THE
PHYSICAL REVIEW
A Journal of Experimental and Theoretical Physics Established by E. L. Nichols in 1893
VOL. 60, NO. 2 JULY 15, 1941 SECOND SERIES

Fission Yield by Fast Neutrons*

M. AGENO,† E. AMALDI,† D. BOCCIARELLI,‡ B. N. CACCIAPUOTI,† AND G. C. TRABACCHI‡
Istituto di Sanità Pubblica, Rome, Italy
(Received April 30, 1941)

The fission cross section of uranium was measured for neutrons produced in the following reactions: Rn+Be, D+C, D+D, D+Be, D+B, D+Li. From the fact that the mean cross section has about the same value for neutrons of the D+D, D+Be, D+B reactions, it was concluded that the fission cross section has a value σ_f which remains nearly constant between 1 and 10 Mev. For neutrons of the Rn+Be and D+C reactions the fission cross sections seem to be respectively, about $\frac{1}{5}\sigma_f$ and $\frac{1}{8}\sigma_f$. Finally, for neutrons of the D+Li reaction the mean cross section is equal to $1.4\sigma_f$. This fact was interpreted by N. Bohr as due to successive transformations which are possible for energies of the impinging neutrons larger than 10 Mev. A similar increase of the fission cross section was observed also in case of thorium, in very good agreement with the theoretical prediction by Bohr.

ROBERTS, Meyer, Hafstad[1] and Ladenburg, Kanner, Barschall, Van Voohris[2] have performed a few measurements on the dependence of the fission cross section on the energy of the impinging neutrons. The first group of researchers have compared qualitatively the fission yield of neutrons produced in different reactions. Ladenburg and others have measured the fission cross section of U and Th for neutrons of 2.4-Mev energy (D+D) and found, respectively, 0.5×10^{-24} and 0.1×10^{-24} cm^2.

These researchers were also able to show that the fission cross section of U remains constant for energies of the impinging neutrons between 2.1 and 3.1 Mev in agreement with the calculations of Bohr and Wheeler.[3] According to these authors, the fission cross section is negligible for excitation energies of the compound nucleus smaller than a critical value E_f. For excitation energies close to E_f, the fission cross section increases very rapidly to a value σ_f which remains constant also for excitation energies considerably larger than E_f.

The theoretical values of E_f for U^{238}, Pa^{232} and Th^{233} are, respectively, 0.7, 1, 1.7 Mev. The experimental values seem to be slightly lower.[4] In case of U the fission can be produced also by slow neutrons. Bohr[5] pointed out, on theoretical considerations, and Nier, Booth, Dunning, Von Grosse[6] confirmed experimentally, that the isotope responsible for this phenomenon is U^{235};

* Our measurements on uranium fission were performed in April–May, 1940. The measurements on thorium and protoactinium fission were performed in January, 1941.
† Istituto Fisico della R. Università.
‡ Istituto di Sanità Pubblica.
[1] R. D. Roberts, R. C. Meyer and L. R. Hafstad, Phys. Rev. **55**, 416 (1939).
[2] R. Ladenburg, M. H. Kanner, H. Barschall and C. C. Van Voorhis, Phys. Rev. **56**, 168 (1939).
[3] N. Bohr and J. A. Wheeler, Phys. Rev. **56**, 426, 1065 (1939).
[4] R. O. Haxby, W. E. Shoupp, W. E. Stephens, W. H. Wells and M. Goldhaber, Phys. Rev. **57**, 1088 (1940).
[5] N. Bohr, Phys. Rev. **55**, 418 (1939).
[6] A. V. Grosse, E. T. Booth and J. R. Dunning, Phys. Rev. **56**, 382 (1939).

our experiments, however, do not refer to this problem.

In this paper we give an account of some experiments on the dependence of the fission cross section on the energy of the impinging neutrons.[7] Our measurements are more quantitative than those of Roberts and others and refer to a larger range of energy than the experiments of Ladenburg and others. We did not perform any absolute measurement of the cross section; the energy of the impinging neutrons was in all experiments larger than 0.2 Mev.

