Mass of the Meson by the Method of Momentum Loss

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Observations on the curvature of the track of a meson before and after traversal of a known amount of matter allow a determination of the mass of the particle. A discussion is presented of the accuracy and advantages of this method of measuring mass, the principle of which is well known but has so far been little used. In this connection the stopping power for fast particles has been computed on the basis of the experiments reported by R. R. Wilson in the preceding paper. Discussion of the formula used is contained in the appendix. Curves are presented for the dependence of range on energy and momentum for swift hydrogen and helium nuclei as well as for mesons. The method of momentum loss is illustrated and values are obtained for the mass of the meson from measurements published by Anderson and Neddermeyer in 1934. The evidence available from these mass determinations and those published by other authors shows the necessity for many more mass determinations before one can decide whether there is a distribution of masses of the meson or not.

INTRODUCTION

MEASUREMENT of the range of a meson and of the curvature of its track by a magnetic field, i.e., its momentum, are well known to suffice for a determination of its mass. For this purpose use is made of the theoretical relation between initial momentum and total path length. In a variant of this method one applies the differential form of the momentum-distance relation, and measures the change in magnetic rigidity of the particle over a small portion of its path. This procedure has been employed by several observers, and Corson and Brode[1] have discussed it in some detail.

The longer is the segment of path length considered, and the higher is the atomic number of the traversed material, the more surely one can exclude the possibility that the particle in question is an electron. But then one can no longer directly apply the differential expression for the rate of change of momentum with distance. Still it is in principle possible also in this case to determine the mass of the particle, as Neddermeyer and Anderson[2] remark in a recent review of methods of measuring this quantity. From an experimental point of view it is only necessary to observe the curvature before and after traversal of a plate of known thickness. However the procedure in question has been but slightly used.

This way of determining mass by the method of momentum loss appears to deserve further emphasis for three reasons:

(a) The necessary measurements of curvature of track in a magnetic field and of thickness of material traversed are among the most direct in the whole field of cosmic-ray research. Difficult determinations of the density of ionization along the track are avoided.

(b) Accuracy in this method, as in almost all other methods of determining the mass, requires that the velocity of the meson be not close to the speed of light. In the acceptable range of velocities, however, practically every meson whose curvature can be measured before and after penetration of the plate will permit an evaluation of its mass. Contrast this attainable abundance of data with the rarity of those collisions of a meson with an electron in which the energy transfer is sufficient to allow a determination of mass from the laws of conservation of momentum and energy.

(c) The law for the stopping power of matter for fast particles has an exceedingly reliable theoretical basis.[3]

The question whether all mesons have the same mass is so important that it appears worth while to give a qualitative discussion of the

[1] D. R. Corson and R. B. Brode, Phys. Rev. **53**, 776 (1938).
[2] S. H. Neddermeyer and C. D. Anderson, Rev. Mod. Phys. **11**, 200 (1939).
[3] See in this connection E. J. Williams, Phys. Rev. **45**, 729 (1934).

resolving power of the "method of momentum loss." If the energy of the meson is large in comparison with its rest energy, the loss of energy, ΔE, in a given thickness of material will be practically independent of energy, according to the theory of stopping power. In the relativistic relation between losses of energy and momentum, $\Delta E \sim v dp$, the velocity v will be essentially a constant, the speed of light. We will therefore find fast particles losing momentum of the same order of magnitude whatever be their mass. Consequently the resolving power is poor for mesons of relativistic velocity (momentum $\gg 100$ Mev/c; $H\rho \gg 4\times 10^5$ gauss cm).

This insensitivity of the momentum loss at relativistic velocities to the mass of the particle conversely makes it possible to test the law of stopping even for mesons whose precise masses are unknown. It is only necessary that their energies be sufficiently great: $H\rho > 10^6$ gauss cm. Measurements under this condition of the change in curvature on traversal of a plate were made by J. G. Wilson[4] and were found to verify a theoretical expression for the absolute rate of loss of energy not very different from (1) below.

For non-relativistic speeds the rate of energy loss per unit of distance may be written very roughly in the form

$$dE/dx \sim \text{constant}/v^2.$$

From this it follows that the mass, μ, of the particle is given by

$$\mu \sim p(dp/\text{const. } dx)^{\frac{1}{3}}.$$

The percentage error in μ will be given by the sum of (a) the percentage error in the momentum, p, or corresponding magnetic rigidity, $H\rho$, of the particle; (b) one-third the percentage error in the measurement of the momentum loss (if that loss is small compared to p); (c) one-third the percentage error in the thickness of the stopping material; (d) one-third the percentage error in the theoretical constant of the stopping power formula.

