermore, the method of calculation employed in the literature has often not been as direct as possible. A second purpose of the paper is therefore to present explicit equations, in what appears to be the most convenient form, for the several desired quantities in terms of the experimentally determined quantities. Many of these equations, so far as I know, have not previously been published or employed. Aside from their convenience for calculation, such explicit equations are needed in order to determine correctly, by propagation of errors, the probable error of the final result, as a function of the probable errors assigned to the various directly measured quantities.

The original purpose of this work was to calculate a new value of R_∞. Such a calculation incidentally results in an evaluation of e/m, and in order to make the paper reasonably complete, I have accordingly examined several additional papers that yield a value of e/m (but *not* of R_∞), from measurements in the spectra of H, D and He^+. The paper concludes with a brief discussion of all precision values of e/m that have been obtained in recent years, and a calculation of the present most reliable value of this important constant.

THEORY

In a two-particle atom such as neutral hydrogen, the electron and nucleus each revolve around the common center of mass. If the mass of the nucleus were infinite, the common center of mass would coincide with the center of the nucleus. In this hypothetical case we are concerned with R_∞, the so-called Rydberg constant for infinite mass, for which Bohr derived the theoretical expression $2\pi^2e^4m/h^3c$, where R_∞ is expressed, as usual, as a wave number (cm^{-1} units) and e is in e.s.u. The variation in the value of any actual Rydberg constant, such as R_H or R_D, from that of R_∞ is due solely to the failure of the center of mass of such an atom to coincide with the center of the nucleus.

R_∞ is now given in terms of the three specific Rydberg constants of interest to us by the equations

$$R_\infty=R_H(1+m/m_H), \tag{1}$$

$$R_\infty=R_D(1+m/m_D), \tag{2}$$

$$R_\infty=R_{He}(1+m/m_{He}) \tag{3}$$

where m = mass of electron, m_H = mass of hydrogen *nucleus*, etc. Since only a ratio of masses is involved, we may choose any desired unit of mass. For convenience we accordingly express mass in atomic weight units, on the *physical* scale, and Eqs. (1)–(3) then become

$$R_\infty=R_H(1+E/H^+), \tag{1'}$$

$$R_\infty=R_D(1+E/D^+), \tag{2'}$$

$$R_\infty=R_{He}(1+E/He^{++}) \tag{3'}$$

where E = atomic weight of electron, $H^+=H-E$, $D^+=D-E$, and $He^{++}=He-2E$, all on the physical scale of atomic weights.

Any *two* of these last three equations are sufficient, in order to evaluate E and R_∞. Furthermore, knowing E and R_∞ from any two equations, we can use the third equation to evaluate its own specific Rydberg constant. We can also express this third Rydberg constant explicitly in terms of the other two specific Rydberg constants. Such equations are given in this paper.

We first present various explicit equations for E. Since we have

$$e/m=F/E, \tag{4}$$

where F = Faraday, each explicit equation for E has a corresponding explicit equation for e/m. The unit of e/m depends on the system of units in which F is expressed, and we shall use e.m.u., since e/m is customarily given in such units.

From Eqs. (1′) and (2′)

$$E=\frac{H^+D^+(R_D-R_H)}{D^+R_H-H^+R_D}. \tag{5}$$

Similarly, from Eqs. (1′) and (3′)

$$E=\frac{H^+He^{++}(R_{He}-R_H)}{He^{++}R_H-H^+R_{He}}. \tag{6}$$

Now the actual *measured* wave numbers (s), after proper correction to give the "Balmer line" wave numbers, are directly related to R. Thus for the H_α and the D_α lines,

$$s_H=(5/36)R_H,\quad s_D=(5/36)R_D. \tag{7}$$

Therefore, putting Eq. (7) in Eq. (5), we get

$$E=\frac{H^+D^+(s_D-s_H)}{D^+s_H-H^+s_D}. \tag{8}$$

This last equation has been used by R. C. Williams[1] in his evaluation of E and e/m. A similar equation for He^+ and H is not of interest, since "corresponding" lines of He^+ and H have not been precisely measured or, as a more direct statement, the lines of He^+ and H that have been measured precisely are connected with their Rydberg constants by *different* proportionality factors. Hence Eq. (6) must be used, in evaluating E.

Equations (5), (6) and (8) are not in the most convenient form for calculation, nor for showing clearly how the assumed errors in the experimental quantities affect the resulting error in E. To get more convenient equations, we combine Eqs. (1′) and (2′) in a way that employs the familiar relations, if $A/B=C/D$, then

$$\frac{A-B}{B}=\frac{C-D}{D}, \quad \text{or} \quad \frac{A-B}{A}=\frac{C-D}{C}. \tag{9}$$

We thus get, from Eqs. (1′) and (2′), in place of Eq. (5),

$$E=\frac{\mathrm{H}^+\mathrm{D}(R_\mathrm{D}-R_\mathrm{H})}{R_\mathrm{H}(\mathrm{D}-\mathrm{H})} \tag{10}$$

or, using Eq. (7), we get

$$E=\frac{\mathrm{H}^+\mathrm{D}(s_\mathrm{D}-s_\mathrm{H})}{s_\mathrm{H}(\mathrm{D}-\mathrm{H})}. \tag{11}$$

The *proportional* errors in D, H, D−H, and s_H are much smaller than the error in the very small difference $s_\mathrm{D}-s_\mathrm{H}$. Thus Eq. (11) shows that, to a close approximation, the proportional error in E equals merely the proportional error in $s_\mathrm{D}-s_\mathrm{H}$. This equation also shows that, for the purpose of calculating E, it is not necessary to make a precise measurement of s_D and of s_H. Any approximate value from the literature may be used, and the entire experiment consists in measuring, as precisely as possible, the small *difference* $s_\mathrm{D}-s_\mathrm{H}$. The same statement applies to e/m, whose value follows from that of E, by merely assuming a value of F in Eq. (4). Thus, putting Eq. (4) in Eqs. (10) and (11) we obtain

$$e/m=\frac{R_\mathrm{H}F(\mathrm{D}-\mathrm{H})}{\mathrm{H}^+\mathrm{D}(R_\mathrm{D}-R_\mathrm{H})} \tag{12}$$

and

$$e/m=\frac{s_\mathrm{H}F(\mathrm{D}-\mathrm{H})}{\mathrm{H}^+\mathrm{D}(s_\mathrm{D}-s_\mathrm{H})}. \tag{13}$$

By combining Eqs. (1′) and (3′), using the relations of Eq. (9), we get, in place of Eq. (6),

$$E=\frac{\mathrm{H}^+\mathrm{He}^+(R_\mathrm{He}-R_\mathrm{H})}{R_\mathrm{H}(\mathrm{He}^+-\mathrm{H})}. \tag{14}$$

The corresponding equation for e/m is

$$e/m=\frac{R_\mathrm{H}F(\mathrm{He}^+-\mathrm{H})}{\mathrm{H}^+\mathrm{He}^+(R_\mathrm{He}-R_\mathrm{H})}. \tag{15}$$

This last equation is Eq. (8), p. 46 of G.C.1929,[2] while Eq. (14) is Eq. (7) of the same page. In that paper the value of e/m was calculated from Houston's data[3] on H and He^+. In Eq. (15), the probable error in e/m is due almost entirely to the probable errors assigned to F and to the *small* difference, $R_\mathrm{He}-R_\mathrm{H}$. The errors in R_H, He^+, H^+ and He^+-H are comparatively negligible. It should also be noted that in the case of $H^+=H-E$, $He^+=He-E$, etc., only an approximate value of E need be used. The precise value of E is then given by Eq. (14) or Eq. (11), etc.

Just as Eqs. (1′) and (2′), or (1′) and (3′), have been solved simultaneously for E, so they can be solved for R_∞. But in the literature it has been standard practice to get R_∞ from Eq. (1′), with a value of E obtained by *assuming* values of F and e/m, in Eq. (4), i.e.,

$$R_\infty=R_\mathrm{H}\left(1+\frac{F}{(e/m)\mathrm{H}^+}\right). \tag{16}$$

I used Eq. (16) on p. 49 of G.C.1929, but one does not thus get the most reliable value of R_∞, since the measurements on He^+ (or on D) are thereby ignored—unless one adopts just the value of E yielded by the experiment. The following equations for R_∞, which give its value directly in terms of the experimental quantities, have not, so far as I know, previously appeared in the literature.

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[1] R. C. Williams, Phys. Rev. **54**, 568 (1938).

[2] R. T. Birge, Rev. Mod. Phys. **1**, 1 (1929). This paper is always referred to as G.C.1929.

[3] W. V. Houston, Phys. Rev. **30**, 608 (1927).

From Eqs. (1′) and (2′), by eliminating E, we get

$$R_\infty=\frac{R_\mathrm{H}R_\mathrm{D}(\mathrm{D}-\mathrm{H})}{\mathrm{D}^+R_\mathrm{H}-\mathrm{H}^+R_\mathrm{D}}. \tag{17}$$

This equation corresponds to Eq. (5) for E, and again by using Eq. (7) we get

$$R_\infty=\frac{36}{5}\left\{\frac{s_\mathrm{H}s_\mathrm{D}(\mathrm{D}-\mathrm{H})}{\mathrm{D}^+s_\mathrm{H}-\mathrm{H}^+s_\mathrm{D}}\right\} \tag{18}$$

which corresponds to Eq. (8) for E. Just as in the case of Eqs. (5) and (8), we can get a more convenient form of Eqs. (17) and (18) by using the relations of Eq. (9). Thus from Eqs. (1′) and (2′) we get

$$R_\infty-R_\mathrm{H}=\frac{R_\mathrm{D}-R_\mathrm{H}}{1-(R_\mathrm{D}\mathrm{H}^+/R_\mathrm{H}\mathrm{D}^+)}. \tag{19}$$

In order to evaluate R_∞ we therefore need to calculate accurately only the *small* difference $R_\infty-R_\mathrm{H}$, instead of the large factors of Eq. (17).

In terms of wave numbers Eq. (19) becomes

$$R_\infty=\frac{36}{5}\left\{s_\mathrm{H}+\frac{s_\mathrm{D}-s_\mathrm{H}}{1-(s_\mathrm{D}\mathrm{H}^+/s_\mathrm{H}\mathrm{D}^+)}\right\}. \tag{20}$$

Equations (19) and (20) thus represent more convenient forms, for accurate calculation, of Eqs. (17) and (18), and in fact Eqs. (19) and (20), when multiplied out, go directly into Eqs. (17) and (18).

In a similar way, from Eqs. (1′) and (3′), we get

$$R_\infty-R_\mathrm{H}=\frac{R_\mathrm{He}-R_\mathrm{H}}{1-(R_\mathrm{He}\mathrm{H}^+/R_\mathrm{H}\mathrm{He}^{++})}, \tag{21}$$

which corresponds to Eq. (19).

