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Range and Ionization Measurements on High Speed Protons

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The relative specific ionization along an initially mono-energetic 4-Mev proton beam has been measured. The stopping power of aluminum relative to air is found to be 1.48 mg/cm² per cm of air at 15°C and 76 cm Hg, and is also found to be independent of the energy in the interval from 1.5 Mev to 4.0 Mev. That this should be so is adequately explained in the interpretation by J. A. Wheeler. The stopping power of a variety of metal foils was also measured and the results are given. From the above measurements it is possible to calculate I, the mean excitation energy of atoms with many electrons (a constant which enters in Bethe's formula for the rate of loss of energy of heavy particles due to ionization) to be 0.85 $Z(me^4/2\hbar)$ or 11.5 Z in electron volts.

A. INTRODUCTION

IN connection with an experiment on proton-proton scattering[1] in which the small Berkeley cyclotron (4-Mev protons) was used, it was desirable to obtain a mono-energetic proton beam of known energy. The raw beam of the cyclotron is inhomogeneous in energy by as much as twenty percent.[2] This difficulty was circumvented by appropriately placing three one-mm slits about 30 cm apart and in the fringing magnetic field of the cyclotron.[3] Although this decreased the intensity of the original beam by a factor of about one thousand, there was ample current left for scattering and range experiments. To determine the energy of this so obtained mono-energetic beam, it was decided for reasons of expediency and accessibility to measure the ionization in air produced by the protons as a function of their range. It is felt that the results of these measurements, although terminated before their completion, are of sufficient interest to warrant publication.

B. RELATIVE IONIZATION BY HIGH ENERGY PROTONS

Figure 1 illustrates the apparatus and method of measuring the relative ionization produced by the protons. Because of the difficulty of keeping the beam current constant, it was necessary to use two ionization collectors as shown; one to measure the specific ionization at various points along the proton path, and another, which was kept in a fixed location, to serve as a monitor ion collector. The protons emerge from the vacuum system through a one-half mil Cellophane foil and first pass through the monitor ion collector. Then they pass through air until they reach the movable shallow ion collector. This collector consists of a thin aluminum foil (2.16 mg/cm²) located 0.5 mm in front of an insulated circular

* The experimental part of this paper was done in August, 1940 at the Radiation Laboratory of the University of California.

[1] R. R. Wilson and E. C. Creutz, Phys. Rev. **59**, 916 (1941).

[2] R. R. Wilson J. App. Phys. **11**, 781 (1940).

[3] E. C. Creutz and R. R. Wilson, Phys. Rev. **59**, 916 (1941).

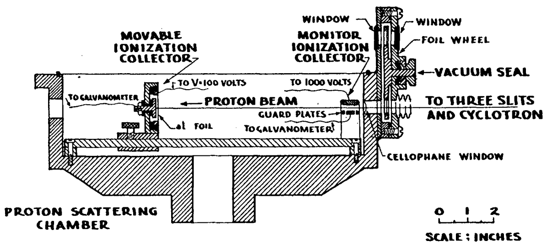

FIG. 1.

brass button of $\frac{3}{4}''$ diameter. The foil was raised to a potential of a few hundred volts with respect to the button, and the ionization current to the button was measured with a sensitive galvanometer. The monitor collector consisted simply of two parallel plates (with guard plates), one of which was raised to a potential of about a thousand volts and the other which led to a galvanometer that measured the ion current. The voltages on the collectors were adjusted high enough so that the ionization currents were very nearly independent of voltage.

The ratio I of the ionization current in the shallow ion chamber to that in the monitor collector, normalized to unity at its maximum value, is plotted in Fig. 2 for various positions of the shallow ionization collector. It is seen that the points fall along a typical Bragg curve. To determine if the results were influenced by heating of the air due to the energy loss of the protons (current about 10^{-10} amp. and of cross section 1×2 mm), a run was made with one-half the proton current used before. Within the accuracy of the measurements, the points were unchanged from their previous values. The temperature of the air as well as the atmospheric pressure was read at each measurement so that the results could be reduced to standard values. Because of multiple scattering in the air, the proton beam, as indicated by the fluorescence it produced on a ZnS screen, spread out to nearly the full aperture of the movable ion collector at its greatest distance from the entrance foil, and it is possible that the points near the end of the Bragg curve are slightly influenced by this effect.