For our purpose it would be very convenient to have homogeneous groups of neutrons and repeat the measurements of the fission cross section for different values of their energy. Unfortunately only the neutrons emitted in the $D+C$ and $D+D$ reactions are approximately homogenous (we use a thick target). In all other reactions, obtained by bombarding light elements with deuterons, the spectrum of the emitted neutrons is rather complicated and extends from zero to several million electron volts of energy. If we assume, however, the theoretical dependence of the fission cross section on the energy of the impinging neutrons, we must expect to find the same experimental mean cross section independently of the spectrum of the neutrons, provided it extends only above the critical energy E_f. If the neutron spectrum extends from zero to several million electron volts, in order to check the theory of Bohr and Wheeler it will be sufficient to know the percentage α of the total number of impinging neutrons which have an energy larger than E_f. The remaining fraction $1-\alpha$ of the impinging neutrons does not give any contribution to the experimental mean cross section. Therefore the ratio of the number of fission processes produced in a given amount of uranium to the number of impinging neutrons is proportional to $\alpha\sigma_f$. By measuring this ratio with different neutron spectra it is possible to check, in outline if not in detail, the constancy of the fission cross section for energies larger than E_f. The method would also permit, in principle, the determination of the critical energy E_f. However, this determination would require the knowledge of the low energy part of the spectrum of the neutrons with a much higher accuracy than is now available.

[7] M. Ageno, E. Amaldi, D. Bocciarelli, B. N. Cacciapuoti and G. C. Trabacchi, Ricerca Scient. **11**, 302 (1940); M. Ageno, E. Amaldi, D. Bocciarelli and G. C. Trabacchi, Ricerca Scient. **11**, 413 (1940); E. Amaldi, D. Bocciarelli and G. C. Trabacchi, Ricerca Scient. in press.

2. Principle of the Method

The best method which permits a measurement of the number of neutrons emitted from a source independently of their energy, consists in slowing down all neutrons in a hydrogenated substance and then measuring the density of thermal neutrons by means of a convenient detector. The integral of the density of thermal neutrons over all the volume of the hydrogenated substance is proportional to the total number of the emitted neutrons. In order to use this method of measuring the number of the impinging neutrons, the experiments on uranium were performed on the following principle. Let us consider a spherical shell, coated with uranium and having its center on the target. The ratio of the number n of fissions produced in the uranium layer to the number of impinging neutrons is proportional to $\alpha\sigma_f$. The number of impinging neutrons can be measured by substituting for the uranium layer a water cylinder so large that all neutrons emitted from the target are slowed down to thermal energy.

We point out the necessity of performing both measurements (of the total number of fissions and of the density of thermal neutrons) in all directions with respect to the direction of the deuterons impinging on the target, because the slowing down process in water partially cancels the anisotropy of the neutrons impinging on the uranium layer.

3. Experimental Data

Our neutrons were produced by bombarding with deuterons accelerated in the 1100-kv tube of the Istituto di Sanità Pubblica,[8] different targets, namely D, Li, Be, B, C. The deuterium target was obtained by adsorbing about 3 liters of D_2 in 20 g of zirconium powder at the temperature of 200°C.[9] This powder was cooled and compressed in a brass cup of 3.5-cm diameter and

[8] E. Amaldi, D. Bocciarelli, F. Rasetti and G. C. Trabacchi, Ricerca Scient. **10**, 623 (1939).

[9] F. H. Penning and J. H. A. Moubis, Physica **4**, 1190 (1937).

2-mm depth which was placed in the lower part of the tube. We have used this method, which presents several inconveniences, among them the low intensity and the rather complicated preparation, because the use of a target of heavy water with liquid air must be excluded on account of the geometry of the arrangement.

Li, Be and C were used in form of disks of the elements; B in form of a disk of B_2O_3. The electrodes of the ionization chambers used for counting the number of fissions in different heavy elements were two disks of 5-cm diameter, 3 cm apart from one another. One of the electrodes was connected to the grid of the first tube of an amplifier and the other was maintained to a voltage of 3800 volts by means of a stabilized system. This electrode was coated with a layer of the heavy element; for U and Th we used thick layers of U_3O_8 and ThO_2.

For Pa we evaporated on a silver electrode 3.5 cc of a solution of 7 percent H_2SO_4 containing less than a hundred micrograms of Pa.[10] By substituting for the electrode coated with a heavy element a blank electrode, we could check the absence of electrical disturbances. The primary ions produced in the gas of the ionization chamber by the fission products gave rise to a weak multiplication on account of the relatively high voltage applied to the chamber. The pulses were recorded by means of a scale-of-four or a scale-of-six and were simultaneously observed by means of an oscillograph.

As the lower part of the tube was cooled by means of a thin layer of water (4 mm thick), we surrounded completely the lower part of the tube and the ionization chamber with Cd of 0.52 g/cm^2 thickness in order to absorb all neutrons of thermal energy. These, however, in our conditions, were negligible; in fact by removing the cadmium layer we could not observe any increase of the number of counts.