One can, of course, get from Eqs. (1′) and (3′) the less convenient form, corresponding to Eq. (17). It is

$$R_\infty=\frac{R_\mathrm{H}R_\mathrm{He}(\mathrm{He}^+-\mathrm{H})}{\mathrm{He}^{++}R_\mathrm{H}-\mathrm{H}^+R_\mathrm{He}}. \tag{22}$$

This completes the equations for evaluating R_∞. But, as already noted, one can also express any one of the three Rydberg constants explicitly in terms of the other two. We shall give the result only for R_He in terms of R_H and R_D, and then only in the form that uses the relations of Eq. (9). This result is

$$R_\mathrm{He}-R_\mathrm{H}=\frac{\mathrm{D}(R_\mathrm{D}-R_\mathrm{H})(\mathrm{He}^+-\mathrm{H})}{\mathrm{He}^+(\mathrm{D}-\mathrm{H})} \tag{23}$$

or, in terms of wave numbers,

$$R_\mathrm{He}=\frac{36}{5}\left\{s_\mathrm{H}+\frac{s_\mathrm{D}-s_\mathrm{H}}{\left(\dfrac{\mathrm{He}^+(\mathrm{D}-\mathrm{H})}{\mathrm{D}(\mathrm{He}^+-\mathrm{H})}\right)}\right\}. \tag{24}$$

We have already noted how the error in E depends upon the errors in s_H and s_D. Let us now determine, from Eq. (20), the corresponding error in R_∞. We note in the first place that the denominator, lying under $(s_\mathrm{D}-s_\mathrm{H})$ as numerator, has a value close to 0.5, since $s_\mathrm{D}\mathrm{H}^+/s_\mathrm{H}\mathrm{D}^+$ equals approximately 0.5. Hence the assigned probable errors in s_D and s_H, as they occur in this denominator, produce a negligible effect on R_∞, as compared to these same errors, applied to the *small* difference $s_\mathrm{D}-s_\mathrm{H}$.

For a similar reason the small probable errors in H^+ and D^+ may be neglected. Thus for the purpose of calculating the probable error of R_∞ one can, to a very close approximation, merely assume that the denominator lying under $s_\mathrm{D}-s_\mathrm{H}$, in Eq. (20), is free from error, and has the value 0.5. The equation then becomes

$$R_\infty=\frac{36}{5}\{2s_\mathrm{D}-s_\mathrm{H}\}. \tag{25}$$

From the assigned errors in s_D and s_H, we now calculate by propagation of errors the resulting error in $2s_\mathrm{D}-s_\mathrm{H}$, and the *proportional* error in this quantity is seen to equal the proportional error in R_∞.

In particular, if both s_D and s_H are assigned the *same* absolute error e, then the absolute error in $2s_\mathrm{D}-s_\mathrm{H}$ is $e\sqrt{5}$, and since $s_\mathrm{D}\sim s_\mathrm{H}$, or $2s_\mathrm{D}-s_\mathrm{H}\sim s_\mathrm{H}$, the *proportional* error in $2s_\mathrm{D}-s_\mathrm{H}$ (*and* in R_∞) equals $e(5)^{\frac{1}{2}}/s_\mathrm{H}$. In other words, the proportional error in R_∞ is, to a close approximation,[4] $\sqrt{5}$

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[4] This method of calculating the probable error in R_∞ is essentially equivalent to writing out the rather complex expression for the probable error, as given rigorously by the law of propagation of errors [R. T. Birge, Am. J. Phys. **7**, 351 (1939)] and then neglecting various terms of the expression that are found to be negligibly small compared to the remaining terms. It must be remembered, in this connection, that Eq. (25) is *not* sufficiently accurate for the calculation of a precision value of R_∞.

times the assigned proportional error in s_H (or in s_D).

In a similar way, the denominator of Eq. (24) is very closely $\frac{2}{3}$, and Eq. (24) is therefore approximately

$$R_{He} = \frac{36}{5}\left(\tfrac{3}{2}s_D - \tfrac{1}{2}s_H\right) \qquad (26)$$

from which one can quickly calculate the probable error in R_{He}, resulting from the assigned probable errors of s_D and s_H. In particular, if both s_D and s_H have the same assigned error, the proportional error in R_{He} is $(5/2)^{\frac{1}{2}}$ times the proportional error of either, and hence only $(1/2)^{\frac{1}{2}}$ as great as the resulting proportional error in R_∞.

One can obviously express either R_∞ or R_{He} in terms of its divergence from R_D, instead of from R_H. Thus Eq. (19), or its more convenient form, Eq. (20), may be replaced by

$$R_\infty = \frac{36}{5}\left\{ s_D + \frac{s_D - s_H}{\dfrac{s_H \cdot D^+}{s_D \cdot H^+} - 1} \right\}. \qquad (27)$$

But Eqs. (20) and (27) are merely different forms of the same equation and hence necessarily yield identical values of R_∞, and also identical probable errors.

The calculated values and calculated probable errors given in the following sections of the paper have been obtained by the use of the equations given in this section,—in particular, Eqs. (11), (13), (14), (15), (20), (21), (24), (25), and (26).

Index of Refraction of Air

In G.C.1929 the adopted value of R_∞ was 109,737.42±0.06 cm^{-1}. This value, as already stated, was based on Houston's data.[3] With the same data but with more recent values of the auxiliary constants, I obtained,[5] in August 1939, the value 109,737.45±0.06 cm^{-1}. Since then there has appeared an important paper by Drinkwater, Richardson and Williams,[6] who obtained $R_\infty = 109{,}737.264$ cm^{-1}, with the auxiliary constants of G.C.1939. I have now recalculated their data and obtain 109,737.303±0.017 cm^{-1}. The causes of this revision appear in the detailed discussion to follow. It is, however, immediately obvious that the new value of R_∞ deviates by a surprisingly large amount from the 1929 value. In fact most of the rather large probable error of ±0.06 cm^{-1} in the 1929 result arises from the necessary uncertainty of the value of the international meter in terms of the red cadmium wave-length. This wave-length is 6438.4696 I.A., *defining* the I.A. system of wave-lengths, but it is $(6438.4696 \pm 0.0020) \times 10^{-8}$ cm in the c.g.s. system.[7]

The probable error in the 1929 value of R_∞, *in the* I.A. *system*, was believed to be only about 0.02 cm^{-1}, as contrasted with the change of 0.12 cm^{-1} now suggested. One important cause for this large change, as DRW point out, is the new value of the index of refraction of air used by them. I shall therefore, before proceeding further in the calculation of R_∞, summarize the present experimental data on the dispersion of air. Such a

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[5] As a preliminary to a new detailed article on the general constants, I compiled, in August 1939, a mimeographed list of constants, together with a brief explanation of the more important changes. This list will be called G.C.1939. As a result of the appearance of new data, as well as of numerous suggestions received since then, many changes have been made in the list. The present article gives the detailed explanation of the change in R_∞.

[6] J. W. Drinkwater, Sir Owen Richardson, and W. E. Williams, Proc. Roy. Soc. **A174**, 164 (1940). To be referred to as DRW.

[7] The ±0.0020 estimate of error is the value that is needed to give Houston's estimate[3] of the absolute error in R_H and R_{He}, as contrasted with the error in their difference, used on p. 46 of G.C.1929. J. E. Sears and H. Barrell, Trans. Roy. Soc. **A233**, 143 (1934) give a list of four values of the wave-length of the red cadmium line, as measured directly in terms of the international standard meter. The unweighted average is $(6438.4696 \pm 0.0004) \times 10^{-8}$ cm. In a later report J. E. Sears, Science Progress **31**, 209 (1936), adds two more recent measurements by Kösters and Lampe, at the Reichsanstalt. The new unweighted average is 6438.4690 (Sears gives 6438.4693, and the origin of his average is not evident), but the probable error is still ±0.0004. L. J. Briggs, Rev. Mod. Phys. **11**, 111 (1939), reproduces Sears' table, but without the original references. In spite of the remarkable agreement shown by the various measurements of the red cadmium line (the *average deviation* from the mean is only one part in seven million), it seems necessary to adopt the larger uncertainty of $\pm 0.0020 \times 10^{-8}$ cm, because of the unsatisfactory character of the defining scratches on the international meter. W. E. Williams, Nature **135**, 496 (1935), gives a photograph of one of the terminal scratches on substandard meter No. 26, magnified 300 diameters. The scratch, which is actually a *series* of rather irregular parallel furrows, is over two mm wide (i.e., 0.006 mm on the actual meter bar, or 6 parts in 10^6 of its length). Thus the adopted probable error in the wave-length of the red cadmium line corresponds to an error of only 1/20 of the width of such a terminal scratch. The fact that the average deviation of the results of different observers corresponds to only 1/40 of the width is chiefly evidence of a uniformity of *interpretation* of the terminal scratches on the international meter bar.

summary seems especially advisable, because thus far no tabulated data have been published in the form needed in spectroscopy.

Since 1918 all measured wave-lengths in spectroscopy have been reduced to vacuum by the use of Meggers and Peters' formula[8] for the dispersion of air. The well-known tables of Kayser[9] are based on their data. But since 1918 the national physical laboratories of Germany, France and England have made new measurements, with almost identical results, but the new results differ considerably from those of Meggers and Peters. The German work[10] has been published only in abstract form, and the paper giving the French work[11] is not readily available. Fortunately, however, all of the results are summarized in the report on the English work[12] although, as just noted, not explicitly in the form needed by spectroscopists.

All standard spectroscopic wave-lengths are based on the wave-length of the red Cd line, in "normal" air, at 15°C, 760 mm. "Normal" air is at present taken to mean air free from water vapor, but containing 0.03 percent of CO_2. Now all of the recent published results on the index of refraction of air refer to CO_2-free air. To reduce them to "normal" air I have added 0.044×10^{-6} to the value of μ, in accordance with the data of Barrell and Sears.[13] Their published equation for the 20°C value of μ for CO_2-free air[14] has been reduced to 15°C by the use of their own measured value of $\alpha=0.003674$. Perard's results[11] have been changed from 0°C to 15°C by means of his measured value of $\alpha=0.003716$. Kösters and Lampe's results[10] have been changed from 20° to 15° by means of their quoted $\alpha=0.00367$. The exact constitution of the air used by Meggers and Peters[8] was not measured, and it is now merely assumed to have been "normal" air, so that no correction for CO_2 content needs to be made.

One further recent investigation on the dispersion of air is that of Bender.[15] His results differ from those of references 10, 11 and 12 even more than do those of Meggers and Peters, and in the opposite sense. His equation for μ at 0°C has been reduced to 15°C by the Barrell and Sears value of the observed α for air (0.003674), in place of the coefficient of a perfect gas (0.003661) used by Bender himself.

Remembering that all published results, except those of Meggers and Peters, refer to CO_2-free air, we then get, finally, for the dispersion of "*normal*" air at 15°C, 760 mm pressure,

$$(\mu-1)\times10^6=a+b/\lambda^2+c/\lambda^4 \qquad (28)$$

where λ is the wave-length in air, in microns, and the coefficients have the following values.

	a	b	c
Reference 8 (M and P)	272.643	1.2288	0.03555
Reference 10 (K and L)	272.742	1.5017	0.01834
Reference 11 (Perard)	272.860	1.4002	0.02994
Reference 12 (B and S)	272.581	1.5453	0.01268
Reference 15 (Bender)	273.531	1.4946	0.03990

One can get a proper comparison of these various results only by noting the resulting values of μ, at various parts of the visible spectrum. Such typical values of $(\mu-1)\times10^6$ are as follows.