These considerations on the whole favor the "method of momentum loss" for non-relativistic velocities.

[4] J. G. Wilson, Proc. Roy. Soc. **A172**, 517 (1939).

Range-Momentum Relation

A more complete treatment of the method in question requires a discussion of the law of stopping power. Fortunately, radiative losses can be neglected for mesons of much greater than electronic mass as for protons and alpha-particles, even for the highest velocities ($v=0.9c$) and greatest atomic numbers (Pb; $Z=82$) which will be of interest. The so-called "radiation unit" or "unit length" of the radiative theory will have a value for mesons greater than that for electrons (0.4 cm in the case of Pb) approximately in the ratio of the squares of the two masses. Thus the mean energy dE_r lost in the form of radiation by a meson of 150-Mev energy and of mass 200 m traversing 1 cm of lead will be given by[5]

$$dE_r/E \sim dE_r/150 \text{ Mev} \sim (1 \text{ cm}) \ln 2/(200)^2\, 0.4 \text{ cm},$$

whence $dE_r \sim 0.01$ Mev. This value is trifling in comparison with the loss of ~ 10 Mev by ionization, which alone therefore need be considered.

According to the theory of stopping power, the rate of dissipation of energy through ionization and excitation by a fast particle of velocity v and of not too great charge number, z, will be given by the relation,[6]

$$dE/dx = (4\pi N e^4 z^2/mv^2) \times \{\ln[2mv^2/I(1-v^2/c^2)] - (v^2/c^2)\}. \quad (1)$$

Here N represents the number of electrons per cm^3 of stopping material, I is a certain mean energy of excitation for these electrons and m is the electronic mass. For atomic hydrogen,[7] $I=1.103(me^4/2\hbar^2)$; for helium,[8] $I=3.19(me^4/2\hbar^2)$. For air ($Z=7.22$), Livingston and Bethe[9] determined the mean excitation energy from the observed stopping power: $I=5.92(me^4/2\hbar^2)$. The experiments of R. R. Wilson reported in the preceding paper permit a similar evaluation of the mean excitation energy for aluminum:[10] $I=11.02(me^4/2\hbar^2)$. Elements of high atomic num-

[5] For a more detailed treatment of the radiation loss, see H. J. Bhabha, Proc. Roy. Soc. **A164**, 257 (1938).
[6] For literature and discussion see Appendix.
[7] H. A. Bethe, Ann. d. Physik **5**, 325 (1930).
[8] E. J. Williams, Proc. Camb. Phil. Soc. **33**, 179 (1937).
[9] M. S. Livingston and H. A. Bethe, Rev. Mod. Phys. **9**, 267 (1937).
[10] R. R. Wilson, Phys. Rev. **60**, 749 (1941) and discussion at end of Wilson's paper by J. A. Wheeler.