In some of the experiments we will describe below, we measured the number of neutrons impinging on heavy elements by means of the activity of 44 sec. induced in Rh by slow neutrons. All these measurements were performed with three Rh detectors 5×5.5 cm^2 of area and 0.36 g/cm^2 thick. The activity of these detectors was measured with an ionization chamber containing carbon dioxide at 2.5 atmos. pressure connected to an Edelman electrometer. The activities given in the tables are defined as the total activity (namely the activity measured from zero time to infinite time) of the detector after 1 minute irradiation. We assume the activity of a preparation of 85 g of U_3O_8 equal to unity. This standard was 42.1 times larger than the standard preparation used in other researches.[11]

4. Experiments with Uranium

In the case of uranium, we performed two experiments based on the principle described in Section 2.

(a) In the first experiment we measured the number of fissions produced in uranium by means of the ionization chamber described in Section 3 and the density of thermal neutrons by means of the activity of 44-sec. period of Rh. Instead of measuring both these quantities in all directions in order to evaluate their integral with respect to θ and φ, the experiments were actually performed only at $\theta=90°$ and $\theta=0°$ with the direction of the impinging neutrons.

From the following simple geometrical consideration it is easy to recognize that the largest contribution to the above-mentioned integrals comes from the measurements at 90°, while the measurements in the forward direction give only a small contribution. If we call l the linear dimension of the detector (Rh or electrode coated with uranium) and r its distance from the center of the target, the solid angle subtended by the detector is l^2/r^2 for each direction θ, φ. In order to evaluate the integral with respect to the azimuth φ, we must multiply the data at $\theta=0°$ by 1 and the data at $\theta=90°$ by the factor $\delta=2\pi r\times l/l^2$. As the distance r of the detector from the center of the target was 5.7 cm, we find for the Rh detector (5×5.5 cm^2) $\delta=6.5$ and for the uranium layer (disk of 5-cm diameter) $\delta=9.1$.

As will appear below, the measurements in forward directions give results slightly different from the measurements at $\theta=90°$, showing that both the number of fissions per minute and the activity of the Rh detector are functions changing only very slowly with the angle θ. It seems,

[10] We are indebted to Professor A. V. Grosse of the University of Chicago for this protoactinium preparation.

[11] E. Amaldi and E. Fermi, Phys. Rev. **50**, 899 (1936).

TABLE I. ($\theta=90°$) *summary of data.*

Reaction	Energy of D^+, Mev	a	$K\times10^{-3}$	n	$n\times10^3/Ka$
$D+C$	0.85	184±5	0.41±0.03	3.2±0.5	0.042
$D+C$	0.85	273±2	0.41±0.03	5.2±0.7	0.046
$D+C$	1.0	669±7	0.46±0.03	11.4±1	0.037
$D+C$	1.0	621±6	0.46±0.03	9.3±0.9	0.033
$D+D$	0.50	80.7±2	0.96±0.05	18.2±1	0.24
$D+Be$	0.85	2217±23	1.03±0.05	580±7	0.25
$D+B$	0.85	514±11	0.76±0.05	101±2	0.26
$D+Li$	0.75	1627±30	1.11±0.03	660±8	0.37
$D+Li$	0.60	780±6	1.11±0.03	310±5	0.36

therefore, reasonable to substitute the integrals of these two quantities with the mean value of the results at $\theta=90°$ and $\theta=0°$ weighted with the above-mentioned factors. As the difference of the data obtained by this method from the data at 90° is of the same order of magnitude as the error of the experimental results, we shall use in the discussion the data obtained at 90°. We will, however, give also the data obtained in the forward directions in order to justify the approximations used.

The measurements were actually performed as follows: with the ionization chamber placed in a well-reproducible position with respect to the target at a distance $r=5.7$ cm at 90° (or 0°) we counted the number n of fissions per minute. Then we removed rapidly the ionization chamber and surrounded the lower part of the tube containing the target with a cylindrical vessel of 28-cm diameter and 25-cm depth filled with water. A Rh detector in the same position as previously occupied by the coated electrode of the ionization chamber (at 5.7 cm at 90° or 0°) was irradiated for 1 minute and its activity a was measured. These two measurements with U and Rh were repeated alternately many times in order to check the constancy during the experiment of the intensity of the emitted neutrons. The activity a of the Rh detector, however, is not simply proportional to the total number of neutrons emitted from the target because the spatial distribution of slow neutrons in water has a different distance dependence for different neutron sources. Therefore, at a later time, we placed around the target a vessel of $38\times80\times28$ cm³ for the experiments at 90° and a cylindrical vessel 70 cm deep and 50 cm in diameter for the measurements at $\theta=0°$ and we measured the activity $A(r)$ of the Rh detector as a function of the distance from the target (in this paper by r we mean the root-mean-square distance between points on the detector and on the target). Then we calculated both at 90° and 0° the ratio

$$K=\int_0^\infty A(r)r^2dr \Big/ A(5.7).$$

The product $K\cdot a$ is proportional to the total number of neutrons as long as we neglect the anisotropy of the emission from the target.

