

TABLE I. Scattering for various resonance levels.

NEUTRON ENERGY*	SCATTERING CROSS SECTIONS				MEASURED VALUES
	$E_r = -0.3$ ev	$E_r = -0.1$ ev	$E_r = -0.05$ ev		
C 0.026 ev	14×10^{-24} cm ²	26×10^{-24} cm ²	37×10^{-24} cm ²	28×10^{-24} cm ²	
D 0.84 ev	12.8	12.9	13.0	12	
I 27 ev	11.05	11.05	11.05	11	

* W. J. Horvath and E. O. Salant, Phys. Rev. **59**, 154 (1941).

section for thermal neutrons which decreases rapidly as the neutron energy increases to 0.84 ev, and becomes almost constant for neutrons with ev in the order of tens. This behavior will be expected if we assume a resonance level at negative energy not far from zero. The scattering cross sections were calculated as a function of neutron energy after the formulation of Bethe.⁴ We assumed that the resonance level is at -0.3 ev, -0.1 ev and -0.05 ev, respectively. The results are shown in the table. The following three assumptions are made in the process of calculation: (1) almost all of σ_N^N for the I group are due to potential scattering; (2) although $\lambda_r \Gamma_N$ loses its meaning in the present case and we must come back to $\lambda \gamma$ Eq. (265a), (reference 4) the former quantity is roughly constant for various nuclei and positions of levels. Thus $\lambda_r \Gamma_N$ was put equal to 0.25×10^{-12} on averaging the values for the levels of Rh, In, and Ag; and (3) Γ was put equal to 0.1 ev. (Small variation in this value does not influence the results greatly.) A resonance level at -0.1 ev fits the experimental results the best. A level at negative energy has already been suggested by Bethe⁴ from the absorption ratio for the C and D neutrons and from the temperature effect. Moreover, Hg has another level at 9 ± 7 ev after the investigation of Nonaka.⁵ It will no doubt be expected that the hump in the scattering curve due to this level is negligibly small; the absorption curve, however, will be greatly changed and the experimental absorption ratio for the C and D neutrons is more naturally understood by assuming two levels, one at -0.1 ev and another at 9 ± 7 ev rather than a single level at -0.3 ev (presumably -2.2 ev) as assumed by Bethe.

In conclusion, I express my sincere thanks to Professor S. Nishikawa and to Professor S. Tomonaga for their valuable suggestions and discussions.

¹ M. Kimura, Proc. Phys. Math. Soc. Japan **22**, 391 (1940).

² M. Kimura, Sci. Papers Inst. Phys. Chem. Res. **39**, 153 (1939).

³ H. B. Hanstein, Phys. Rev. **59**, 489 (1941).

⁴ H. A. Bethe, Rev. Mod. Phys. **9**, 152 (1937).

⁵ I. Nonaka, Proc. Phys. Math. Soc. Japan **21**, 594 (1939).

The Limiting Form of Poisson's Distribution

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IN a recent article E. Rodgers¹ has shown for Poisson's distribution (The law of rare events) that the probability of a deviation not exceeding the standard deviation converges to the corresponding probability for a Gaussian (normal) distribution. This theorem, the proof of which is rather complicated, is a special case of the following relation:

The Poisson distribution (x) of a variable x

$$w(x) = m^x e^{-m} : x! \quad (1)$$

converges with increasing values of m , where m stands for the mean value, towards the normal distribution. Since the Poisson law plays an important role in radioactivity, the proof, which is very simple, may be of interest for the physicists.

Defining the variable z by

$$x = m(1+z) \quad (2)$$

we obtain for large values of m which implies large x , from Stirling's formula

$$w(x) = \frac{m^{m(1+z)} e^{-m}}{[2\pi m(1+z)]^{\frac{1}{2}} m^{m(1+z)} (1+z)^{m+mz} e^{-m-mz}} \\ = e^{mz(1+z)^{-m-mz-\frac{1}{2}}} : (2\pi m)^{\frac{1}{2}}$$

If

$$m \gg \frac{1}{2}$$

the factor $\frac{1}{2}$ which appears in the exponent may be neglected. The usual expansion gives

$$w(x) = \frac{1}{(2\pi m)^{\frac{1}{2}}} e^{mz - (x+mz)(z-\frac{1}{2}z^2 \dots)} \\ = \frac{e^{-mz^2/2}}{(2\pi m)^{\frac{1}{2}}}$$