The striking characteristics of the curve are the pronounced decrease of ionization with energy, the very sharp maximum, and the rapid linear decrease of the curve after the maximum. The curve can be fit adequately, except for the last two cm, by an equation of the form

$$I = A/(R_0 - R), \qquad (1)$$

where A and R_0 are constants with values 4.2 and 27 cm, respectively. The sharp maximum is an indication of how mono-energetic the proton beam was. The intersection of a straight line fitted to the linear part near the end of the Bragg curve with the range axis gives the *extrapolated ionization range*. It can be seen that this quantity is very precisely determined from such an

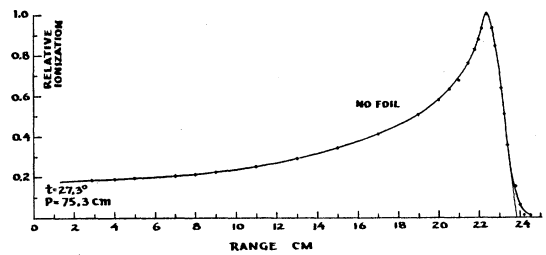

FIG. 2.

ionization range curve. However, the relation between the extrapolated ionization range and the mean range, from which can be obtained the energy,[4] is not yet accurately known for protons. The difference between the two ranges can be calculated if one knows how the ionization varies along the path of an individual proton, especially near the end of the range. Rado[5] has attempted to do this with the available data. Inasmuch as these data are so widely variable from one observer to another, he was unable to reach any definite value. His work does suggest, however, that 0.6 cm should be subtracted from the extrapolated ionization range of 23.2 cm, as read from Fig. 2, to give 22.6 cm as the mean range. When this range is reduced to 15°C and 76-cm Hg pressure, and when the stopping power of the foils is added, we obtain 4.00 Mev for the energy of the protons as given by the Cornell range energy relation.[5] The determination of the stopping power of the foils will be explained in the next section.

It is felt that obtaining an ionization range

[4] M. S. Livingston and H. A. Bethe, Rev. Mod. Phys. 9, 264 (1937).

[5] Rado, Thesis, Massachusetts Institute of Technology (1939).

curve would be a convenient and accurate method of determining the energy of a proton beam if a relation between the energy and the extrapolated ionization range existed. It is hoped to obtain such a relation experimentally by using the magnetic analyzer developed at Princeton for measuring proton energy.

C. The Stopping Power of Aluminum and of Other Metals

A rotating foil wheel was located as shown in Fig. 1 so the protons could pass through any one of the several aluminum foils of different thicknesses. The thickness of each foil was determined by measuring its area and weight with an accuracy of about one percent. The area of each foil was about one square cm. A chemical analysis of the aluminum from which the foils was cut showed it to be 99.35 percent aluminum, 0.35 percent iron, and 0.25 percent silicon.[6]

With each foil in place, the same type of measurements were made as described in the previous section. The results of this series of measurements are shown in Fig. 3. All the data were taken at one time. The curves all seem to have the same shape as that obtained with no foil, except that they are shifted to the left. This indicates that the straggling does not differ appreciably in air and aluminum. In Fig. 4 the

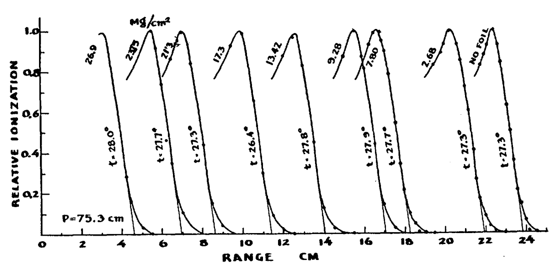

Fig. 3.

stopping power of each foil, determined by subtracting the extrapolated ionization range obtained with the foil from that without a foil, is plotted. The extrapolated ranges were reduced to the same temperature and pressure (300°K and 73.31-cm Hg) before the subtractions were made. Figure 4 shows that the stopping power of each foil when plotted against its weight per mg falls surprisingly close to a straight line that passes through the origin. This is interpreted as meaning that the stopping power relative to air is independent of the energy in the interval from 1.5 to 4.0 Mev. The slope of the straight line of Fig. 4 gives the specific stopping power to be 1.48 mg/cm^2 of aluminum per cm air when reduced to 15°C and 76 cm Hg.

In the same way, the stopping powers of several other metal foils were measured. In these

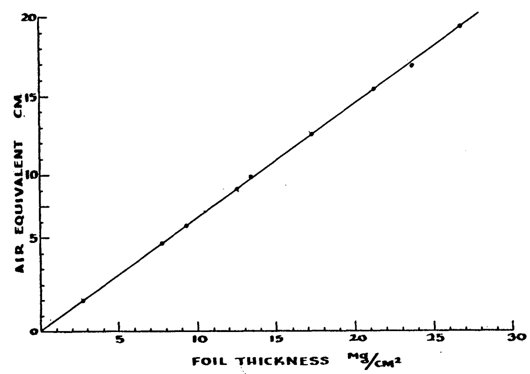

Fig. 4.

cases only one foil of each metal was available. The ionization range curves are shown in Fig. 5, and in Table I is listed the specific stopping power for each of the foils measured and the quantity q which is explained in the interpretation.