The results of our measurements are summarized in Table I ($\theta=90°$) and Table II ($\theta=0°$). In the third column we give the activity a of the Rh detector, in the fourth column the factor K multiplied by 10^{-3} and in the fifth column the number n of fissions per minute. The sixth column contains the quantity $n10^3/Ka$ calculated from the preceding data.

As it appears from Table II, we also performed an experiment with the photo-neutrons of Rn+Be. As the emission of neutrons in this case is certainly isotropic we performed this experiment only in one direction. We put in a brass tube having the same shape of the lower part of the tube a layer of beryllium powder 5 cm thick in which we plunged a small glass tube containing initially 500 millicuries of radon. The geometry of the experiment was the same as that used with the tube in the forward direction.

As the observed effect was very weak, we counted the number of pulses given by our ionization chamber, alternately with and without the γ-ray source close to the chamber. We counted 75 pulses (corrected to 79 for the decay of radon) in 980 minutes with the γ-ray source, against 1 pulse in 709 minutes without the γ-ray source.

We shall discuss in Section 6 the results obtained with photo-neutrons and neutrons of the $D+C$ reaction. From the data of Table I it appears that the quantity $n\times10^3/Ka$ has about the same value for neutrons of the $D+D$, $D+Be$

TABLE II. ($\theta=0°$) *summary of data.*

Reaction	Energy of D^+ Mev	a	$K\times10^{-3}$	n	$n\times10/Ka$
$D+C$	0.85	241±2	0.34±0.03	6.9±0.8	0.084
$D+C$	1.0	705±11	0.47±0.03	17±2	0.051
$D+D$	0.50	26.4±0.5	1.1 ±0.06	12.2±1	0.42
$D+Be$	0.85	2790±40	0.90±0.05	669±6	0.27
$D+B$	0.85	640±5	0.71±0.04	164±3	0.36
$D+Li$	0.75	2400±50	1.07±0.01	1070±7	0.42
γ Rn+Be		3.95±0.04	0.39±0.04	0.08±0.009	0.052

and $D+B$ reactions while for neutrons of the $D+Li$ reactions this quantity seems to be about 1.4 times larger.

(b) We thought it convenient to compare the mean cross section for fission of the neutrons from the $D+Be$ and $D+Li$ reactions by means of an improved method. On account of the low intensity it was impossible to use this second method in the case of the $D+D$ and $D+B$ reactions.

Experiment (a) is open to some criticism because of the anisotropy of the emitted neutrons for which we did not correct the experimental data. Therefore we mounted the ionization chamber with the electrode coated with uranium on a wooden arm which could turn around the center of the target. The distance of the U_3O_8 layer from the center of the target was 11 cm. By means of this disposition we measured the number of fissions n per minute for θ equal to 0°, 22°.5, 45°, 67°.5, 90°, 112°.5, 135°. The ratio

$$s=\frac{\sum n(\theta)\sin\theta}{n(90°)\sum\sin\theta}$$

represents the factor by which we must multiply the number n of fissions per minute observed in the 90° direction and given in column 5, Table I, in order to correct in a complete way for the anisotropy of the emitted neutrons. In Table III, column 4, we give the values of this factor s for the $Be+D$ and $Li+D$ reactions. In column 5 we give the values of the quantity ns/a calculated by using for n and a the values already given in Table I and reproduced in columns 2 and 3 of Table III.

In order to measure the total number of neutrons emitted from the target we dissolved 30 kg of $MgCl_2$ in 100 liters of water and put this solution in the form of a water cylinder 50 cm high and 50 cm in diameter, around the target. This solution was irradiated 40 min. in order to activate the 37-min. period of Cl. During the irradiation the solution was continuously mixed by blowing with a small pump some air close to the bottom of the container.

TABLE III.

REACTION	a	n	s	$\frac{n\cdot s}{a}$
$Be+D$ 0.85 Mev	2217±23	580±7	1.04±0.03	0.272±0.010
$Li+D$ 0.75 Mev	1627±30	660±8	0.98±0.03	0.398±0.015
$Li+D$ 0.60 Mev	780±6	310±5	0.98±0.03	0.390±0.014

TABLE IV.