Reference	8	10	11	12	15
λ6438.47	275.814	276.471	276.412	276.383	277.369
λ5085.82	277.925	278.822	278.711	278.745	279.906
λ4358.32	280.097	281.156	281.061	281.068	282.505

It will be noted that references 10, 11 and 12 agree almost perfectly, whereas the results of reference 8 are low and those of reference 15 are high. In all of the calculations of this paper I have used the Barrell and Sears' 1939 dispersion curve (reference 12), since their work is the most recent, and appears to be also the most thorough.

In 1934 Sears and Barrell,[7] in connection with their measurement of the red Cd line in terms of the standard meter, made a determination of μ for air, for the red Cd line only. The value that they obtained, for "normal" air, is 276.494.[16] This gives 1.7802A for the wave-length correction of the red Cd line to vacuum. Hence if, in agreement with Sears and Barrell,[7] we adopt 6438.4696×10^{-8} cm as the best value of the wave-length in "normal" air, the best value in

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[8] W. F. Meggers and C. G. Peters, Bull. Bur. Stand. **14**, 697 (1918).

[9] H. Kayser, *Tabelle der Schwingungszahlen* (Leipzig, 1925).

[10] Kösters and Lampe, Physik. Zeits. **35**, 223 (1934).

[11] A. Perard, Trav. Bur. int. Poids Mes. **19**, 1 (1934).

[12] H. Barrell and J. E. Sears, Phil. Trans. **238**, 1 (1939).

[13] Reference 12. See p. 54.

[14] Reference 12. See p. 30.

[15] D. Bender, Phys. Rev. **54**, 179 (1938).

[16] Sears and Barrell give μ only for CO_2-free air, but their published values of λ6440.25099 in vacuum and λ6438.4708 in "normal" air correspond to this value of $(\mu-1)\times10^6$, as contrasted with their 1939 value of 276.383, as already given.

vacuum is 6440.2498×10^{-8} cm on the basis of their 1934 work. On the other hand, the 1939 work of Barrell and Sears[12] gives 1.7795A as the correction, and hence 6440.2491×10^{-8} cm as the wave-length in vacuum. This 1939 work evidently appeared just as the DRW paper[6] was ready for the press. Hence all of the published DRW results are based on the 1934 value of $\lambda 6440.2498$ as the vacuum wave-length, and to these they have added adjusted values corresponding to the 1939 value of $\lambda 6440.2491$. In discussing their data, it is more convenient to leave it in its original form, and finally to make this small adjustment, just as DRW have done. It amounts to an *increase* of each wave number by 1.687 parts in 10^7. This increase is 0.0016_{56} cm^{-1} for the H_α and D_α lines.

EXPERIMENTAL DATA OF DRW, AND CALCULATIONS

In order to obtain a value of R_∞, DRW[6] measured *in vacuum* with a reflection echelon the wave-lengths of the H_α and D_α lines. This instrument consisted of 40 plates, each 6.88 mm thick, and had a calculated resolving power of 0.018 cm^{-1}, or of $846{,}000 (=\lambda/\Delta\lambda)$ at the wave-length used. The chief objection to this type of instrument is doubtless the limited field (not over two spectral orders) and the consequent radical warping of the true intensity distribution by the "envelope" of the diffraction pattern. However, when the theoretical correction had been made for this warping, the resulting corrected microphotometer curves were not only extremely self-consistent, but they also satisfied the various known or expected intensity relations. Hence the final experimental results appear to be quite reliable. The wave-lengths in vacuum of the H_α and D_α lines were measured directly in terms of the red Cd line, so that, as already noted, only the vacuum wave-length of that line need be known. DRW publish only the resulting wave numbers in vacuum of the various lines measured.

The theoretical pattern of the fine structure components of the H_α (and D_α) line and the energy level diagram from which the pattern is obtained, are given by DRW on page 165 of their paper. Each of the levels lies *below* the "Balmer" level by an amount,[17]

$$\Delta T = \frac{R\alpha^2 Z^4}{n^3}\left(\frac{1}{j+\frac{1}{2}} - \frac{3}{4n}\right) \tag{29}$$

where R_{H} should be used for H_α and R_{D} for D_α, with $Z=1$. In the case of He 4686 one uses R_{He} and $Z=2$. The "Balmer" lines are given by the simple formula

$$s = RZ^2\left(\frac{1}{n''^2} - \frac{1}{n'^2}\right). \tag{30}$$

The problem is thus to obtain in some way the wave number of some *one* fine structure component, and then to apply the necessary correction to give the wave number of the Balmer line, which for H_α and D_α corresponds to $n'=3$, $n''=2$. The component chosen by DRW[6] and by Houston[3] is number one, for which $j'=5/2$, $j''=3/2$. This, the strongest of the predicted five components, cannot be resolved from components 4 and 5, which should lie to the long wave-length side at distances of approximately 0.036 and 0.144 cm^{-1}, with theoretical intensities 1/9 and 1/45 of that of component 1. The "center of mass" of these three components accordingly lies 0.00637 cm^{-1} to the red of component 1, if $1/\alpha$ is taken as 137. Houston[3] used a correction of 0.0056 cm^{-1}, and Williams and Gibbs[18] pointed out that, on the foregoing simple assumptions, 0.0063 cm^{-1} should have been used.

The problem is, however, much more involved, since each fine structure component has its own intensity distribution, due mainly to the Doppler effect. The half-width depends on experimental conditions, and must be determined in each case more or less empirically.[19] But both Williams and Gibbs[18] and DRW[6] agree that if the upper part of the intensity curve is used to locate its "center line," component 5 has a negligible influence and this center line lies almost exactly 0.0036 cm^{-1} to the red of component 1. Incidentally this is just

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[17] See H. E. White, *Introduction to Atomic Spectra* (McGraw-Hill, 1934), p. 137.

[18] R. C. Williams and R. C. Gibbs, Phys. Rev. **45**, 491 (1934).

[19] Thus R. C. Williams and R. C. Gibbs, Phys. Rev. **45**, 475 (1934) find half-widths of 0.180 cm^{-1} and 0.171 cm^{-1} for the components of H_α and D_α, respectively. Their paper gives a detailed account of the method used by them in analyzing the microphotometer curves. Different investigators have used somewhat different methods.

the correction gotten on the simple "center of mass" theory, when the very weak component 5 is ignored.

In subsequent work R. C. Williams[1] found a half-width of 0.165 cm^{-1} and a correction of 0.0041 cm^{-1} for the effect of component 4 of H_α. He then makes an additional correction of -0.0012 cm^{-1} for the effect of component 3, which consists really of two coincident components, of total intensity about $\frac{1}{8}$ that of component 1, and which lies 0.221 cm^{-1} to the violet of component 1. DRW give only the position of their 1,4 complex and state that the effect of component 3 has already been taken into account in determining the position of this complex. Williams[1] uses slightly different corrections for H_α and D_α whereas DRW use the same correction for both. A general study of the evidence indicates that 0.0036 cm^{-1} is probably the best correction to be applied to the data of both Houston[3] and DRW,[6] and for both the H_α and the D_α lines. As a matter of fact, whether 0.0036 or 0.0056 cm^{-1} is chosen for the correction makes very little difference in the resulting value of the Rydberg constant but, as will be shown presently, Houston's data are internally more consistent when 0.0036 cm^{-1} is used.

Houston also used component 1 of the H_β line to make an additional evaluation of R_H. The simple "center of mass" procedure here gives 0.00269 cm^{-1} as the necessary correction to the 1,4,5 complex. Williams and Gibbs[18] erroneously publish this last result as 0.0029 cm^{-1}, but they find by the type of empirical analysis used for H_α that 0.0024 cm^{-1} is the required correction. This latter figure happens to be just the correction used by Houston,[3] and I have accordingly adopted it.

We now start the explicit calculation of the DRW data on H_α and D_α. The wave number of the 1,4 complex of H_α was obtained from 10 different plates, with an unweighted average of $15{,}233.0654\pm0.0003_2$ cm^{-1}, where the stated uncertainty is merely the standard probable error of a mean, and hence is based wholly on internal consistency of the data. To allow for possible systematic errors I have arbitrarily included an additional error of 0.0010 cm^{-1}, which may add to or subtract from the preceding error. Hence the resulting uncertainty (square root of sum of squares of the errors) is $\pm0.0010_5$ cm^{-1}. DRW base all uncertainties, throughout their paper, on the root-mean-square (r.m.s.) error of a *single* result (or single plate in this case), with no further allowance for errors of any kind. There seems to be no particular logic in such a procedure since according to it a series of observations yields a result no more reliable than a single observation. In this case their published uncertainty is ±0.0014 cm^{-1}.

We now add 0.0036 cm^{-1} to the wave number just calculated, and thus get 15,233.0690 cm^{-1} for the wave number of component 1. This value is now to be corrected to get the Balmer line. The necessary (negative) correction[20] is, by Eq. (29), $R_H\alpha^2(1/64-1/324)=0.07327$ cm^{-1}, if $1/\alpha=137$. It seems best, wherever possible, to use the value of α determined by the data themselves but, as is well known, it has not been possible to get a very reliable value of α from the analysis of H_α and D_α. On the other hand, the magnitude of the correction is so small in this case that it is almost immaterial what value of α is used. As a matter of fact, these last two facts are connected logically. Thus in the case of λ4686 the correction is large and it is important what value of α is used. But there the fine structure components are more widely spaced and it is possible to get a fairly reliable value of α from their analysis.

With the Balmer line correction just given, we get $15{,}232.9957_3$ cm^{-1} for the Balmer line itself. This result, however, is in terms of the 1934 index of refraction of air.[7] It becomes 15,232.9974 cm^{-1} in terms of the 1939 index,[12] and this last quantity I denote s_H. Thus[21]

$$s_H=15{,}232.9974\pm0.0003_2\ \mathrm{cm}^{-1}$$
$$\text{or}\quad \pm0.0010_5\ \mathrm{cm}^{-1}.$$

Multiplying s_H by 7.2 (the reciprocal of $1/4-1/9$ in Eq. (30)) we get

$$R_H=109{,}677.581_2\pm0.002_3\ \mathrm{cm}^{-1}$$
$$\text{or}\quad \pm0.007_5\ \mathrm{cm}^{-1}.$$

DRW give $R_H=109{,}677.583\pm0.010$ cm^{-1}.

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[20] DRW write 0.072–0.0036 as the total correction to be subtracted, but give this difference as 0.0694, which happens to be nearly correct. (0.0733–0.0036 = 0.0697.)

[21] I give the calculated results of DRW in general with two probable errors. The first is based merely on the internal consistency of the data, whereas the second (which is *adopted*) includes some allowance for systematic error. When only one probable error is stated, it is the latter of the two.