D. Interpretation of the Results[7]

Bethe has given a well-known formula[8] for the rate of loss of energy per unit of path length by heavy particles of non-relativistic velocity. For protons his relation reduces to

$$-dE/dx = (4\pi NZe^4/mv^2)\ \ln(2mv^2/I). \qquad (2)$$

Here v is the speed of the particle, and e and m represent the electronic charge and mass, N gives the number of atoms per unit of volume, Z is the atomic number of the stopping material and I is a certain mean excitation energy characteristic of the atom in question. The value of I varies with atomic number in a complicated way for light atoms. On the other hand, for atoms containing many electrons, Bloch has shown[9] on the basis of

[6] The foils and analysis were kindly supplied by the Reynolds Metals Company.

[7] Discussion contributed by J. A. Wheeler.

[8] H. A. Bethe, Ann. d. Physik **5**, 325 (1930).

[9] F. Bloch, Zeits. f. Physik **81**, 363 (1933).

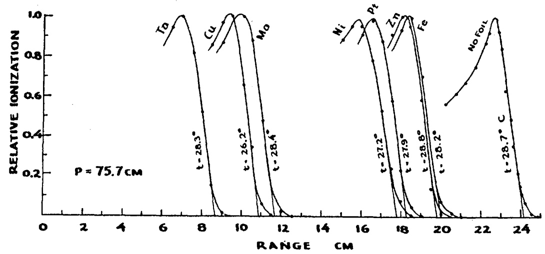

FIG. 5.

the statistical atomic model that the mean excitation energy should be proportional to the atomic number, Z. No one has as yet computed from first principles the constant of proportionality,[10] which therefore has to be found by comparison with experiments. This Bloch did.[9] He found $I/Z = 0.961(me^4/2\hbar^2) = 13.1$ ev. Unfortunately the velocities at which measurements were available in 1933 were limited to 1.92×10^9 cm/sec. (7.6 Mev alpha-particle; same speed as K electrons of element $Z=9$). In the meantime further theoretical work of Livingston and Bethe[4] has emphasized the importance of the corrections which must be introduced into the simple relation (2) when the velocity of the particle is low enough to be comparable to the speed of the bound electrons in the stopping material. One would arrive at a misleading estimate of Bloch's constant if he were to look apart from such corrections even in discussing the present experiments of Wilson, performed with protons of velocity ranging up to $v = 2.77 \times 10^9$ cm/sec. Thus if Bethe's simple formula applied, and I were exactly proportional to Z, one would expect there to exist a linear relationship between relative stopping power and logarithm of atomic member:

$$\frac{(-dE/NZdx)_{\text{material}}}{(-dE/NZdx)_{\text{air}}} = \frac{\ln(2mv^2/\text{constant}) - \ln Z}{\ln(2mv^2/\text{constant}) - \ln 7.22}. \quad (3)$$

The left-hand side of relation (3) may be represented by the letter q:

$$q = \frac{(-dE/NZdx)_{\text{material}}}{(-dE/NZdx)_{\text{air}}} = \left[\frac{(NZdx)_{\text{air}}}{(NZdx)_{\text{material}}}\right]_{\text{for equivalent stopping}}$$

$$= \frac{\text{atomic weight of material}}{\text{twice its atomic number}} \times \left(\frac{\text{mg/cm}^2\text{ of air}}{\text{mg/cm}^2\text{ of material}}\right)_{\text{for equivalent stopping}}. \quad (4)$$

This quantity has been computed from the experimental data in Table I and is plotted in Fig. 6 as a function of $\ln Z$. It is seen that q is a practically linear function of $\ln Z$. Without further consideration one would be led to the false conclusion that he could obtain a fairly accurate value of Bloch's constant from the intercept of the straight line with the horizontal coordinate axis: $\ln(2mv^2/\text{constant}) = 6.40$, whence I/Z = effective mean of initial and final energy of proton, divided by 276,000. Actually, however, the mean energy of the protons employed in the experiments differed considerably (2–4 Mev) from element to element, so that on the basis of Bethe's simple theory one should not expect the points in Fig. 6 to come at all close to a straight line, as they do. Moreover, for a definite element, such as aluminum, the stopping power relative to air is seen from Fig. 4 not to vary with energy as it should according to (3).