REACTION	i	b	i/b	$\frac{ns}{a}\frac{i}{b}$ ×10⁴	$\frac{d\sigma_{Li}}{\alpha\sigma_{Be}}$
1.	Be+D ..8312±116	408±11	20.4±0.6	133	1.37±0.09
	Li+D ..6063±107	280±6	21.6±0.6	182	
2.	Be+D ..8247±115	388±6	21.2±0.5	128	1.49±0.09
	Li+D ..5197±100	252±3	20.6±0.5	191	
3.	Be±D ..7474±112	1430±30	5.23±0.01	520	1.34±0.08
	Li±D ..8128±114	1440±30	5.65±0.01	697	
					1.40±0.05

It is clear that the activity of a given amount of the solution is proportional to the integral with respect to the distance from the target as well as to the angles θ, φ, of the density of slow neutrons and therefore to the total number of emitted neutrons. Therefore, after the irradiation we placed the solution in a small glass vessel having the shape of a hollow cylinder 20 cm long which could be put on a thin-walled Geiger counter of 20-mm diameter. Then the counting system was started 3 min. after the end of the irradiation and we measured the total number of pulses observed in 40 min. We were very careful to keep constant both the time of observation as well as the 3 minutes of delay because our solution showed a rather complicated decay curve due to the superposition of many periods. In fact our $MgCl_2$ was not very pure and contained also some potassium. Therefore the zero effect of the counter (63 pulses/min.) was always measured by putting on the counter our thin-walled glass vessel filled with non-activated solution.

In Table IV, column 2, we give the total number i of pulses observed in 40 min. in three successive experiments; the zero effect has already been subtracted. In order to check the constancy of the emission of neutrons during the irradiation and in order to be able to compare results obtained with different reactions, we measured, every fifth minute, the activity of a Rh detector, after 1-min. irradiation, in a well-reproducible position with respect to the target in the solution of $MgCl_2$.

In column 3 of Table IV we give the mean value b of these activities.

In experiments 1 and 2 the Rh detector was placed at 13.2 cm from the target at 90° with the direction of the impinging deuterons: in experiment 3 it was put in the same direction but at a distance of only 5.7 cm. As our curves of the activity as a function of the distance r show that the ratio $A(5.7)/A(13.2)$ has the same value for neutrons of $D+$Li and $D+$Be reactions, the data of experiment 3 can be compared with that of experiments 1 and 2. We checked also that the presence of the salt did not change appreciably the spatial distribution of slow neutrons.

In column 4, Table IV, we give the values of the quantity i/b and in column 5 a quantity proportional to $\alpha\sigma$, namely the ratio of the data of column 5, Table III, to the data of column 4, Table IV, $(ns/a)/(i/b)\times 10^4$.

Finally in column 6, Table IV, we give for each of the three experiments the ratio $\alpha\sigma_{\mathrm{Li}}/\alpha\sigma_{\mathrm{Be}}$. We find for this quantity the value 1.4 as from experiment (a).

5. Experiments with Thorium and Protoactinium

In a first attempt we performed with Th an experiment similar to experiment (a) (on U) described in Section 4. The precision of this experiment was too low and did not allow, therefore, any definite conclusion.

Recently we again worked on this subject and we thought it more convenient to compare the fission cross section of Th and Pa with the fission cross section of U with neutrons produced in different reactions. For this purpose we used two similar ionization chambers, as described in Section 3, connected to two linear amplifiers, one with the electrode coated with uranium, the other with the electrode coated with Th (or Pa).

When these two ionization chambers were put in a given position with respect to the target, we counted by means of a scale-of-six and a scale-of-four the number of fissions observed in a given time, respectively in U and Th (or Pa). The ratio $n_{\mathrm{U}}/n_{\mathrm{Th}}$ of these numbers of pulses was measured with neutrons produced in the $D+$Be and $D+$Li reactions. The experiment was repeated, alternatively, five times with these two targets.

In the experiments on Th the two layers of U and Th were placed symmetrically in a horizontal plane, below the lower part of the tube, at 10 cm from the center of the target. In Table V we give results obtained in five successive experiments and the corresponding mean values.

Table V. *Ratio of number of fission in* U *and* Th.

$D+$Be	$D+$Li
4.46	3.17
4.57	2.82
4.04	3.53
4.37	3.03
4.19	3.45
4.34±0.10	3.20±0.13

From these data we conclude that the fission cross section of Th increases, with increasing energy of the impinging neutrons, more than the fission cross section of U by a factor

$$\frac{4.34\pm 0.10}{3.20\pm 0.13}=1.35\pm 0.08.$$

As mentioned above, we tried a similar experiment with Pa.