The D_α line is now to be similarly treated, in order to evaluate R_D. The 1,4 complex of this line is measured by DRW on 7 plates, with an unweighted average of $15{,}237.2096 \pm 0.0003_2$ cm^{-1}. The probable error happens to be identical with that for H_α and I have again allowed 0.0010 cm^{-1} for systematic error. As already discussed, a correction of $+0.0036$ cm^{-1} is again to be applied to get component 1, and the correction for the Balmer line is 0.07329 cm^{-1}, the small change from that for H_α being due to the use of R_D in place of R_H. With the additional correction for the 1939 index of refraction, one thus gets finally

$$s_D = 15{,}237.1416 \pm 0.0003_2 \text{ cm}^{-1} \quad \text{or} \quad \pm 0.0010_5 \text{ cm}^{-1}$$

and

$$R_D = 109{,}707.419_3 \pm 0.002_3 \text{ cm}^{-1} \quad \text{or} \quad \pm 0.007_5 \text{ cm}^{-1}.$$

DRW give $R_D = 109{,}707.421 \pm 0.009_2$ cm^{-1}.

We now calculate R_∞ directly from the observed values of s_H and s_D, by the use of Eq. (20). For this purpose we need values of the atomic weights of H and D, and for convenience I give at this point all the auxiliary constants that will be needed in the remainder of the work. They are (all on the physical scale)

$H = 1.00813 \pm 0.00001_7$,
$D = 2.01473 \pm 0.00001_9$,
$He = 4.00389 \pm 0.00007$,
$C = 12.01465 \pm 0.00023$,
$F = 9651.4_0 \pm 1.0$ abs. e.m.u.,
$E = 0.0005486$ (used in getting H^+, D^+ etc.),
$p = 1.00048 \pm 0.00002$ (one int. ohm $= p$ abs. ohm),
$q = 0.99986 \pm 0.00002$ (one int. amp. $= q$ abs. amp.),
$pq = 1.00034 \pm 0.00003$ (one int. volt $= pq$ abs. volt),
$c = (2.99776 \pm 0.00004) \times 10^{10}$ cm/sec.

The values of the atomic weights and of p are those used in a previous letter[22] on e/m, but the value of q has been decreased by a significant amount.[23]

Equation (20) then gives

$$R_\infty = 109{,}737.302_7 \pm 0.005_2 \text{ cm}^{-1} \quad \text{or} \quad \pm 0.016_8 \text{ cm}^{-1}.$$

The method of calculating the probable error in R_∞, from the assigned probable errors in s_H and s_D, has been discussed earlier in the paper. Equation (20) gives just the result that would be obtained by calculating e/m from Eq. (12), and then using this value of e/m in Eq. (16). This latter procedure was used by DRW, but they obtained $109{,}737.264_4 \pm 0.020$ cm^{-1}, although they used the latest values of the auxiliary constants, as just listed. The discrepancy is due to two errors in their use of Eq. (16). The atomic weight of neutral H was used in place of the correct H^+, and the atomic weight of the mixture of hydrogen isotopes (H and D) was used in place of the correct isotope of mass 1. I noticed the latter error, when the DRW paper first appeared, and calculated corrected values, which DRW later published.[24] Unfortunately I did not notice the

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[22] R. T. Birge, Phys. Rev. **54**, 972 (1938).

[23] The values of p and q are now in a very satisfactory state. An entire paper would be required to cover the details, but the values here adopted are obtained as follows. (1) H. L. Curtis, R. W. Curtis, and C. L. Critchfield, J. Research Nat. Bur. Stand. **22**, 485 (1939) obtain $q = 0.99986$ in terms of the N.B.S. int. amp. This becomes 0.999868 in terms of the mean int. amp. (2) P. Vigoureux, N.P.L. Collected Researches **24**, 173 (1938) gets $q = 0.99986$ in terms of the N.P.L. int. amp. which becomes 0.99985 in terms of the mean int. amp. From these two results I get as a final average, $q = 0.99986 \pm 0.00002$, in terms of the mean int. amp. (3) H. L. Curtis, C. Moon, and C. M. Sparks, J. Research Nat. Bur. Stand. **21**, 375 (1938) obtain as the weighted average of this and a previous investigation, $p = 1.000468$ in terms of the N.B.S. int. ohm, or 1.000472 in terms of the mean int. ohm. (4) Two investigations at the N.P.L., one by L. Hartshorn and N. F. Astbury, Phil. Trans. **A236**, 423 (1937), and the other by P. Vigoureux, N.P.L. Collected Researches **24**, 277 (1938) each give $p = 1.00050$ in terms of the N.P.L. int. ohm, or 1.000508 in terms of the mean int. ohm. A slightly higher accuracy is claimed for the N.B.S. work, and I accordingly adopt as a final weighted average, $p = 1.00048 \pm 0.00002$, in terms of the mean int. ohm.

The value of F given in reference 22 is 9651.31 ± 0.80 abs. e.m.u., on the physical scale. A slightly more precise calculation shows that this figure should have been given as 9651.33 e.m.u. The present adopted value is only slightly larger, but actually two large and almost compensating changes have been made. In the first place the value of q has been changed from 0.99993 to 0.99986. This change *decreases* the value of F by 0.67 e.m.u. On the other hand, I have now adopted a certain weighted average of the values of F given by the silver and the iodine voltameters, in place of merely the silver result. This very involved and controversial subject will be discussed in detail elsewhere. The change *increases* the value of F by 0.73 e.m.u. A very small change (from 1.00027 to 1.000272) has also been made in the ratio of the physical and chemical scales of atomic weights. The net result of these various changes is an increase of 0.07 e.m.u. in the value of F.

The atomic weights are the standard values given by M. S. Livingston and H. A. Bethe, Rev. Mod. Phys. **9**, 245 (1937), except that for carbon, which I have derived from the latest experimental results.

[24] DRW, Proc. Roy. Soc. **A175**, 345 (1940).

other and more serious error until I had started the systematic calculations of this paper.[25]

In addition to R_∞, the data of DRW allow the calculation of E and of e/m. The value of E may be obtained directly from Eq. (11), with the result

$$E=(5.48647\pm 0.00059)\times 10^{-4}$$
$$\text{or}\quad \pm 0.00099.$$

The error in E depends directly on that in (s_D-s_H) and any systematic error in this *difference* is likely to be less than that in either s_D or s_H. Hence the second quoted probable error in E is based on an assumed systematic error of only 0.0006 cm^{-1} in s_D-s_H, in place of the 0.0010 cm^{-1} systematic error already adopted for s_D and s_H, separately.

Then from Eq. (4), or directly from Eq. (12),

$$e/m=(1.75913\pm 0.00027)\times 10^{7}\ \text{e.m.u.}$$
$$\text{or}\quad \pm 0.00037.$$

DRW, using Eqs. (8) and (4), get $E=(5.48646\pm 0.00050)\times 10^{-4}$, and $e/m=(1.7591\pm 0.0004)\times 10^{7}$ e.m.u. For this last result they used $F=9651.31$ abs. e.m.u.

A discussion of the causes for the deviation of DRW's value of R_∞ from my 1929 value is postponed until Houston's data[3] have been recalculated. But it should be noted at this time that there is a second method for calculating R_∞ from data such as that of DRW. Instead of using Eq. (20), we may use Eq. (1′), or its equivalent, Eq. (16), with some "outside" value of e/m and of F (to give E of Eq. (4)). R_∞ may also be calculated from Eq. (2′), and Eqs. (1′) and (2′) will then yield *different* values of R_∞ unless the adopted value of e/m happens to be identical with the value yielded by Eq. (12). In the case of DRW, the value of e/m given by Eq. (12) does happen to satisfy this condition very closely.[26]

Even in this situation, the resulting probable error in R_∞ will be different, according as Eq. (1′) or (2′) is used, and each error will be different from that resulting from the use of Eq. (20). Furthermore, with equally accurate values of R_D and R_H, the former—used in Eq. (2′)—will lead to the more accurate value of R_∞. This necessary fact is shown clearly by the actual results. With the "outside" values of $e/m=(1.7591\pm 0.0005)\times 10^{7}$ and $F=9651.4_0\pm 1.0$, and with $R_H=109{,}677.5812\pm 0.0075$, we get from Eq. (16)—or from Eqs. (1′) and (4)—

$$R_\infty=109{,}737.303_6\pm 0.019_5\ \text{cm}^{-1},$$

whereas with $R_D=109{,}707.4193\pm 0.0075$ we get, from Eqs. (2′) and (4),

$$R_\infty=109{,}737.303_2\pm 0.011_8\ \text{cm}^{-1}.$$

The value of R_∞ obtained from R_D alone is thus apparently more accurate than that obtained by Eq. (20) from s_H and s_D simultaneously, even though the "outside" value of e/m has been assigned a slightly larger error (± 0.0005) than the calculated error (± 0.00037) of the "inside" value, and the use of Eq. (2′) involves also the error in F, whereas Eq. (20) does not. It must be realized, however, that the use of Eq. (20), rather than of Eqs. (2′) and (4) with "outside" values of e/m and F, eliminates the possibility of certain types of systematic error (in particular, errors in F), and hence I believe that the result yielded by Eq. (20), i.e.,

$$R_\infty=109{,}737.303\pm 0.017\ \text{cm}^{-1},$$

is probably the best to adopt. It is, in fact, the final result, as adopted in this paper, except that the probable error should be increased to ± 0.05 cm^{-1} to allow for the uncertainty in the relation of the I.A. to the c.g.s. system of length.

Finally, from DRW's values of R_H and R_D it is possible to calculate any other R, such as R_{He}. This last constant is given by Eq. (24), directly in terms of the original data s_H and s_D, with the result

$$R_{He}=109{,}722.263_5\pm 0.012\ \text{cm}^{-1}.$$

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[25] This confusion regarding the isotopes of hydrogen is a direct consequence of the unfortunate situation that exists in respect to the symbols in use. In writing equations such as those used in this paper, it is obvious that H and D are more convenient than H^1 and H^2, since double superscripts are thereby avoided. On the other hand, the symbol H represents, to the chemist, the atomic weight of the mixture of hydrogen isotopes H^1 and H^2, just as C represents the mixture of C^{12} and C^{13}. In fact, in setting up a partial table of atomic weights (on the physical and on the chemical scale), as I have done in G.C.1939, it seemed really necessary to use H^1 and H^2. For if H and D are used, how is the atomic weight of the mixture to be symbolized? Thus there are definite objections to each of the two sets of symbols.

[26] The present most probable value of e/m is considered later in this paper.