Both of the difficulties just mentioned are resolved when one notes that the speed of a K electron of aluminum is 2.8×10^9 cm/sec., or practically equal to that of the proton. This

TABLE I. *Specific stopping power.*

Foil	$\frac{\text{MG}}{\text{CM}^2}$	Total Stopping by Foil in CM	Specific Stopping Power in $\frac{\text{MG}}{\text{CM}^2}$ per CM	Energy Interval in Mev	q
Al	17.30	11.72	1.48	1.5–4.0	0.864
Cu	25.35	12.55	2.02	2.6–4.0	0.665
Fe	7.47	4.07	1.83	3.7–4.0	0.718
Mo	27.85	11.87	2.34	2.7–4.0	0.597
Ni	11.48	5.97	1.92	3.4–4.0	0.668
Pt	17.80	5.40	3.30	3.4–4.0	0.465
Ta	47.20	14.72	3.20	2.2–4.0	0.475
Zn	8.29	4.13	2.01	3.7–4.0	0.668

[10] An approximate theoretical estimate was, however, made by H. Jensen, Zeits. f. Physik 106, 620 (1937).

circumstance requires the introduction of the corrections of Livingston and Bethe[4] into the right-hand side of (3), which then, quite independent of Bloch's theory of excitation energies, becomes:

$$\frac{\ln(2mv^2/I_{\mathrm{Al}}) - (C_k/Z)_{\mathrm{Al}}}{\ln(2mv^2/I_{\mathrm{air}}) - (C_k/7.22)_{\mathrm{air}}}. \quad (5)$$

For I_{air} Livingston and Bethe find 80.5 ev or $5.92(me^4/2\hbar^2)$ from the observed absolute stopping power.[4] Their Fig. 28 gives the corrections C_K. Wilson's Fig. 4 shows that 27.8 mg/cm² of Al are equivalent to 20 cm air (300°K, 75.31 cm pressure). This comparison gives for the ratio (5) the value $(26.97/26)(20\times1.166/27.8)=0.870$, constant over the range of velocities investigated. These data are used in Table II to compute I_{Al}.

The values in the last column come out to be strikingly consistent with one another. Since corrections for binding of the L electrons will be least important at the higher energies, it is reasonable to take 8.83 as best value of $\ln(2mc^2/I_{\mathrm{Al}})$. This result gives $I_{\mathrm{Al}}=150$ ev $=11.0(me^4/2\hbar^2)$. Table II suggests why the stopping power of aluminum is constant relative to air over a considerable interval of energy: the correction for binding of the K electrons is decreasing for one element, increasing for the other. The complicated nature of the compensation shows how difficult it is to give any simple interpretation to the linear relationship in Fig. 6.

TABLE II. *Determination of mean excitation energy of aluminum from stopping power relative to air observed by Wilson for protons of various energies.*

Energy Mev	ln $(2mc^2/I_{\mathrm{air}})$	$(C_k/7.22)_{\mathrm{air}}$	Numerator of (5)	$(C_k/Z)_{\mathrm{Al}}$	ln $(2mc^2/I_{\mathrm{Al}})$
2.0	4.00	0.11	3.38	0.04	8.87
2.5	4.22	0.10	3.58	0.05	8.86
3.0	4.40	0.10	3.73	0.05	8.83
3.5	4.55	0.09	3.87	0.06	8.83
4.0	4.69	0.08	4.00	0.06	8.82

As basis for fixing the constant of proportionality between I and Z there is really available at present only the data for air and aluminum:

$$\text{Air,}\quad I/Z=11.15\ \text{ev}=0.820(me^4/2\hbar^2),$$
$$\text{Aluminum,}\quad I/Z=11.54\ \text{ev}=0.848(me^4/2\hbar^2).$$

Since Bloch's statistical theory of the proportionality is the more nearly correct the higher is the atomic number, it seems to be reasonable in the light of the evidence to write for the mean excitation energy of atoms with many electrons

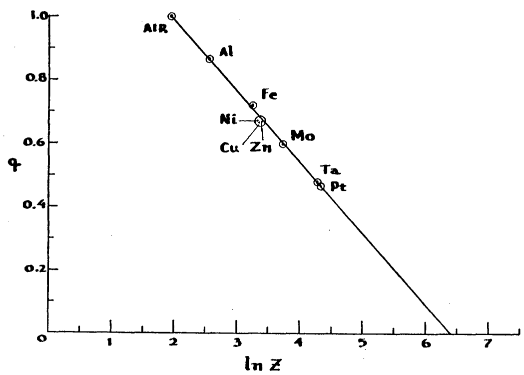

FIG. 6.

$$I/Z=11.5\ \text{ev}=0.85(me^4/2\hbar^2).$$

The constant in (6), about 10 percent lower than that given by Bloch, is employed in the following paper to determine the range-energy relation for fast particles in lead.

In conclusion the author takes pleasure in expressing his gratitude to Professor Ernest O. Lawrence for the privilege of using the cyclotron. I am especially grateful to Professor John A. Wheeler for pointing out the importance of the measurements and for contributing the interpretation. Acknowledgment is made to the Research Corporation for financial support in this project.