Although the electrode coated with this element was placed as close as possible to the target, and neutrons produced in the $D+$Be reaction were used, we observed only 1 fission every second or third minute. On account of the larger number of the impinging neutrons, in the case of the $D+$Li reaction we observed about 2 or 3 fissions per minute. We counted, altogether, 120 counts with neutrons produced in the $D+$Be reaction and 345 counts with neutrons produced in the $D+$Li reaction and found that the ratio of fission cross section of U and Pa is slightly higher with neutrons of the $D+$Li than with neutrons of the $D+$Be reaction. The difference is, however, of the same order of magnitude as the statistical error and therefore can be considered only as an indication.

6. Discussion

Let us first consider the data obtained in the case of U with neutrons of the $D+$D, $D+$Be and $D+$B reactions, given in Table I, column 6.

We see that the mean cross section obtained with neutrons of the $D+$Be reaction (0–4.6 Mev) is equal to the cross section obtained with neutrons of the $D+$D reaction which, in our condition, gave an approximatively homogeneous

group of neutrons of 2.6 Mev. From this fact we conclude that the percentage α of the neutrons, produced in the $D+$Be reaction, having an energy larger than the critical energy for fission E_f of U is about equal to one: $\alpha\sim 1$ and that the fission cross section remains practically constant for energies of the impinging neutrons between E_f and the upper limit of the spectrum of the $D+$Be reaction. In fact, by correcting for the proton-neutron elastic cross section the data of Bonner and Brubaker,[12] which extend on the low energy side to 0.6 Mev, we find that the neutrons having energy below 1 Mev, represent only about 5 percent. As the theoretical value of the critical energy E_f is 0.7 Mev we conclude that we must expect $\alpha\sim 1$ as we found experimentally.

In a similar way we can discuss the result obtained with the neutrons of the $D+$B reaction. This spectrum has an upper limit at 13.5 Mev and was measured, on the low energy side, down to 2 Mev.[12] The extrapolation of the spectrum to zero energy is, in this case, rather uncertain and therefore we cannot evaluate the percentage α of neutrons having energy higher than E_f. We can, however, presume that α will differ not very much from 1, since E_f is rather low. Therefore, considering the fact that the mean fission cross section obtained in this case is slightly higher than that obtained with neutrons of the $D+$D and $D+$Be reactions (the difference is of the same order of magnitude as the experimental errors) we can conclude that the fission cross section of uranium is practically constant up to energies of the impinging neutrons higher than 10 Mev.

In order to have some information on the value of the fission cross section for energies of the impinging neutrons of the order of 0.5 Mev we carried out a similar experiment with neutrons of the $D+$C reaction for 1-Mev and 0.85-Mev energy of the impinging ions. At 90° the energies of the emitted neutrons are, respectively, 0.55 and 0.40 Mev.[13] As we see from Table I, for higher energy of the impinging neutrons we find a lower value of the fission cross section. This very strange result can be explained as due to contamination of the C target with deuterium. Such a contamination, not very important at 1-Mev energy of the impinging deuterons, cannot be neglected at lower energy, as the yield of the $D+$C reaction decreases more rapidly than that of the $D+$D reaction.

We can try to correct the data obtained with neutrons of the $D+$C reaction in the following way: let us call N_D' the number of neutrons of the $D+$D reaction emitted from our contaminated carbon target, and $N_C(0.85)$, $N_C(1)$ the numbers of neutrons of the $D+$C reaction emitted, respectively, for 0.85- and 1-Mev energy of the impinging deuterons. Assuming N_D and the fission cross section σ_f as constant in this small interval of energy, we have that the sums $N_C(0.85)+N_D$, $N_C(1)+N_D$ represent the total numbers of emitted neutrons and therefore are proportional to the rhodium activity integrated to all the volume. On the other side the quantities $N_C(0.85)\sigma_f+N_D 0.25$, $N_C(1)\sigma_f+N 0.25$ are equal to the observed cross sections. Solving these four equations we find

$$\sigma_f\sim 0.03,\quad N_D\sim 28,\quad N_C(0.85)\sim 38,\quad N_C(1)\sim 280.$$

We conclude by this rather indirect method that the fission cross section of U for energies of the impinging neutrons of 0.5 Mev is about $\frac{1}{8}(=0.03/0.25)$ of the cross section obtained between 1 and 10 Mev. This value seems to be considerably higher than the one given by other authors.[3] Therefore we thought it convenient to perform a similar experiment with photo-neutrons emitted from a Rn+Be source, having 0.200-Mev energy.