Substituting x for z , according to (2), we obtain

$$w(x) = \frac{e^{-(x-m)^2/2m}}{(2\pi m)^{\frac{1}{2}}} \quad (3)$$

a Gaussian distribution with standard deviation $\sigma = m^{\frac{1}{2}}$. Rodgers' theorem follows at once from this general property. The convergence, established by L. von Bortkiewicz,² is well known in statistics. Our method does not enable one to calculate the probability of a deviation falling in the interval $m \mp \sigma$ as a function of m , as was done by Rodgers.

¹ Eric Rodgers, Phys. Rev. **57**, 735 (1940).

² L. von Bortkiewicz, *Das Gesetz der kleinen Zahlen* (Leipzig, 1898).

Long-Lived Activity of Rhodium

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ON bombarding rhodium with slow neutrons, Fermi and his co-workers¹ found two β -activities decaying with half-lives of 44 sec. and 4 min., respectively, and Pontecorvo² concluded that these two activities are due to nuclear isomers of Rh¹⁰⁴. This conclusion was confirmed by some other investigators.^{3,4}

Pool, Cork and Thornton⁵ showed that the bombardment of rhodium with fast neutrons gives rise to an activity of 1.1-hour period. We repeated the experiments by bombarding powdered metallic rhodium (Kahlbaum) with slow neutrons (through paraffin) from Be+D, and fast neutrons from Li+D produced in our cyclotron under similar conditions previously reported.⁶ A Lauritsen type electroscopes was used to observe the activity. In both cases the 1.1-hour period of Pool *et al.* could not be ob-

served at all; the irradiated samples, however, showed the existence of very long periods. Then the samples were irradiated intermittently for a long time and their long-lived activities examined. The results are as follows.

1. Slow neutron bombardment. Beside the well-known two periods (44 sec. and 4 min.) two-week activities were found, their periods being about 20 hours and 70 days, respectively. But chemical separation showed that these activities were to be attributed to iridium impurity, which have very large capture cross sections for slow neutrons.

2. Fast neutron bombardment. A long-lived activity decaying with a period of 210 ± 6 days was found. Chemical separation showed that the carrier of this activity is rhodium and is neither ruthenium nor masurium.

With a thin walled G-M counter and magnetic field it was found that the emitted particles were both negative and positive electrons and that the intensity ratio of these particles was $e^-/e^+ = 1.2$. These particles were accompanied by a considerable amount of γ -rays. With the Al absorption and the range energy relation⁷ the upper limit of the energy of these mixed β -rays was found to be 1.1 ± 0.1 Mev.

From the above observations it can be concluded that the corresponding isotope of this long-lived activity is Rh¹⁰², produced from the abundant stable isotope Rh¹⁰³ by the reaction ($n, 2n$), and this Rh¹⁰² can emit either positive or negative electrons and goes over to Pd¹⁰² or Ru¹⁰², respectively. A more detailed report will be published shortly in the Scientific Papers of the Institute of Physical and Chemical Research.

In conclusion the writer wishes to express his best thanks to Professor H. Nagaoka for his interest and kind encouragement throughout this work, and to Dr. Y. Nishina for his kind suggestions and discussions, and to Mr. T. Shirai for chemical separations of the samples.

¹ E. Fermi *et al.*, Proc. Roy. Soc. **149**, 522 (1935).

² B. Pontecorvo, Phys. Rev. **54**, 542 (1938); Nature **141**, 785 (1936).

³ H. Reddemann, Naturwiss. **26**, 125 (1938).

⁴ E. C. Crittenden, Jr., Phys. Rev. **56**, 709 (1939).

⁵ M. L. Pool, J. M. Cork and R. L. Thornton, Phys. Rev. **52**, 239 (1937).

⁶ O. Minakawa, Phys. Rev. **57**, 1189 (1940).

⁷ N. Feather, Proc. Camb. Phil. Soc. **34**, 599 (1938).

Isotopic Weights of Sulphur and Titanium

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WITH the mass-spectrograph of Bainbridge-Jordan type, the isotopic weights of S³², S³⁴, Ti⁴⁶, Ti⁴⁷, Ti⁴⁸, Ti⁴⁹ and Ti⁵⁰ were measured by the doublet method.