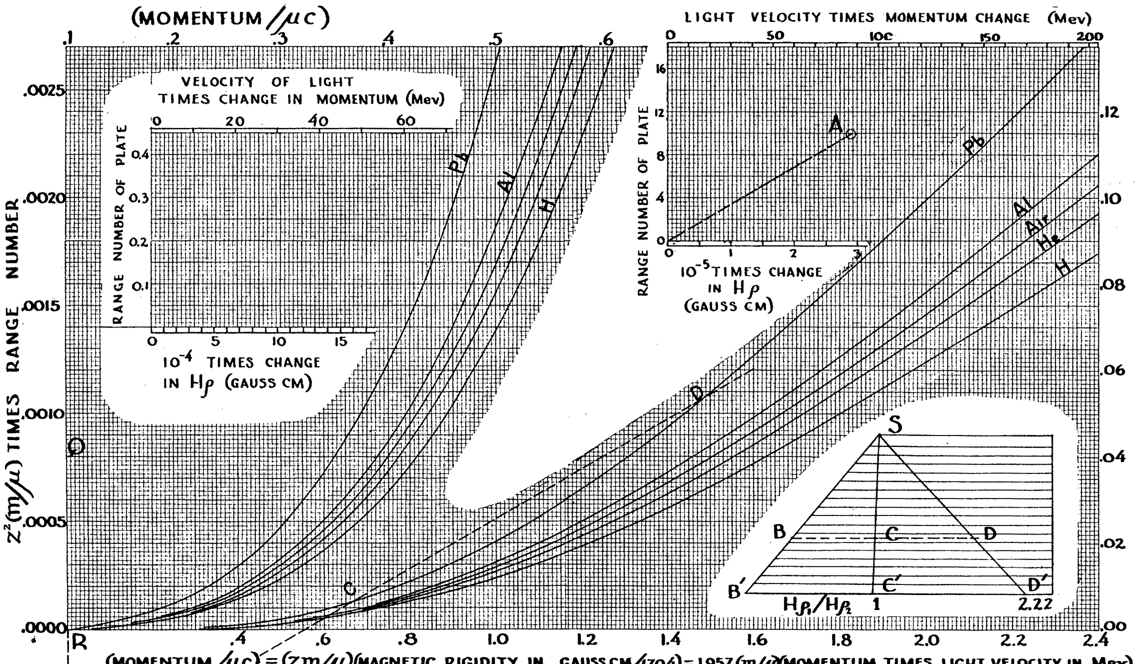

FIG. 1. Calculated relation between range and momentum for hydrogen and helium nuclei and for mesons. For the stopping materials computations were based on values of the mean ionization energy mentioned in the text. For Pb and Al the ionization energy was determined from Bloch's formula with the constant of proportionality based on the data of R. R. Wilson in the preceding paper. To interpolate for the stopping power of a compound substance such as $PbCl_2$ note that the logarithm of the effective atomic number is $(82 \ln 82+34 \ln 17)/(82+34)$. Given that a meson ($z=1$) with a given magnetic rigidity, $H\rho$, of 528,000 gauss cm passes through a 3.70-cm lead plate, corresponding to a range number of 9.92, and emerges with a rigidity of 238,000 gauss cm. We compute the change in $H\rho$ and plot the point A in the upper diagram. The secant CD must be parallel to OA but the positions of C and D individually remain to be found. A large scale transparent model is constructed of the template in the lower right corner of the figure. Lines SC and SD are drawn such that $D'B' : C'B' = DB : CB = 528{,}000$ gauss cm: 238,000 gauss cm. $=2.22$. With its rulings parallel to OA and with corresponding points $C'D'$ or CD, etc., kept on the curve for lead, the template is moved until the prolongation B' of $C'D'$ or B of CD, etc., lies on the line QR extended. The graphical solution so found is indicated by the points CD on the range curve. From the location of D it is seen that $1.49=(m/\mu)(H\rho_1/1704)=310\ (m/\mu)$. This result gives $\mu=209\ m$ for the mass of the meson as determined from the given observations.

ber are most effective in distinguishing mesons from electrons in the method of momentum loss, but for these no direct determination of the mean excitation energy is available. Fortunately (a) the stopping power is not very sensitive to the value of I and (b) Bloch's theory of excitation energies[11] permits a satisfactory estimate of this quantity. According to Bloch, the mean excitation energy for atoms containing many electrons is proportional to the atomic number Z. For the constant of proportionality he gave the value $I/Z=0.96(me^4/2\hbar^2)=13.1$ ev on the basis of the experiments available in 1933, but the data reported in the preceding paper by Wilson lead to a more reliable estimate of Bloch's constant:[10]

$$I/Z=0.85(me^4/2\hbar^2)=11.5 \text{ ev}.$$

For atoms to which the Thomas-Fermi model applies this formula gives

$$\ln(2mc^2/I)=11.391-\ln Z. \tag{2}$$

Let us now state the connection between range and energy or momentum in the form which is convenient for general use and at the same time suited for determining the mass of the meson. The range of a particle of mass μ and charge number z may be considered to depend on any one of the following quantities:

$$\begin{aligned}
&\text{kinetic energy:} && E=\mu c^2(\cosh\theta-1)\\
&\text{momentum:} && p=\mu c\sinh\theta\\
&\text{magnetic rigidity:} && H\rho=\mu(c^2/ze)\sinh\theta\\
&\text{velocity:} && v=c\tanh\theta.
\end{aligned} \tag{3}$$

Better than any of these quantities for inde-

[11] F. Bloch, Zeits. f. Physik **81**, 363 (1933).