We can compare the result obtained with photo-neutrons with the data obtained for $\theta=0°$ with neutrons of the $D+$Be reaction. In fact, as we see from the data given in Tables I and II and as it appeared from the measurements of the angular distribution,[14] the neutrons emitted in the $D+$Be reaction are nearly isotropic. Therefore from the data of Table II we can conclude that the fission cross section for photo-neutrons is about $\frac{1}{5}(=0.05/0.27)$ of the cross section obtained with neutrons having energies between 1 and 10 Mev.

These rather high values of the fission cross section found both at 0.2 and 0.5 Mev energy of the impinging neutrons seem to indicate that the

[12] T. W. Bonner and W. M. Brubaker, Phys. Rev. **50**, 308 (1936).
[13] T. W. Bonner, Phys. Rev. **53**, 496 (1938).
[14] M. Ageno, E. Amaldi, D. Bocciarelli and G. C. Trabacchi, Atti R. Acc. Ital. In press.

fission cross section decreases, for energies below the critical energy E_f, less rapidly than expected.[3] In this connection we point out that, in the meantime, Haxby, Shoupp, Stephens and Wells[4] have measured the critical energy E_f for U and found 0.35 ± 0.1 Mev.

Let us now compare the results obtained with $D+$Be and $D+$Li reactions and given in Table I. We see that, in case of the $D+$Li neutrons, the mean cross section is larger by a factor 1.4 than in case of the $D+$Be neutrons. This result is confirmed by experiment (b) whose results are summarized in Table IV. The interpretation of this fact is due to N. Bohr[15] who pointed out that, for sufficiently high energy of the impinging neutrons, a new process becomes possible, as follows: The capture of the impinging neutron by a nucleus U^{238} gives rise to an excited U^{239}. The excitation energy is given by the sum of the energy E of the impinging neutrons and of the binding energy W_n of a neutron in the nucleus, which is equal to 5.2 Mev. The probability of fission of the compound nucleus U^{239} is determined by the competition with other disintegrative processes, among which the most important is the evaporation of a neutron. The ratio of the probabilities of fission and of evaporation of a neutron, is for U^{239} according to Bohr and Wheeler equal to 1 : 4. For energies of the impinging neutrons of the order of a few Mev after the evaporation of a neutron the residual nucleus U^{238} has such a low excitation energy that fission is impossible.

If the energy of the impinging neutrons is sufficiently high, however, the excitation energy of the residual nucleus U^{238} can be larger than its critical energy for fission. Also, in this case, the fission probability is determined of course by the competition with the evaporation of a neutron. As U^{238} is a nucleus containing an even number of particles, the ratio ρ of the probabilities of fission and evaporation will be larger than in case of U^{239}. According to Bohr we have $\rho=3:1$. Therefore, if we indicate by σ_0 the cross section for formation of the compound system U^{239}, the fission cross section will be

$$\sigma_f=\tfrac{1}{5}\sigma_0, \tag{1}$$

when the impinging neutrons are of low energy (a few Mev).

For sufficiently high energies of the impinging neutrons, two different fission processes are possible: namely, the direct fission of U^{239} with cross section σ_f, and the fission of the system U^{238} residual of the evaporation of a neutron from U^{239}. The cross section corresponding to the second process is given by

$$\sigma_f'=\tfrac{4}{5}\tfrac{3}{4}\sigma_0, \tag{2}$$

where $\frac{4}{5}\sigma_0$ represents the cross section for evaporation of a neutron from U^{239} and the numerical factor $\frac{3}{4}$ corresponds to the above given ratio ρ of the probability of fission and evaporation of a neutron for the residual system U^{238}. Therefore for high energy of the impinging neutrons, the fission cross section must be equal to

$$\sigma_f+\sigma_f'=4\sigma_f. \tag{3}$$

Let us now combine this theoretical result of Bohr with our data obtained with $D+$Li neutrons. Assuming, roughly, that the successive fission process of U^{238} takes place only above a definite energy E' of the impinging neutrons with cross section given by (2), we can indicate by α' the fraction of impinging neutrons having energy higher than E'. The mean cross section we must expect to find experimentally with a continuous spectrum of neutrons is given by

$$\sigma_f(1-\alpha')+(\sigma_f+\sigma_f')\alpha'=\sigma_f+\alpha'\sigma_f'=\sigma_f(1+3\alpha').$$

Comparing this theoretical result with the ratio of the fission cross sections obtained with neutrons, respectively, of the $D+$Li and $D+$Be reactions we conclude

$$1+3\alpha'=1.4;\quad \alpha'=0.13.$$

From the inspection of the spectrum of neutrons of the $D+$Li reaction[16] we see that about 13 percent of the emitted neutrons have energies larger than 12 Mev. This value is slightly higher than theoretically expected. In fact, taking into account that in the case of U^{238} the critical energy E_f for fission is equal to 5.7 Mev, we could expect that the successive fission

[15] N. Bohr, Phys. Rev. **58**, 864 (1940). We are indebted to Professor Bohr for sending us his manuscript.