In addition to the work on R_∞ and e/m, DRW[6] also make a detailed study of the fine structure of H_α and D_α. Such work should lead to a fairly precise value of the fine structure constant. However, as is well known, repeated investigations of this subject have shown rather definitely that the position and intensity of certain components, notably component 3, are not in agreement with theory. Pasternack[27] has tried to explain the observed discrepancies by assuming a displacement of the 2^2S level by about 0.03 cm^{-1}. DRW, however, believe that a slight contamination of the fine structure components of D_α by H_α components, and of both D_α and H_α by molecular lines, is sufficient to account for all observed irregularities. It is just because of such irregularities, which have been known for many years, that so much work has been done on the fine structure of H_α and D_α. There is as yet no reason to suppose that the method we have just used in calculating R_H, R_D, and R_∞ is affected by the observed irregularities, but because of them it has not been possible to get a reliable value of α, as already stated. Furthermore, because of the great amount of experimental work on the subject, a separate article is needed to cover it adequately, as also stated earlier in this paper. Fortunately no similar irregularities have been found in the case of the λ4686 line of He$^+$, and from this line one can obtain a reasonable value of α, as will be shown presently.

Data of Houston, and Calculations

As already noted, the value of R_∞ given in G.C.1929 was based entirely on Houston's experimental value[3] of R_H, with the use of "outside" values of e/m and F in Eq. (16). Houston's experimental value[3] of R_{He} was not used at all, but it should have been used. It is therefore of interest to calculate the most reliable value of R_∞, by the use of *all* of his data, and with the latest auxiliary constants, and to compare such a revised value with that obtained by DRW. In 1929 I merely accepted Houston's calculated value of R_H, but I will now recalculate it, just as I have done in the case of the DRW work.

Houston calculated R_H from the measured wave-length, *in air*, of component 1 (also called I_a) of H_α and of H_β, and he calculated R_{He} from the measured wave-lengths of components I_a and II_b of the λ4686 line of He$^+$. The work was done with a Fabry-Perot interferometer, and all wave-lengths were measured against the λ5015 helium line, whose wave-length in air was taken as 5015.6750 I.A. This is also the present accepted wave-length, but the discrepancy that still exists —as will presently be shown—between his value of R_∞ and that of DRW, must be ascribed in the main, according to DRW, to some error in this accepted wave-length.

Houston, in his reduction to vacuum, naturally used the dispersion curve of Meggers and Peters. We shall now use Barrell and Sears[12] 1939 dispersion curve and this change in the dispersion of air alone makes each calculated wave-length in vacuum about 0.004A larger, a by no means negligible change. As already noted, I will use +0.0036 cm^{-1}, in place of Houston's +0.0056 cm^{-1}, for the correction to the 1,4,5 complex of H_α to get the wave number of component 1, but for H_β I retain his correction of +0.0024 cm^{-1}. The only additional quantity needed is the "Balmer" line correction, which for component 1 of H_β is $R_H\alpha^2(1/64-7/3072)=0.0780$ cm^{-1}.

Two further points should be noted. (1) Houston measured the H_α line on 5 plates, and the average wave-length in air of the 1,4,5 complex, as given correctly in his Table I, is 6562.8473 I.A. This wave-length, however, is written and used as 6562.8475 in the calculations of his Table III, thus introducing a very small error. (2) The corresponding complex of H_β was also measured on 5 plates, and the average, as correctly stated in his Table I, and correctly *used* in his Table III, is 4861.3578 I.A. Because of a purely typographical error this wave-length appears in his Table III as 4861.3678 and DRW, overlooking the true situation, point out (reference 6, p. 166) that his stated wave number of H_β in vacuum, in Table III, does not correspond to the stated wave-length in air, on the basis of the Meggers and Peters' dispersion curve.

With these changes I then obtain, by the same process as was used for DRW,

$$s_H=15{,}233.0113 \text{ cm}^{-1} \text{ from } H_\alpha,$$
$$s_H'=20{,}564.5643 \text{ cm}^{-1} \text{ from } H_\beta,$$

where s_H and s_H' denote the "Balmer" lines.

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[27] S. Pasternack, Phys. Rev. 54, 1113 (1938).

Multiplication of s_H by 7.2 and of s_H' by 16/3 gives R_H, and the results are

$$R_H = 109{,}677.6813 \text{ cm}^{-1} \text{ from } H_\alpha$$
$$= 109{,}677.6762 \text{ cm}^{-1} \text{ from } H_\beta.$$

The close agreement of these two values is a really remarkable verification of the entire theory of the hydrogen atom. The discrepancy is only $+0.0051$ cm^{-1} (5 parts in 10^8), as contrasted with a *negative* discrepancy of 0.021 cm^{-1} resulting from Houston's own calculated values of 109,677.754 and 109,677.775. With his own correction of $+0.0056$ cm^{-1} in the case of H_α, and with Meggers and Peters' dispersion curve, I obtain 109,677.755 from H_α and 109,677.771 from H_β (discrepancy -0.016 cm^{-1}) as the values of R_H that Houston should have gotten from his data. If now the correction is changed to $+0.0036$ cm^{-1}, the discrepancy is *increased* in magnitude from -0.016 to -0.030 cm^{-1}. With the 1939 dispersion curve, however, the corresponding discrepancies are $+0.0195$ cm^{-1} and $+0.0051$ cm^{-1}. Thus the use of the $+0.0036$ cm^{-1} correction decreases the discrepancy between the values of R_H from H_α and H_β *only* when the 1939 dispersion curve is used. With Meggers and Peters' curve the discrepancy (now of opposite sign) is *increased*. Our final conclusion is, then, that in order to get a consistent value of R_H from Houston's data on H_α and H_β, it is necessary to use the new 1939 dispersion curve and *also* the new $+0.0036$ cm^{-1} correction. This fact may be considered as internal evidence in favor of the use of both.

Houston gives relative weights of 3 to 1 to the two values of R_H from H_α and H_β, and with this weighting I get the weighted average

$$R_H = 109{,}677.6800 \text{ cm}^{-1},$$

which is 0.079 cm^{-1} smaller than his published average, but 0.099 cm^{-1} *larger* than the DRW result. If now we use the Meggers and Peters' dispersion curve, but leave everything else unchanged, the weighted average rises to

$$R_H = 109{,}677.7493 \text{ cm}^{-1},$$

which is 0.168 cm^{-1} larger than the DRW result. The use of Houston's $+0.0056$ cm^{-1} correction raises this figure by only 0.0098 cm^{-1}, and increases the 0.168 cm^{-1} discrepancy by a like amount to 0.178 cm^{-1}. Thus the use of the newer values for the dispersion curve of air decreases the discrepancy between Houston's and DRW's values of R_H only from 0.168 to 0.099 cm^{-1}, and the other changes that have been made in Houston's calculations are considerably smaller.

The final unexplained discrepancy between the results of DRW and Houston may be stated more clearly as follows. Houston measured 6562.8473 I.A. as the wave-length in air of the 1,4 complex of H_α. DRW's published wave number in vacuum, when reduced to wave-length in air by the 1939 dispersion curve, is 6562.8533 I.A.—just 0.006A larger. This difference is many times the apparent probable error of the measurement but, as already noted, Houston's result is based on the λ5015 helium line, whereas the DRW result is based on the λ6438 cadmium line.

This completes the discussion of Houston's value of R_H, and we now consider his value of R_{He}, which is based on the measured wave-lengths of components I_a and II_b of the λ4686 line. For this line $R = R_{He}$, $Z = 2$, $n' = 4$ and $n'' = 3$, in Eqs. (29) and (30). For the I_a component, $j' = 7/2$, $j'' = 5/2$. For the II_b component $j' = 5/2$, $j'' = 3/2$. These are the two strongest components, out of a total of 8 predicted components, and are of nearly equal intensity. Chu[28] gives a diagram of the fine-structure pattern. Because of the great intensity of the two components used, as compared to the remaining components, Houston[3] used merely the observed wave-lengths, without any additional correction (but see footnote 30). Chu[28] later made a more detailed analysis of the microphotometric curves. His results will be discussed presently.

Houston obtained component I_a on 7 different plates, with an average wave-length in air of 4685.8030 I.A., and a mean deviation of 0.0026 I.A. The II_b component was measured also on 7 plates, with an average of 4685.7030 I.A., and mean deviation 0.0012 I.A. In this case we can use the difference of these two wave-lengths to obtain an *experimental* value of the fine structure constant α. With the usual dispersion curve,[12] one obtains for the wave number difference, in vacuum, $\Delta s = 0.45533$ cm^{-1}. This should equal $(101/1296)R_{He}\alpha^2$, thus giving $1/\alpha = 137.038$,

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[28] D. Y. Chu, Phys. Rev. 55, 175 (1939).

which all recent experimental evidence indicates is close to the true value. With this value of α, the negative correction of the $\mathrm{I_a}$ component to give the "Balmer" line is $16R_{\mathrm{He}}\alpha^2(1/324-1/1024)=0.19723$ cm^{-1}. The negative correction for the $\mathrm{II_b}$ component is $16R_{\mathrm{He}}\alpha^2(1/108-7/3072)=0.65256$ cm^{-1}. When one thus uses the value of α derived from the data, the two resulting values of the "Balmer" line are necessarily identical, and the result is

$$s_{\mathrm{He}}=21{,}324.89082 \text{ cm}^{-1}.$$

Multiplying by 36/7 (the reciprocal of 4(1/9 $-$1/16) in Eq. (30)), we get $R_{\mathrm{He}}=109{,}722.295_7$ cm^{-1}, which is 0.0322 cm^{-1} larger than the value already calculated indirectly from DRW's data. The agreement is, accidentally, much better than that between the two *directly* calculated values of R_{H}, by Houston and by DRW. Houston himself, using the Meggers and Peters' dispersion curve and an "outside" value of α, got from the two components slightly different values of R_{He} and a weighted average value[29] of $109{,}722.403\pm0.004$ cm^{-1}.

From Houston's values of $R_{\mathrm{H}}=109{,}677.6800$ and $R_{\mathrm{He}}=109{,}722.2957$, as just recalculated, we can now obtain E and e/m by means of Eqs. (14) and (15). The results are $E=5.47829\times10^{-4}$ and $e/m=1.7617_6\times10^{7}$ e.m.u. No probable error is calculated for this value of e/m, since it is not used in the final average, as discussed later. If Houston's H_α correction of 0.0056 cm^{-1} were used, we would get 1.7621_8. In G.C.1929 (p. 47) I got, from the same data, $e/m=1.7608\pm0.0008$. In 1938 I obtained[22] $1.7601_5\pm0.0008$. These changes indicate very well the effect of varying values of auxiliary constants, and varying methods of reduction of the same data.

We can also obtain R_∞, by means of Eq. (21). The result is $R_\infty=109{,}737.3125$ cm^{-1}, which is only 0.010 cm^{-1} above the adopted DRW result. Houston[3] himself gives $R_\infty=109{,}737.424$ cm^{-1} and this value was *adopted* in G.C.1929 (p. 49). In the case of DRW we have calculated R_∞ also from R_{H} and from R_{D} separately, using Eqs. (1′) and (2′) and outside values of e/m and F to get E from Eq. (4). The result from R_{D} is the more accurate. In a similar way, we can get R_∞ from Houston's value of R_{H} and from his value of R_{He}, by the use of Eqs. (1′) and (3′). These two new values are 109,737.4024 cm^{-1} and 109,737.3351 cm^{-1}, respectively. Both of these values are higher than that obtained from Eq. (21), and differ considerably from the DRW result. This shows that the previous close coincidence in the values of R_∞ from the DRW and the Houston work is purely accidental, and is due to the fact that real discrepancies in the values of R_{H} and e/m (or E) happen nearly to cancel each other, in the calculation of R_∞.