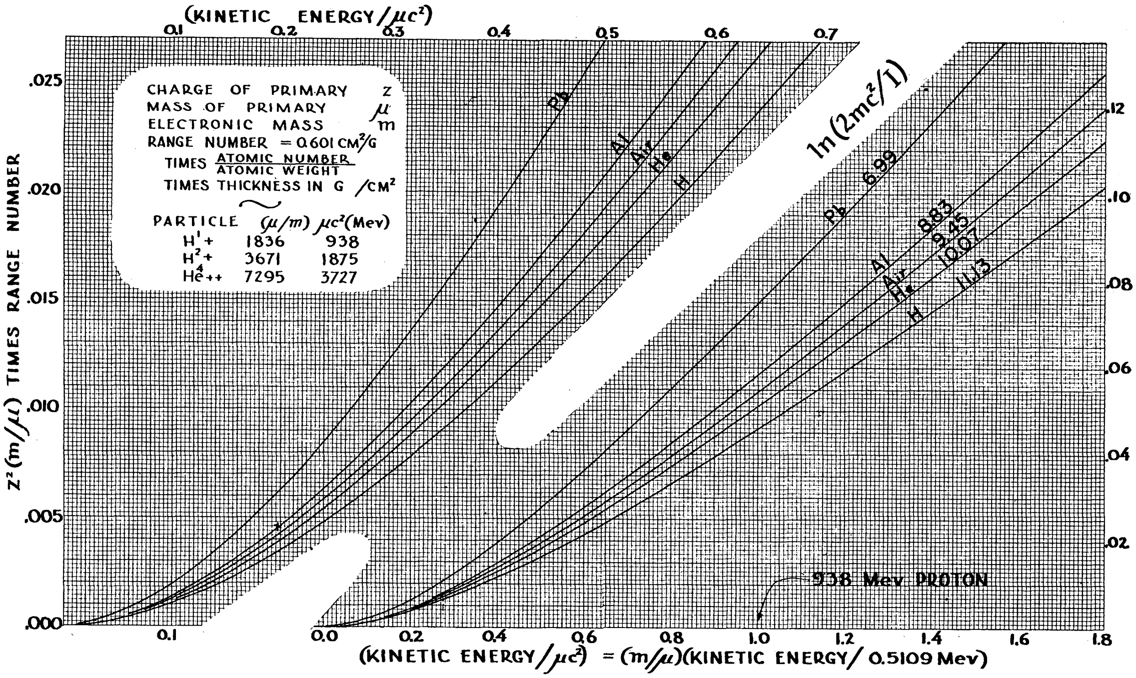

FIG. 2. Calculated relation between range and energy for hydrogen and helium nuclei and for mesons. The point x corresponds to an alpha-particle with the following properties: energy $=0.2\times3727=745$ Mev; range in aluminum $=0.0046\times(7295/4)\times(26.97/13\times0.601\ \mathrm{cm^2/g})=29\ \mathrm{g/cm^2}$ or 10.7 cm.

pendent variables, however, are the dimensionless numbers $\sinh\theta$ and $\cosh\theta$, used in Figs. 1 and 2, respectively. If the dependent variable or range is expressed in g/cm² it will fluctuate in an irregular way from element to element because of the lack of any precise relation between atomic number Z, and atomic weight, A. For this reason let us express the range in terms of a dimensionless "range number," defined as follows:

$$\begin{aligned}\text{"range number"} &= 4\pi\cdot\text{Avogadro's number}\\ &\cdot(e^2/mc^2)^2\cdot(Z/A)\cdot(\text{range in g/cm}^2) \qquad (4)\\ &=(0.601\ \mathrm{cm^2/g})\cdot(Z/A)\cdot(\text{range in g/cm}^2).\end{aligned}$$

In the case of CH_4, the expression Z/A will have the value $(6+4)/(12+4)$. In the case of dry air at 15°C, at a pressure of $76\times13.6\times980$ dynes/cm², division of the range by 2714 cm gives the range number. For lead of density 11.3 g/cm³ range number equals thickness multiplied by 2.68 cm⁻¹.

In terms of the "range number," relation (1) takes the form $(z^2m/\mu)\cdot(\text{range number})=$

$$\int^{\theta}\frac{d(\cosh\theta+\cosh^{-1}\theta-2)}{\ln(2mc^2/I)+2\ln\sinh\theta-\tanh^2\theta}. \qquad (5)$$

Numerical integration gives the curves[12] drawn in Figs. 1 and 2. The constant of integration was fixed in the case of air so as to give a range of 238 cm for 15 Mev protons;[9] it was estimated in the case of the other substances listed. Negligible on the scale of the figures is the slight dependence of the constant of integration in (5) upon the charge of the primary, a dependence which arises principally from the phenomenon of capture and loss of electrons at the very end of the range. Of course, the constant of integration has no effect on the determination of the mass of the meson by the method of momentum-loss.