[16] T. W. Bonner and W. H. Brubaker, Phys. Rev. **47**, 973 (1935); **48**, 742 (1935). We used the data of these authors instead of that of W. E. Stephens, Phys. Rev. **53**, 223 (1938), because they are more complete on the low energy side (recoil protons were observed down to 0.5 Mev instead of 3 Mev).

of U^{238} can take place for energies of the impinging neutrons larger than

$$E' = 5.7 + 2.6 = 8.3 \text{ Mev},$$

where the second term represents the energy carried out by the evaporating neutron from the compound system U^{239} which was calculated as two times the nuclear temperature. The difference between our experimental data and the theoretically expected value of E' is certainly due, in great part, to experimental errors.

As it was pointed out by Bohr, we must expect a similar behavior of the fission cross section in case of Th and Pa. For Th, Bohr gives the following values of the partial cross section for direct fission of Th^{233} and successive fission of Th^{232}

$$\sigma_f = \frac{1}{25}\sigma_0, \quad \sigma_f' = \frac{8}{25}\sigma_0.$$

In the reasonable assumption that the successive fission process of Th^{232} begins to be possible for about the same energy E' of the impinging neutrons as found in case of uranium, we must expect that the mean experimental cross section for fission of Th obtained with the $D+$Li neutrons be

$$1 + 8\alpha' = 1 + \gamma \times 0.13 = 2$$

times larger than the same quantity measured with neutrons of the $D+$Be reactions. Combining the data of Table IV and the result of Section 5, we obtain

$$(1.40 \pm 0.05) \times (1.35 \pm 0.08) = 1.9 \pm 0.2,$$

which seems to agree very well with the theoretical expectation.

Finally we point out that, although our data in case of Pa do not permit any definite conclusion, they seem to indicate that by increasing the energy of the impinging neutrons the fission cross section increases less than the fission cross section of uranium, as we must expect according to the data given by Bohr.

JULY 15, 1941 PHYSICAL REVIEW VOLUME 60

The Nuclear Magnetic Moments of C^{13}, Ba^{135} and Ba^{137} *

ROBERT HENDERSON HAY
Columbia University, New York, New York

(Received May 28, 1941)

The nuclear gyromagnetic ratios for C^{13}, Ba^{135} and Ba^{137} have been measured by the molecular beam magnetic resonance method. The signs of the nuclear magnetic moments have also been determined. For C^{13} the g value is 1.402 ± 0.004 and the moment is positive. When this fact is considered with the results of theoretical treatments of the C^{13} nucleus, $I = \frac{1}{2}$ seems to be the preferable value for C^{13}. The resultant nuclear magnetic moment is $+0.701 \pm 0.002$ n. m. The moments for both Ba^{135} and Ba^{137} are positive. The g values are 0.5575 ± 0.0017 and 0.6236 ± 0.0019 respectively. Using $I = \frac{3}{2}$ for both nuclei, the corresponding nuclear magnetic moments are $+0.8363 \pm 0.0026$ and $+0.9354 \pm 0.0029$, respectively. The ratio of the moments is 1.1174 ± 0.0010. All g values are referred to that for Li^7 of 2.1688. Careful examination of the barium beam intensity as a function of homogeneous field strength with constant oscillating field frequency revealed only two resonance minima.

INTRODUCTION

DIRECT experimental data on the nuclear spin and magnetic moment of C^{13} have more than intrinsic interest, for this isotope is one of the light nuclei to which theoretical calculations can be applied with some hope of success. Rose and Bethe[1] have used the Hartree model with Thomas spin-orbit coupling to predict a value of $\frac{1}{2}$ for the nuclear spin while Sachs,[2] on the other hand, obtains the value $\frac{3}{2}$

* Publication assisted by the Ernest Kempton Adams Fund for Physical Research of Columbia University.

[1] M. E. Rose and H. A. Bethe, Phys. Rev. **51**, 205 (1937).
[2] R. G. Sachs, Phys. Rev. **55**, 825 (1939).