DATA OF CHU, AND CALCULATIONS

Chu,[28] as already noted, has analyzed the fine structure of He λ4686 and has calculated a value of R_{He}. His work was done under the direction of Houston, with similar apparatus to that used by Houston. The wave-length measurements of both men are based on the λ5015 helium line. Hence it seems legitimate to combine Chu's value of R_{He} with Houston's value of R_{H}, to get a value of R_∞, and also of e/m.

In Houston's work the two strong maxima of the intensity curve of λ4686 were identified with components $\mathrm{I_a}$ and $\mathrm{II_b}$. In Chu's work a careful analysis of the intensity curve was carried out, in much the same way as was done by Williams and Gibbs[19] for H_α and D_α. In the case of λ4686, the fine structure extends over so large an interval, 2.43 cm^{-1}, that with the usual orders of interference of the Fabry-Perot interferometer, there is a great deal of overlapping of orders. By varying the order of interference, the resulting intensity pattern can be made to vary markedly. The corrections necessary to apply to the two strong maxima were calculated by Chu, from his analysis of the intensity curves, for each order of interference used, and these corrections, as might be expected, also varied greatly. In fact, for the $\mathrm{II_b}$ component the corrections varied from +0.0012A to $-$0.0029A. This variation is really an advantage, for if the final corrected wave-length is constant from plate to plate, it is very good evidence that the procedure employed is funda-

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[29] Each of Houston's stated errors appears to be based on external consistency, but since only two values are available in each case, internal consistency would yield a far more reliable estimate. I have not carried through a calculation of probable error such as I did in the case of the DRW results, because I am adopting the DRW results. The sole purpose of the present calculation is to get a correct comparison with the DRW results, by the use of the same auxiliary constants and the same method of calculation in each case.

mentally sound. This result was actually attained. Thus the resulting wave-lengths[30] of the II_b component, on the four plates used, varied only by 0.0009A. It was more difficult to measure the I_a component and on one plate (No. 126) its wave-length differs by 0.0037A from the average of the remaining three plates. Chu states that the position of the maximum on the microphotometric tracing of this plate was particularly uncertain.

We shall first follow Chu in accepting the results of all four plates, and we shall also use his assigned weights, which run from 4 to 1. Then for the I_a component the average wave-length in air is 4685.8012A, giving $s=21{,}335.09626$ cm^{-1} for the wave number in vacuum (with the 1939 dispersion curve of air). For the II_b component the corresponding results are 4685.7017A and 21,335.54931 cm^{-1}. These wave-lengths are respectively 0.0018A and 0.0013A larger than Houston's values. The wave number difference is, from Chu's data, 0.45305 cm^{-1}, from which we get $1/\alpha=137.383$. As we have seen, Houston's corresponding result is $1/\alpha=137.038$. Chu does not give explicitly his value of $1/\alpha$ (137.383), but I had calculated it and until the recent work of Christy and Keller[31] I had considered it the best experimental determination, with an assigned probable error of ±0.4. It is to this value by Chu that I had reference in my own recent letter[32] on the subject.

With Chu's value of α, the "Balmer" corrections for the I_a and II_b components are now 0.19625 cm^{-1} and 0.64930 cm^{-1}, respectively. The necessarily identical value of the Balmer line, from either component, is $s_{\mathrm{He}}=21{,}334.90001$ cm^{-1}, and from this we get

$$R_{\mathrm{He}}=109{,}722.342_9 \text{ cm}^{-1},$$

which is 0.047 cm^{-1} larger than Houston's value, and 0.079 cm^{-1} larger than DRW's indirect value. Chu himself gives $109{,}722.43\pm0.04$ cm^{-1}, using Meggers and Peters' dispersion curve.

We can now combine Houston's value of $R_{\mathrm{H}}=109{,}677.6800$ cm^{-1} and Chu's value of $R_{\mathrm{He}}=109{,}722.3429$ cm^{-1}, to get, from Eqs. (14), (15), and (21)

$$E=5.484086\times10^{-4},$$
$$e/m=1.7598_9\times10^{7} \text{ e.m.u.},$$
$$R_\infty=109{,}737.3756 \text{ cm}^{-1}.$$

It has just been noted that component I_a was observed to be very ill-defined on one of Chu's plates (No. 126), and the results given by this plate are in definite disagreement with those of the remaining three plates. The disagreement is shown clearly by calculating $1/\alpha$ from each plate separately. The results are

Plate	$1/\alpha$
118	137.314
124	136.971
126	139.790
134	136.971

The weighted average value, omitting No. 126, is 137.079 and if I had noticed these facts earlier, I would certainly not have taken Chu's weighted average of all four plates (137.38) as the best experimental value of $1/\alpha$. This seems to be a situation where it is clearly justifiable to discard one plate, and hence it would appear that Chu's best value of $1/\alpha$ is more nearly 137.1 ± 0.2, as compared to the value 136.95 ± 0.05 obtained from x-ray data.[31, 32] Although Houston's value (137.04) is also very satisfactory, it is presumably not as reliable as Chu's, because of Houston's less detailed analysis of his intensity curves.

With plate No. 126 omitted, and with $1/\alpha=137.079$, one gets, from Chu's remaining three plates

$$R_{\mathrm{He}}=109{,}722.3280 \text{ cm}^{-1},$$

which is only 0.0156 cm^{-1} larger than Houston's value. Combining this last result with Houston's R_{H}, we obtain

$$E=5.48225\times10^{-4},$$
$$e/m=1.7604_8\times10^{7} \text{ e.m.u.},$$
$$R_\infty=109{,}737.3556 \text{ cm}^{-1}.$$

These results are, I believe, the most reliable that can be obtained from the combined work of Houston and Chu. Their work and the work of DRW furnish all of the experimental data from which one can obtain a precision value of R_∞. In

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[30] It is only fair to remark that Houston[3] also recognized clearly the need for making corrections on each individual plate, to get the true positions of the I_a and II_b components. He tried, however, to vary the order of interference in such a way that the various resulting corrections would, *on the average*, be expected to cancel out.

[31] R. F. Christy and J. M. Keller, Phys. Rev. **58**, 658 (1940).

[32] R. T. Birge, Phys. Rev. **58**, 658 (1940).

adopting a final value of R_∞ one should really consider some weighted average of the two results. But because of the character of the discrepancies, as already discussed, such an average appears to have little meaning. For this reason, and for this reason alone, I have adopted as the final value of R_∞ merely the DRW result.[33]

On the other hand, the value of e/m just given is to be combined with many other values, later in this paper, to get a final best value. Hence it is necessary to adopt a probable error for it. This value of e/m has been calculated from Eq. (15), and the only probable errors that need be considered are those in F and in the difference $R_{\mathrm{He}}-R_{\mathrm{H}}$. The adopted value of F is $9651.4_0 \pm 1.0$ abs. e.m.u., as already stated.

Houston's two values of R_{H} (from H_α and H_β) differ by only 0.0051 cm^{-1}, and hence 0.010 cm^{-1} seems a liberal estimate for the probable error of the weighted average (109,677.6800 cm^{-1}). Chu's value of R_{He}, from 3 plates, differs from that calculated from all 4 plates by 0.015 cm^{-1}. Since the omitted plate is obviously poor, 0.010 cm^{-1} again seems a liberal estimate for the probable error of the 3-plate result (109,722.3280 cm^{-1}). Now *both* Houston and Chu base all their wave-lengths on He λ5015. Hence the most serious sources of possible *systematic* error do not appear in the *difference* $R_{\mathrm{He}}-R_{\mathrm{H}}$. We therefore get $R_{\mathrm{He}}-R_{\mathrm{H}}=44.648_0 \pm 0.014\ \mathrm{cm}^{-1}$, and finally

$$e/m=(1.7604_8 \pm 0.0005_8)\times 10^7 \text{ abs. e.m.u.}$$

as the adopted result from the Houston-Chu work.

Experimental Values of *e/m* from Fine Structure Analysis

a. Work of R. C. Williams

Although we have now covered the experimental data leading to values of R_∞, and incidentally of e/m, there are further similar investigations from which one can obtain experimental values of e/m but not of R_∞. I have not examined this additional work with the thoroughness of the preceding, and the following discussion is rather brief.

One important additional investigation on the fine structure of H_α and D_α is that of R. C. Williams, which was carried out under the direction of R. C. Gibbs. In addition to the papers giving the final results on e/m (reference 1) and on α,[34] and the papers already quoted,[18, 19] there are other earlier reports.[35–37]

In order to calculate e/m, the *difference* of wave-length of component 1 of H_α and D_α was measured with an interferometer of over 250,000 resolving power (3-mm etalon silvered to reflect over 90 percent). The method of analyzing the fine structure is discussed in references 34 and 19. Williams' published average "peak-to-peak" interval in air for the H_α and D_α lines is $4.14716 \pm 0.0004\ \mathrm{cm}^{-1}$. Division by the index of refraction of air (1.00027624) gives 4.146014 cm^{-1} for the interval in vacuum. The use of Meggers and Peters' $\mu=1.0002757$ changes this result only to 4.146017, but Williams gives 4.14610 cm^{-1} as his value in vacuum.

Full details of the corrections to be applied to each complex "peak" to get the position of component 1 are given by Williams in his Table III of reference 1. He finds a *differential* correction of $+0.0009\ \mathrm{cm}^{-1}$, but most of this correction is due to a differential correction of $+0.0012\ \mathrm{cm}^{-1}$ for the effect of component 3. As noted earlier, DRW make no differential correction to their $\mathrm{H}_\alpha-\mathrm{D}_\alpha$ interval, but their printed results include the correction for component 3. Hence there is no real disagreement between Williams and DRW.

Williams' corrected interval is thus 4.14691_4

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[33] Professor Houston has called my attention to the fact that the relations between R_{H}, R_{D}, R_{He} and R_∞, as obtained by various observers, can conveniently be shown on a graph of the same general character as the Birge-Bond diagram. Thus in Eqs. (1′)–(3′), if we use an *assumed* value of $E(=E_0)$, each experimental value of R_z will yield an *apparent* value of $R_\infty(=R_\infty')$. If now these values of R_∞' are plotted against the reciprocal of the nuclear atomic weight (i.e., against $1/\mathrm{H}^+$, $1/\mathrm{D}^+$ etc.) as abscissa, all discrepancies are immediately obvious. If there are no discrepancies, all of the points will lie on one straight line. The extrapolation to zero abscissa of the straight line through any two points gives the value of R_∞ resulting from the use of these two data. On such a diagram, the DRW data give one straight line, and Houston's data give another line with a quite different slope, corresponding to the quite different value of e/m yielded by his data. The Houston-Chu data give a line of intermediate slope. The diagram shows immediately that no compromise line will give a satisfactory representation of the data of any one of the different observers.