Figure 1 illustrates how measurements of magnetic rigidity before and after a meson

[12] As long as they last, large photostats of these curves may be obtained from the Secretary, Department of Physics, Princeton University. We are indebted to Mr. Ralph Thompson for much help with the numerical computations.

TABLE I. *Columns 1, 2 and 3 give measurements of Anderson and Neddermeyer on single cosmic-ray particles. On the preliminary assumption that each particle is a meson, the range-momentum relation allows as described in Fig. 1 a computation of the ratio (μ/m) of unknown mass to electronic mass. Knowledge of the mass and momentum after traversal of the lead plate suffices to determine the factor $1/\beta^2$ = (velocity of light/velocity of particle)^2 which is approximately proportional to the ionization. It is seen that this ionization would be exceedingly large for the last three particles if they were mesons. However, the observed rate of ionization, according to a kind personal communication of Anderson, is certainly not so great for these particles as twice the rate for a fast electron. Therefore the last three particles cannot be mesons of the computed masses. In the absence of information about the ionization of the other particles, we have calculated the "energy decrement number," I, to test whether it is reasonable to assume that these particles are electrons which have experienced radiative losses. According to theory, all values of I between 0 and 1 are equally probable for electrons. Consistently small values of I are therefore an argument against attributing electronic character to a group of particles. The angles θ* elec. *and θ* meson *are defined in the text. Arguments based on scattering indicate that no one of the last five tracks, even if it represents a meson, will permit a mass determination.*

1	2	3	4	5	6	7	8	9	10	11	12	13
	Momentum (in Mev)			Analysis if Electron					Anlaysis if Meson			
Thickness of Lead	cp_1	cp_2	E_t (Mev)	t	I	θ Elec. (Degrees)	$c\Delta p$ (Mev)	Range Number	$\frac{H\rho_1}{H\rho_2}$	$\frac{\mu}{m}$	$\frac{1}{\beta^2}$	θ Meson (Degrees)
1.35	113	86	102	3.1	0.00	33	27	3.6	1.32	150	1.8	44
1.1	220	160	173	2.5	0.01	16	60	3.0	1.33	590	4.6	35
1.1	200	125	138	2.5	0.02	21	75	3.0	1.60	560	6.2	52
1.1	240	220	233	2.5	0.00	12	(20)	3.0	1.09	(310)	1.5	14
1.1	38	6	19	2.5	0.07	430	32	3.0	6.3	[45]	15.7	1700
0.7	63	23	31	1.6	0.27	90	40	1.9	2.7	[110]	6.9	238
1.0	140	20	32	2.3	0.34	123	120	2.7	7.0	[390]	101	1230
1.0	106	26	38	2.3	0.20	95	80	2.7	4.1	[250]	26	475
1.5	110	12	30	3.5	0.09	250	98	4.0	9.2	[220]	88	2350

penetrates a plate of known thickness allow a determination of the mass of the meson.

Distinction Between Electrons and Mesons

In 1934 Anderson and Neddermeyer[13] reported measurements on the momentum loss of single cosmic-ray particles which had traversed ~1 cm of lead without undergoing multiplication. Their observations are reproduced in the first three columns of Table I. As the cascade theory and the work of these and other physicists have since emphasized, many particles of this character at sea level are mesons. On the preliminary assumption that all of the particles are mesons, the entries in column 8 of Table I will represent loss of momentum by ionization alone. Consequently the methods outlined in the caption of Fig. 1 will permit a determination of the mass of each particle. The values in columns 8, 9, and 10 of the table were used in the graphical calculations. The computed ratio of the mass of each particle to that of the electron is given in column 11. No estimate of probable error has been attempted. Those values will generally be the more reliable where the fractional loss in momentum has been the greater.

From the data alone it is not possible to tell which tracks represent mesons and which represent electrons. In fact two possible explanations exist for the change in momentum in any given instance. Either the loss is due to a high rate of ionization, in which case the particle must be moving slowly, and the quotient (momentum/velocity) will be large and will indicate a meson; or the loss is due to radiation, which is only possible if the mass is comparable to that of an electron. There are in principle, however, three qualitative means to distinguish between electrons and mesons: (a) ionization, (b) scattering, and (c) the theory of radiation.