1. Sulphur. (a) S³²: By the electric discharge through the mixture of oxygen and a trace of ethylmercaptane (C₂H₅SH) vapor, we obtained well-matched doublets O¹⁶₂-S³² and C¹²O¹⁶₂-C¹²S³². These mass differences are given in Table I. These two mass differences being quite in good agreement with each other within the experimental error, we take the weighted mean of these results for the mass difference O¹⁶₂-S³² as follows: O¹⁶₂-S³² = 191.1 ± 0.7 . From this difference, the following isotopic weight as

TABLE I.

DOUBLET	NUMBER OF DOUBLETS MEASURED	DIFFERENCE IN MASS ($\Delta M \times 10^4$)
O ¹⁶ ₂ -S ³²	11	191.5 \pm 1.1
C ¹² O ¹⁶ ₂ -C ¹² S ³²	4	189.4 \pm 2.3

well as packing fraction of S³² can be obtained:

$$S^{32} = 31.98089 \pm 0.7 \times 10^{-4}$$

packing fraction of S³² = -5.98 ± 0.02 .

Although this value does not agree with that of Aston, i.e., $31.9823 \pm 3 \times 10^{-4}$,¹ the Brasefield and Pollard's data² of S³²(α, p) Cl³⁵ reaction as well as our recalculated chlorine mass (34.97881) enable us to obtain the following value for S³²,

$$S^{32} = 31.98054 \pm 3.9 \times 10^{-4},$$

which agrees well with our present value.

(b) S³⁴: The SH³²₂-S³⁴ doublet is obtained by the ordinary discharge through the vapor of ethylmercaptane, and the mass difference obtained is as follows:

DOUBLET	NUMBER OF DOUBLETS MEASURED	DIFFERENCE IN MASS ($\Delta M \times 10^4$)
SH ³² ₂ -S ³⁴	11	200.4 \pm 3.2

From the results thus obtained and also from the values S³² = $31.98089 \pm 0.7 \times 10^{-4}$ and H¹ = $1.008131 \pm 0.033 \times 10^{-4}$,³ the following isotopic weight and packing fraction of S³⁴ can be obtained,

$$S^{34} = 33.97711 \pm 3.3 \times 10^{-4}$$

packing fraction of S³⁴ = -6.73 ± 0.10 .

2. Titanium. In the determinations of all five isotopic weights of Ti, we obtained by the discharge through the vapor mixture of titantetrabromide, normal heptane and ethylmercaptane the doublets given in Table II. By using these results, together with H¹ = $1.008131 \pm 0.033 \times 10^{-4}$,

$$C^{12} = 12.003871 \pm 0.33 \times 10^{-4},³$$

and

$$S^{32} = 31.98089 \pm 0.7 \times 10^{-4},$$

TABLE II.

DOUBLET	NUMBER OF DOUBLETS MEASURED	DIFFERENCE IN MASS ($\Delta M \times 10^4$)
C ¹² SH ³² ₂ -Ti ⁴⁶	4	349.0 \pm 9.5
C ¹² SH ³² ₂ -Ti ⁴⁷	6	444.2 \pm 9.4
C ¹² ₄ -Ti ⁴⁸	9	521.6 \pm 4.6
C ¹² ₄ H ₂ -Ti ⁴⁹	19	588.3 \pm 5.1
C ¹² ₄ H ₂ -Ti ⁵⁰	19	694.6 \pm 3.6

TABLE III.

ISOTOPIC WEIGHT	PACKING FRACTION	RELATIVE ABUNDANCE*
Ti ⁴⁶ = $45.96612 \pm 9.5 \times 10^{-4}$	-7.36 \pm 0.21	7.95
Ti ⁴⁷ = 46.96473 ± 9.5	-7.50 \pm 0.20	7.75
Ti ⁴⁸ = 47.96332 ± 4.8	-7.64 \pm 0.10	73.45
Ti ⁴⁹ = 48.96479 ± 5.3	-7.19 \pm 0.11	5.51
Ti ⁵⁰ = 49.96229 ± 3.8	-7.54 \pm 0.08	5.34

* O. Hahn, S. Flügge, and J. Mattauch, Physik. Zeits. **41**, 1 (1941).