[34] R. C. Williams, Phys. Rev. **54**, 558 (1938).

[35] R. C. Gibbs and R. C. Williams, Phys. Rev. **44**, 1029 (1933).

[36] R. C. Williams and R. C. Gibbs, Phys. Rev. **48**, 971 (1935).

[37] R. C. Williams and R. C. Gibbs, Phys. Rev. **49**, 416 (1936), (two abstracts).

cm^{-1} and Eq. (11) then gives E. For convenience in comparing the results of different observers, Eq. (11) may be written as

$$E=(1.3238985\times10^{-4})(s_{\mathrm{D}}-s_{\mathrm{H}}) \qquad (11')$$

with the DRW value of s_{H} (15,232.9974 cm^{-1}) and the atomic weights already listed. Therefore $E=5.49009_3\times10^{-4}$ from Williams' data. Then from Eq. (4), $e/m=1.7579_7\times10^7$ e.m.u. Williams gives $(5.4902\pm0.0005)\times10^{-4}$ and $(1.7579\pm0.0004)\times10^7$.

In the case of DRW we have already found the result $e/m=(1.75913\pm0.00037)\times10^7$, where the probable error was calculated with some care, and includes a certain allowance for systematic error. In order to get a final weighted average value of e/m, it is necessary to assign *relative* weights to the various results. Hence it is only the *relative* probable errors that are important, and in the case of all the remaining results, we ask ourselves, "If 0.00037×10^7 is taken as a fair estimate of the probable error of the DRW value of e/m, what is a proper estimate for the new investigation, when one considers the relative accuracy of the instruments used, the detailed measurements made, etc.?" The probable errors adopted here are therefore frankly purely personal estimates.

For the R. C. Williams result I *adopt*

$$e/m=(1.7579_7\pm0.0005)\times10^7.$$

b. Work of C. F. Robinson

Robinson has as yet published only an abstract[38] of his investigation, which is a continuation of work started by Houston[39] on an analysis of the fine structure of H_α and D_α by a new method. I am, however, indebted to Professor Houston for further details. Robinson's measured separation of component 1 of H_α and D_α, in air, is $\Delta s=(4.1453\pm0.0010)$ cm^{-1}. This becomes 4.14415 ±0.0010 cm^{-1} in vacuum, as contrasted with Williams' 4.14691 cm^{-1} and with Houston's preliminary values[39] of 4.1440 and 4.1399 cm^{-1}. Accepting Robinson's estimate of his own probable error in Δs, and using the probable error in F already given, we obtain

$$E=(5.4864_4\pm0.0013_2)\times10^{-4}$$

and

$$e/m=(1.7591_4\pm0.00046)\times10^7.$$

The resulting probable error in e/m is essentially that *adopted* for Williams' result, and the data just given thus show directly the error one needs to assume in the $\mathrm{H}_\alpha-\mathrm{D}_\alpha$ interval to get my adopted error in e/m. I assume the same accuracy for the work of Williams and of Robinson and *adopt* for Robinson's result

$$e/m=(1.7591_4\pm0.0005)\times10^7.$$

c. Work of Shane and Spedding

The last investigation on the fine structure of H_α and D_α to be discussed, but the earliest in point of time, is that of Shane and Spedding.[40] They used an interferometer with a plate separation of about 7.8 mm. They make an elaborate analysis of their intensity curves, and give their final separations of H_α and D_α components in terms of orders of interference, but not in terms of wave-length or wave number. Each result is accompanied by its corresponding value of e/m. Under the circumstances the simplest procedure appears to be merely to accept their final average value of e/m, which is $(1.7579\pm0.0003)\times10^7$, and then correct it for the change in values of auxiliary constants. They used Eq. (12) and in this equation their adopted values give $F(\mathrm{D}-\mathrm{H})/\mathrm{H}^+\mathrm{D}=4786.383$, whereas my present adopted values give 4785.7526. The value of e/m is then to be reduced in this ratio. But according to Williams and Gibbs,[36] Shane and Spedding did not reduce their wave number interval from air to vacuum, although in Eq. (12) they used the wave number in vacuum of R_{H}. To correct for this error, one should multiply their result by the index of refraction of air (1.000276), thus giving 1.0001444 as the resulting factor for the two almost compensating corrections. The revised result, with my own adopted probable error, is

$$e/m=(1.7581_5\pm0.0006)\times10^7 \text{ e.m.u.}$$

EXPERIMENTAL VALUES OF e/m BY OTHER METHODS

In 1938 I obtained,[22] as the best average value of e/m from all methods, $(1.7591\pm0.0003)\times10^7$ abs. e.m.u. The first four determinations listed then comprise the work on H, D, and He$^+$, and revised values for all four have now been ob-

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[38] C. F. Robinson, Phys. Rev. **55**, 423A (1939).

[39] W. V. Houston, Phys. Rev. **51**, 446 (1937).

[40] C. D. Shane and F. H. Spedding, Phys. Rev. **47**, 33 (1935).

tained, together with the new value by DRW.[6] The remaining six determinations given in the 1938 letter, together with one new determination, will now be considered. No attempt is made at critical discussion, and in fact my only object now is to explain, as briefly as possible, why my present adopted value, or its assigned probable error, deviates from that given by the author. In general it is difficult, in the case of the methods now to be considered, for an outsider to make any intelligent evaluation of the probable error, and hence I have, in general, merely accepted that published by the author. It is quite possible that, *relative* to the adopted probable errors in the preceding sections of the paper, all of the probable errors to follow should be slightly increased.

When e/m is obtained from fine structure analysis, the value of F_iq is involved, where F_i is the value of the Faraday in international electric units, and $F(=F_iq)$ its value in absolute units. Now as already discussed in footnote 23, I have, subsequent to the 1938 work[22] on e/m, *raised* the value of F_i and *lowered* that of q by almost equal amounts. Hence the resulting values of e/m are virtually unchanged, although additional small changes have now been made in certain instances. On the other hand, in the case of five of the six remaining methods that have been used in a precise determination of e/m, F_i does not appear at all in the equation, but q appears in the denominator. Hence in all these cases e/m is *raised* by about 0.00012×10^7 e.m.u., as a result of the *lowering* of q from 0.99993 to 0.99986. In the sixth method (refraction of x-rays), F_iq occurs in the denominator and again the value of e/m is virtually unchanged.

a. Zeeman effect

A number of investigations on the value of e/m from Zeeman effect have been carried out by Houston and co-workers. The results of all of their work are tabulated by Kinsler and Houston.[41] These comprise the final results of work on Zn, Cd, He, and Ne. They give as a final weighted average, $e/m=(1.7570\pm0.0007)\times10^7$. It is not apparent how the probable error was obtained. Accepting their own probable errors for the four individual results, I obtain as the weighted average $1.7569_{46}\pm0.00017$ by external consistency, and ±0.00047 by internal consistency. Since, however, it is well to make some allowance for systematic errors, I will increase this latter error to ±0.0007, to agree with their own assigned error. The values of the auxiliary constants are not stated, but in an earlier paper[42] the values $q=0.99995$ and $c=299{,}796$ km/sec. were used, and I believe were used also in all succeeding papers of the series. In 1934 I recommended[43] $c=299{,}776\pm4$ km/sec., and the inclusion of more recent results leaves unchanged both this average and its probable error. In the Zeeman effect work the change in c decreases e/m by almost the same amount that the change in q increases it. The exact corrected result is 1.7569_{87}. Hence I adopt

$$e/m=(1.7570\pm0.0007)\times10^7 \text{ e.m.u.}$$

b. Refraction of x-rays

Bearden,[44] in an elaborate investigation, obtained $e/m=(1.7601\pm0.0003)\times10^7$. All of his auxiliary constants agree with those I was using at the time. Hence the only essential change now to be made is that resulting from the very small changes in F and C. His stated probable error is very small, but is the result of a detailed consideration of the errors of all factors in the final expressions for e/m. His exact probable error in e/m, based on $F=9651.3\pm0.80$, is $\pm0.0003_{17}$. With the new $F=9651.4_0\pm1.0$ and $\mathrm{C}=12.01465\pm0.00023$, his result becomes $1.7600_6\pm0.0003_3$. In order to make at least a slight allowance for systematic error I *adopt*

$$e/m=(1.7600_6\pm0.0004)\times10^7 \text{ e.m.u.}$$

c. Direct velocity measurements

Two careful investigations by this method have been made, the first by Perry and Chaffee,[45] and the second by Kirchner.[46] Perry and Chaffee obtained six series of results, with an actual average of 1.7609 ± 0.0002, where the error is

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[41] L. E. Kinsler and W. V. Houston, Phys. Rev. **46**, 533 (1934).

[42] J. S. Campbell and W. V. Houston, Phys. Rev. **39**, 601 (1932).

[43] R. T. Birge, Nature **134**, 771 (1934).

[44] J. A. Bearden, Phys. Rev. **54**, 698 (1938).

[45] C. T. Perry and E. L. Chaffee, Phys. Rev. **36**, 904 (1930).

[46] F. Kirchner, Ann. d. Physik **8**, 975 (1931) and **12**, 503 (1932).

TABLE I.

e/m	P. E.	REFERENCE IN TEXT	METHOD	$f(F_ipq)$
1.75913	3.7 ×10^{-4}	6	H_α and D_α	F_iq
1.75797	5.0 ×10^{-4}	1	H_α and D_α	F_iq
1.75914	5.0 ×10^{-4}	38	H_α and D_α	F_iq
1.75815	6.0 ×10^{-4}	40	H_α and D_α	F_iq
1.76048	5.8 ×10^{-4}	3, 28	R_H and R_{He}	F_iq
1.75700	7.0 ×10^{-4}	41	Zeeman effect	$1/q$
1.76006	4.0 ×10^{-4}	44	X-ray refraction	$1/F_iq$
1.76110	10.0 ×10^{-4}	45	Direct velocity	$1/pq$
1.75900	9.0 ×10^{-4}	46	Direct velocity	$1/pq$
1.75982	4.0 ×10^{-4}	48	Magnetic deflection	$1/q$
1.75820	13.0 ×10^{-4}	49	Crossed fields	p/q
1.75870	8.0 ×10^{-4}	51	Busch method	p/q

based on external consistency. They however *adopt* a final probable error of ±0.0010, and I shall do the same. The value of c enters into their formula in a rather complex way and I find, by actual trial, that the change of c from 299,796 to 299,776 produces a change in e/m of only about one part in 10^5, which is negligible. They used $pq=1.00046$ to reduce from int. to abs. volts, and with the new value 1.00034, one gets $e/m=1.7611_1$. I therefore *adopt*

$$e/m=(1.7611\pm0.0010)\times10^7 \text{ e.m.u.}$$

Kirchner[46] obtained, in two separate investigations, 1.7585±0.0012 (revised from the original 1.7598±0.0025) and 1.7590±0.0015. From these results I obtained[47] a weighted average of 1.7587±0.0009, where the error is based on internal consistency. This average has since been quoted in the literature as Kirchner's final result. Kirchner gives corresponding figures in int. and abs. volts that imply an assumed $pq=1.0005$. With our new value, the weighted average becomes 1.7589_6 and I *adopt*

$$e/m=(1.7590\pm0.0009)\times10^7 \text{ e.m.u.}$$

d. Magnetic deflection

Using a new magnetic deflection method devised by E. O. Lawrence, Dunnington[48] has made a most extensive and refined investigation of the value of e/m. His final published result is 1.7597±0.0004. This result is based on $q=0.99993$. With the new value of q one obtains

$$e/m=(1.7598_2\pm0.0004)\times10^7 \text{ e.m.u.},$$

which I adopt.