(a) A meson emerging from the lead plate into the cloud chamber will ionize more than an electron of the same momentum. The relative rate of ionization will be given approximately by the ratio $c^2/v^2 = 1 + (\mu c^2/cp)^2$ which has been computed in column 12 of Table I on the assumption that the particles are mesons. The calculated rates of ionization are on the whole rather different from those to be expected for electrons, and in fact Dr. Anderson informs us that an inspection of the last three tracks shows that

[13] C. C. Anderson and S. H. Neddermeyer, *International Conference on Physics* (London, 1934, Table II, p. 179).

this criterion excludes the possibility that these tracks represent mesons. It will be noted that this test requires only a qualitative knowledge of the ionization, in contrast to those methods of determining the meson mass which depend on an accurate count of numbers of ions.

(b) Particles of the same momentum will suffer the more scattering the greater is their mass. Following the treatment of multiple scattering given by Williams,[14] we may express the arithmetic mean projected angle of deflection due to multiple scattering in lead approximately in the form

$$\theta \text{ in degrees} = (2500 \text{ degrees Mev}/pc)\cdot(\text{thickness in cm})^{\frac{1}{2}}(1+[\mu c^2/pc]^2)^{\frac{1}{2}}. \quad (6)$$

Here there is some ambiguity as to the proper mean value to insert for the momentum, p, which changes in traversing the plate. The scattering is greatest when the particle is traveling most slowly. We have used for p in (6) the momentum after traversal of the plate in order to obtain a rough estimate of the order of magnitude of the scattering to be expected for each of the tracks in Table I. As columns 7 and 13 of this table show, there is not sufficient difference in the expected scattering for the first four particles to distinguish between electrons [$\mu = m$ in Eq. (6)] and mesons (μ from column 11 of table). For the last five tracks the computed scattering is so great that the calculation itself has no meaning. It is clear that these tracks will give no information about the mass of mesons.

Generalizing the foregoing conclusion, we may say that it is important to avoid large scattering in determining the mass of the meson by the method of momentum loss. This condition will be satisfied if the momentum of the particle after traversal of the plate is not too small. On the other hand, the momentum afterward must be considerably less than that before traversal if the measurement of the momentum change is to be accurate. Both conditions will be abundantly fulfilled if the primary momentum is of the order of 100 Mev and the momentum decreases in the plate by a factor of the order of 2.

(c) In addition to ionization and scattering as criteria in selecting acceptable tracks, we have in the theory of radiation a means to analyze the possibility that the particles in Table I are fast electrons. If this is the case, they will have lost energy by ionization at a rate of ~ 12 Mev per cm of lead. We can therefore correct the values, cp_2, of the final energy to obtain the value, E_t, which would have been observed if radiative losses alone occurred. Column 4 of the table gives E_t and column 5 shows the thickness, t, of the plate as measured in terms of the so-called "unit-length" of the theory of radiation. There is to be expected a statistical connection between E_t and t. According to Bethe and Heitler[15] the chance, dN, that an electron emerges from the plate with a final energy E_t (corrected for ionization losses) is given by the expression

$$dN = E^{-1}dE_t(\ln E/E_t)^{t-1}/(t-1))! \quad (7)$$

Let us measure the loss of energy through radiation by an "energy decrement number," I, defined as follows:

$$I = \int^{\ln(E/E_t)} y^{t-1}\exp(-y)dy/(t-1)! = I(\ln E/E_t, t-1). \quad (8)$$

This quantity will be zero if there is no energy loss, unity if the particle loses all of its energy by radiation. It is a purely *experimental* quantity and is easily computed, for extensive tables of the function $I(z, p)$ are available.[16] In terms of the "energy decrement number," relation (6) takes a very simple form:

$$dN = dI. \quad (9)$$

In words, *all values of the energy decrement number between zero and unity are equally probable,* whatever be the original energy of the electron or the thickness of the plate which it penetrates.

The energy decrement number, I, for each particle in Table I has been computed according to the definition in Eq. (7). The entries in column 6 show that the values of I are on the whole quite small, particularly for the first four tracks. In the absence of other information, consistently small values of I will be an argument against attributing electronic character to a group of particles.

[14] E. J. Williams, Phys. Rev. **55**, 303 (1940).

[15] H. Bethe and W. Heitler, Proc. Roy. Soc. **A146**, 83 (1934).

[16] K. Pearson, *Tables of the Incomplete Γ-Function* (London, 1922).