e. Crossed fields

Shaw[49] has used a method of crossed electric and magnetic fields to obtain a published value of $e/m=1.7571\pm0.0013$. He, however, failed to make the reduction from international to absolute electric units, and since the factor V/H^2 appears in his equation (V=potential difference, H=magnetic field), the proper reduction factor is $pq/q^2=p/q=1.00062$ for the values of p and q now chosen. Hence his corrected result is

$$e/m=(1.7582\pm0.0013)\times10^7 \text{ e.m.u.},$$

which I adopt.

f. Longitudinal magnetic field (Busch method)

In 1929 the best work on e/m with the Busch method was that by Wolf.[50] His investigation is discussed on pp. 43–44 of G.C.1929. At that time I assumed that he had not made the correction from international to absolute units, an assumption that was later verified. As in the case of crossed fields, the factor V/H^2 again appears in the equation. In 1929 I corrected Wolf's result from 1.7679 to 1.7689. With the latest p/q value, it becomes 1.7690.

A recent and much more accurate determination by this method has been made by Goedicke.[51] He does not give the values of p and q used, but from certain published figures, his value of q is evidently about 0.99991. I assume that his value of p is 1.00050, since that value is commonly used. On these assumptions, his value of $e/m=1.7586$ should be raised only to 1.7587. He gives 0.0023 as a *maximum* possible error, and following my usual custom I will take 1/3 of this as the probable error. Hence I adopt

$$e/m=(1.7587\pm0.0008)\times10^7 \text{ e.m.u.}$$

for his final result.

SUMMARY OF VALUES OF e/m

We have now discussed and adopted values of e/m from twelve different investigations, covering seven essentially different methods. All of the adopted results and probable errors are listed in Table I. So far as values of the auxiliary constants

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[47] R. T. Birge, Phys. Rev. **42**, 736 (1932).

[48] F. G. Dunnington, Phys. Rev. **43**, 404 (1933) and **52**, 475 (1937).

[49] A. E. Shaw, Phys. Rev. **54**, 193 (1938).

[50] F. Wolf, Ann. d. Physik [4] **83**, 849 (1927).

[51] Egon Goedicke, Ann. d. Physik [5] **36**, 47 (1939).

are concerned, the chief sources of recent change and controversy are the values of F_i (the Faraday in international units), p and q. The last column shows the explicit dependence of e/m on these three constants.

The weighted average of the 12 results is $e/m=1.75920_6$. The probable error by external consistency (r_e) is 0.00019_2, and by internal consistency (r_i) is $\pm 0.00015_8$. The ratio r_e/r_i is 1.213, which is very satisfactory. In 1938 I got[22] for the corresponding results 1.75909 ± 0.00024 (r_e) and $r_e/r_i=1.51$, and recommended 1.7591 ± 0.0003 as the best value to adopt. Although the new average (1.75920_6) is only slightly larger than the value 1.7591, which has been rather widely used for the past three years, it seems best to adopt it.

It is, however, desirable to consider in some detail the internal consistency of the data since, as noted in my 1938 paper,[22] this is a subject that has been under discussion for many years. In that paper I concluded that the annoying previous discrepancy between the "spectroscopic" and the "deflection" values of e/m had virtually disappeared. Bearden[52] definitely disagreed with this conclusion, and even disagreed with my classification of his x-ray refraction value of e/m as a "spectroscopic" result. So far as this last point is concerned, I think it is mainly a question of how the two groups of results are defined.

Although in 1929[2] I gave *two* values of e/m, commonly referred to as the "spectroscopic" and the "deflection" values, I never for one moment believed that there are actually two different values. I assumed then, and I still assume, that any apparent discrepancy is due to errors of theory or of experiment, or both. Certain values of e/m are obtained with spectroscopic equipment and involve measurements of spectral wavelengths. These I have grouped together to obtain a weighted average "spectroscopic" result, and Bearden's result certainly belongs in this class. The other experiments are electric and magnetic in character, and involve linear acceleration and deflection of free electrons over comparatively long paths. The technique and quantities measured are totally different from those in the spectroscopic work. Hence the systematic errors are also likely to be totally different, and to such errors I attribute any outstanding difference in the values of e/m that may exist.

Bearden[52] gives a rather close argument to show that his value of e/m is concerned with "free" electrons, and hence should be classed with the deflection experiments. With this assumption he gets two values of e/m ($1.7583_1 \pm 0.0002_8$ and $1.7598_8 \pm 0.0001_7$), and he states that there is less than one chance in 10^9 of such a discrepancy being due to chance. His data, however, give for the discrepancy 0.00157 ± 0.0003276, or 4.79 times its probable error, and there is one chance in 826 of this occurring. If these last odds are taken at face value, a really serious discrepancy exists, but there seems to be no point in exaggerating the situation.

In considering the question now, I will omit Bearden's x-ray result from *both* groups. Then the first six results in the table comprise the spectroscopic group, and these six give

$$\begin{array}{ll} e/m=1.75880 \pm 0.00028_{20}\ (r_e) & \\ & \text{ratio } 1.355, \\ \quad\text{or} \quad \pm 0.00020_{81}\ (r_i) & \end{array}$$

whereas the last five results of the table comprise the deflection group, and give

$$\begin{array}{ll} e/m=1.75959 \pm 0.00024_{07}\ (r_e) & \\ & \text{ratio } 0.785. \\ \quad\text{or} \quad \pm 0.00030_{65}\ (r_i) & \end{array}$$

With the probable errors based on external consistency (r_e), there is one chance in $6\frac{2}{3}$ that this discrepancy is accidental (the discrepancy being 2.13 times its probable error), and an almost identical result is obtained from the r_i values. Although one would naturally wish that the odds were more nearly even, a discrepancy of this magnitude is not considered by statisticians as of any real significance.

If now we compare Bearden's result (point 7 of the table), with the two averages just found, and use r_e errors, we find that the odds are just even that his result is consistent with the deflection average, but there is only one chance in 12 that his result is consistent with the spectroscopic average. One possible conclusion from these facts is that point 7 is definitely too high. It is, however, worth while to explore the matter further.

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[52] J. A. Bearden, Phys. Rev. 55, 584 (1939).

It has already been mentioned in footnote 23 that the value of F now adopted is almost identical with that advocated in 1938, but the coincidence is only accidental, and is actually the result of two almost compensating changes that have been made.

Since F appears in different equations for e/m in different positions, it is possible to *evaluate* it merely from the e/m data. The resulting value has, however, a larger probable error than the value obtained from electrolysis. Thus the weighted average of points 1–5 inclusive is $e/m=1.75897\pm0.00026_{45}$ (r_e) or $\pm0.00021_{79}$ (r_i). The ratio r_e/r_i is 1.21_4, which is satisfactory. If now we calculate the value of F that will bring this result into agreement with Bearden's point 7, the result is

$$F=9651.40(1.76006/1.75897)^{\frac{1}{2}}$$
$$=9654.39 \text{ abs. e.m.u.},$$

and the new value of e/m is $[(1.76006)(1.75897)]^{\frac{1}{2}}=1.7595_{15}$. This is very close to the "deflection" average of 1.75959, which does not involve F at all. In fact, the value of F that will force the fine structure average (1.75897) to agree with the deflection average is

$$F=(9651.40)(1.75959)/(1.75897)$$
$$=9654.80 \text{ abs. e.m.u.}$$

The two new values of F just computed correspond to 96,531 and 96,535 int. coulombs (chemical scale), values that are far above the direct results given by either the silver voltameter (96,494) or the iodine voltameter (96,511). Such high values of F are clearly ruled out, on the basis of all experimental work with voltameters, but it is certainly worth noting that the value $F=96{,}533$ int. coulombs (chemical scale) will bring into almost perfect agreement *all* values of e/m, except that from Zeeman effect (point 6), which is certainly too low.

My final conclusion is then that, on the basis of all experimental work to date, 1.7592 is the best value of e/m. But if it becomes possible, as a result of the future discovery of new sources of systematic error in F, to raise its value some 30 coulombs above the present adopted result of 96,501.3 int. coulombs (chemical scale), then the consistency of the present e/m results will be greatly improved, and the best value of e/m will become 1.7595 or 1.7596. The weighted average of all 12 results, as already given, is $1.7592_1\pm0.0002$, but just because of this possible uncertainty in regard to the value of F, I shall enlarge the probable error to 0.0005. Hence the final *recommended* value is

$$e/m=(1.7592\pm0.0005)\times10^7 \text{ abs. e.m.u.}$$

or

$$(e/m)c=(5.2736_6\pm0.0015)\times10^{17} \text{ abs. e.s.u.}$$

Then, from the adopted values of e/m and F

$$E=(5.4862_4\pm0.0017)$$
$$\times10^{-4} \text{ (physical scale) (derived).}$$

SUMMARY

As a matter of convenience, the final values adopted in this paper are now listed. A list of adopted values of auxiliary constants has already been given, as well as the individual values of e/m. All of the following results are based entirely on the work of DRW,[6] except the value of e/m and the derived value of E.

$$R_{\mathrm{H}}=109{,}677.581_2\pm0.007_5 \text{ cm}^{-1} \text{ (I.A. scale),}$$
$$R_{\mathrm{D}}=109{,}707.419_3\pm0.007_5 \text{ cm}^{-1} \text{ (I.A. scale),}$$
$$R_{\mathrm{He}}=109{,}722.263\ \pm0.012 \text{ cm}^{-1} \text{ (I.A. scale),}$$
$$R_{\infty}=109{,}737.303\ \pm0.017 \text{ cm}^{-1} \text{ (I.A. scale),}$$
$$\text{or}\quad \pm0.05 \text{ cm}^{-1} \text{ (c.g.s. system),}$$
$$e/m=(1.7592\ \pm0.0005)\times10^7 \text{ abs. e.m.u.,}$$
$$(e/m)c=(5.2736_6\pm0.0015)\times10^{17} \text{ abs. e.s.u.,}$$
$$E=(5.4864_7\pm0.0009_9)$$
$$\times10^{-4} \text{ (physical scale) (direct),}$$
$$E=(5.4862_4\pm0.0017)$$
$$\times10^{-4} \text{ (physical scale) (derived).}$$

In general calculations, the *derived* value of E should be used, in order that it may be consistent with the adopted values of F and e/m.