TABLE II. *Previously published data on the mass of the meson. The methods used for the determination are: curvature in a magnetic field and ionization density (c and i), curvature and range (c and r), change of curvature in a known amount of matter (c and c), curvature and collision with an electron (c and e). The errors given for the momentum and the mass are those calculated by the authors. The references are listed separately. Notes give additional information from original paper.*

	cp (IN MEV)	MASS (μ/m)	METHOD	REF.
1	33	220 ± 50	c and i	1
2	55	$>430(<800)$	c and i	1
3	44	190 ± 60	c and i	1
4	34	160 ± 30	c and i	1
	Note: Values of 1, 3 and 4 are considered reliable; the upper limit of μ/m for particle 2 is set by a minimum range equal to the observed length of track in the chamber, the lower limit is estimated from the lower limit of ionization.			
5	45	250	c and i	2,3
	Note: Mass value considered as "good."			
6	29	~130	c and i	4
	Note: Value only approximate. Corson and Brode, (3), calculate $\mu/m=160$ with a different method for evaluating the ionization loss.			
7	57	~200	c and i	5
	Note: Value "probable," but not very good.			
8	16.5	<200	c and r	14,3
	Note: Observable range was 18 cm, track out of focus, so that ionization density not measurable; if it was 10 times normal, mass would be 125.			
9	16.5	~350	c and r	6,3
	Note: Observed by (6), calculated by (3); according to private communication from Anderson curvature is average value over whole length of track (4 cm), particle besides probably scattered, computed value has "not much meaning."			
10	≤42	<1000	c and r	6,3
	Note: The same critical remark *a fortiori*, as the particle does not stop in the chamber (range >5 cm) and the $H\rho$ value given is only an upper limit.			
11	52	220 ± 35	c and r	7,8
	Note: Considered by Anderson as reliable measurement.			
12	180	<65	c and e	8
	Note: Value calculated from *possible* collision with an atomic electron which coming out of the plate shows an energy of 16 Mev; other interpretation possible.			
13	7.7	120 ± 30	cc	9,3
	Note: Curvature of track changes in the gas by 0.6 cm/cm. Observed length of 5 cm sets a lower limit for the range which gives according to (3) an upper limit of mass as 110 m.			
14	$222\pm3\rightarrow150\pm3$	180 ± 20	cc	10
	Note: Particle loses momentum by penetrating 4.8 cm of Pb; calculated by use of Bhabha's relativistic formula [Proc. Roy. Soc. **A164**, 255 (1938)].			
15	11.6 ± 0.2	170 ± 8	c and r	10
	Note: Radius of curvature is an average over range of 6.5 cm, therefore calculated mass is somewhat doubtful, in spite of small margin of error given. No allowance made for the scattering which is important at such a low velocity.			
16	2.4	20?	c and r	11
17	8 ± 3	100 ± 30	c and r	11
18	8.5 ± 1.5	120 ± 30	c and r	11
19	4 ± 2	55 ± 35	c and r	11
20	12 ± 6	$170+100$	c and r	11
	Note: Values obtained by use of "slow" cloud chamber; masses of particles 17 and 18 considered as better values than those of the other particles. No allowance made for the scattering which is important at such low velocities.			
21	93	240 ± 20	c and e	12
	Note: Value reliable.			
22	$221\rightarrow126$	250 ± 50	cc	13
23	$165\rightarrow87$	170 ± 20	cc	13
	Note: 23 considered reliable.			
24	30	180 ± 25	c and i; c and e	15
	Note: Reliable.			
	Average of reliable values: 180			

[1] E. J. Williams and E. Pickup, Nature **141**, 648 (1938).
[2] D. R. Corson and R. B. Brode, Phys. Rev. **53**, 215 (1938).
[3] D. R. Corson and R. B. Brode, Phys. Rev. **53**, 773 (1938).
[4] J. C. Street and E. C. Stevenson, Phys. Rev. **52**, 1003 (1937).
[5] P. Ehrenfest, Jr., C. R. Paris, **206**, 428 (1938).
[6] C. D. Anderson and S. H. Neddermeyer, Phys. Rev. **50**, 263 (1936), (Figs. 12 and 13).
[7] S. H. Neddermeyer and C. D. Anderson, Phys. Rev. **54**, 88 (1938).
[8] S. H. Neddermeyer and C. D. Anderson, Rev. Mod. Phys. **11**, 191 (1939), (Fig. 19).
[9] A. J. Ruhlig and H. R. Crane, Phys. Rev. **53**, 266 (1938).
[10] Y. N. Nishina, M. Takeuchi and T. Ichimiya, Phys. Rev. **55**, 585 (1939).
[11] H. Maier-Leibnitz, Zeits. f. Physik **112**, 569 (1939).
[12] L. Leprince Ringuet, S. Gorovdetzky, E. Nageotte and R. Richard-Foy, Phys. Rev. **59**, 460 (1941).
[13] J. G. Wilson, Proc. Roy. Soc. **A172**, 521 (1939).
[14] R. B. Brode and M. A. Starr, Phys. Rev. **53**, 3 (1938).
[15] Donald J. Hughes, Phys. Rev. **60**, 414 (1941).

MASS OF THE MESON

The criteria of ionization and scattering exclude the last five tracks in Table I, and the change of momentum is too small for the fourth track to permit a reliable mass determination. Without a qualitative knowledge of the ionization we cannot exclude the otherwise unlikely possibility that the first three tracks represent electrons. If they represent mesons, however, they by no means lead to a unique value for the mass of the meson. Nor do the mass determinations previously published and compiled in Table II allow a decision of the very important question whether the mass of the meson is unique. In this connection we may recall the remark of Anderson and Neddermeyer in 1939[2] that "it has become increasingly likely that a complete interpretation of the experimental data is not to be found in the single assumption of unstable particles with unit charge and a unique mass of the order of 200 electron masses." Also Weisz[17] has pointed out that assumption of a suitable distribution of meson masses allows a reasonable correlation of otherwise discordant estimates of the lifetime of the meson.

Before any statistically reliable conclusions about the uniqueness of mesons are possible, it will be necessary to obtain many more measurements of the masses of penetrating particles. For this purpose the method of momentum loss appears especially satisfactory in view of the considerations presented in this paper.

We are indebted to Dr. Carl D. Anderson for correspondence and to Dr. Marcel Schein for a number of interesting discussions on the properties of the meson.

Appendix. Remarks on Eq. (1) for Stopping Power

The non-relativistic form of relation (1) was first derived by H. Bethe,[18] who has also given the above relativistic formula. Equation (1) does not, however, agree with the relativistic expressions for stopping power as published in various general references.[19] In the derivation one classifies the various energy losses, Q, into two groups according as they are less than or greater than a certain quantity, Q_1, which is small in comparison to mc^2, large in comparison with atomic binding energies, but which is otherwise arbitrary. The rate of loss of energy by collisions in the first group is

$$(2\pi Ne^4Z^2/mv^2)\{\ln[2mv^2Q_1/I^2(1-v^2/c^2)]-(v^2/c^2)\}$$

as Møller[20] and Bethe[20] have both proven. E. J. Williams[21] obtained the same result by the method of impact parameters and was able to show that the relativistic terms in this expression arise from the Lorentz contraction of the field of the primary particle in distant collisions, and should therefore be considered as especially reliable. In collisions of the second group where the energy transfer is large ($Q>Q_1$) and lies between Q and $Q+dQ$, the calculated average rate of loss of energy is

$$(2\pi Ne^4Z^2/mc^2)(dQ/Q)[c^2/v^2-(Q/Q\ \mathrm{max})+\mathrm{correction}].$$

Here $Q\ \mathrm{max}=2mp_0^2/(m^2+\mu^2+2mE_0/c^2)$ is the maximum loss which a primary of momentum p_0 and total energy E_0 can experience in a single encounter. The calculated value of the correction term is in general very large and depends in a striking way upon the spin and magnetic moment assumed for the heavy particle.[22] The calculated correction term is however negligible if the energy of the struck electron in the center-of-gravity frame of reference is small in comparison with mc^2. This condition is satisfied if the energy of the primary in the laboratory frame of reference is small in comparison with $(\mu/m)\mu c^2$, or consequently Q max is considerably less than E_0. Then impacts for which Q is between Q_1 and Q max result in an average rate of loss of energy,

$$(2\pi Ne^4Z^2/mv^2)\{\ln(Q\ \mathrm{max}/Q_1)-(v^2/c^2)\}.$$

If in addition μ is much greater than m, then Q max will be given by $2mp_0^2/\mu^2=2mv^2/(1-v^2/c^2)$, and the total average rate of loss of energy will be represented by expression (1). Relation (1) is valid only if the primary is considerably heavier than an electron; only if its energy is small in comparison with $(\mu/m)\mu c^2$; only if it is moving faster than the bound electrons of the stopping material;[23] only if capture and loss of electrons by it can be neglected and its charge is not too great;[24] only if the energy is not so great that the density of the material has an influence upon its stopping power;[25] only if the average energy loss and most probable energy loss are essentially identical.[26] All these conditions are satisfied in the present application.

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