On the Momentum Loss of Heavy Ions

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In the calculation of the momentum loss of a heavy ion in its passage through matter, an estimate is needed of the charge of the ion as a function of its velocity. The process of capture and loss of electrons in collisions is characterized by the velocity within the ion of the electron or electrons participating. This velocity is roughly proportional to the ion velocity. The factor of proportionality is introduced as a convenient parameter γ. Two methods are used to estimate the charge of the ion: (1) It is supposed that the characteristic velocity is that of the energetically most easily removable electron, which is determined from the Thomas-Fermi model. Under this assumption γ tends to increase with atomic number ($\gamma=1.3$ for $Z=6$ to $\gamma=1.8$ for $Z=55$). (2) It is supposed that the characteristic velocity is that of the outermost electron, which is also calculated from the Thomas-Fermi model. γ is found to decrease with atomic number ($\gamma=0.6$ for $Z=6$ to $\gamma=0.35$ for $Z=55$). The characteristic velocity probably lies between these extremes.

AN energetic heavy particle, such as a fragment produced in nuclear fission or an atom recoiling from the impact of an alpha-particle, becomes ionized as it passes through matter. The electronic stopping cross section is proportional to the square of the particle charge. The proportionality factor is the specific stopping cross section, which is the stopping cross section for a particle of unit charge. The specific stopping cross section has been calculated for air from data on alpha-particles in an earlier paper.[1]

The average charge of the particle must be estimated from the process of capture and loss. This process is very complicated; our knowledge concerning it is limited to little more than the fact that, when an electronic state of the particle is being filled and emptied because of collisions with atoms, the probabilities for capture into that state and loss out of that state are about equal when the average velocity of an electron in that state is approximately equal to the particle velocity. At any particular velocity there are only a few electronic states for which one or the other of the probabilities is not negligible. For that velocity the process is characterized by the average velocity of electrons in those states. This characteristic velocity is roughly proportional to the particle velocity. It has been found convenient to introduce the factor of proportionality as a parameter γ. Thus

$$V_e=\gamma V,$$

where V is the particle velocity and V_e is the characteristic velocity.

In an attempt to calculate the characteristic velocity, it has been supposed that this velocity is equal to the root mean square velocity of the *energetically* most easily removable electron in the Thomas-Fermi model for the ion. This quantity is calculable from the expression

$$V_e^2=\frac{4}{5h^3\pi m^2}\left\{\int_0^{r_1}drr^2(p_0^5-p_1^5)+\int_{r_1}^{r_0}drr^2p_0^5\right\}, \quad (1)$$

where p_0 is the maximum value of the electron momentum and p_1 is its minimum value at the position considered, r_0 is the radius of the ion, and r_1 is so determined that there is one electron in an energy range measured from the top of the Thomas-Fermi distribution. If the electron is supposed limited to an infinitesimal energy range at the top of the distribution, the expression can be written

$$V_e^2=\frac{1}{m^2}\int_0^{r_0}drr^2p_0^3\bigg/\int_0^{r_0}drr^2p_0. \quad (1a)$$

Results of calculations from (1) and (1a) are shown plotted in the heavier curves of Fig. 1.

[1] J. Knipp and E. Teller, Phys. Rev. 59, 659 (1941).

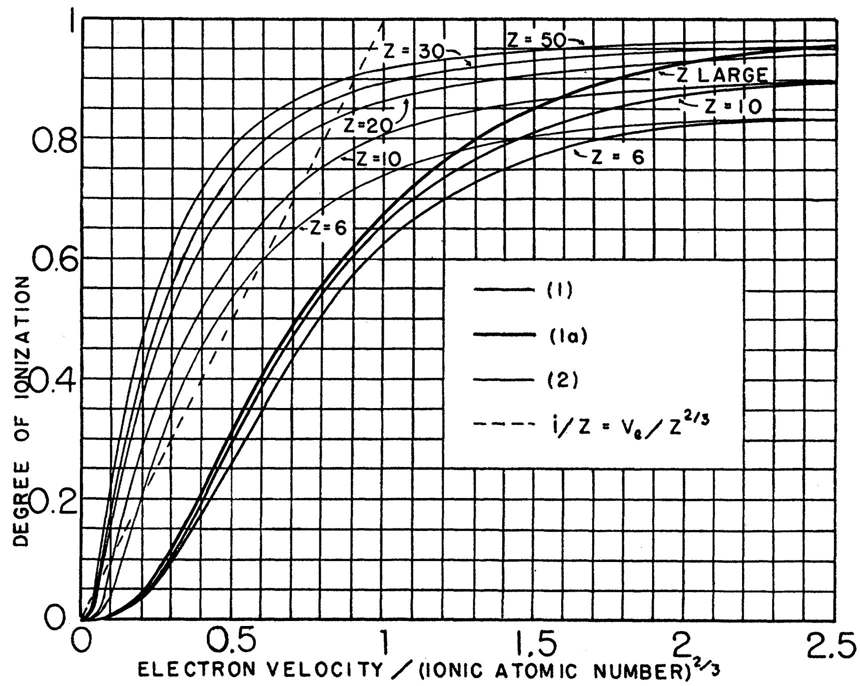

FIG. 1. Curves showing the ratio of ionic to nuclear charge i/Z plotted as a function of the velocity characteristic of the process of capture and loss (in units $Z^{\frac{2}{3}}e^2/\hbar$). The curves (1) are calculated under the assumption that the characteristic velocity is the energetically most easily removable electron of the ion. The very heavy curve (1a) is the limiting curve for very large Z. The curves (2) are calculated by assuming the characteristic velocity to be the velocity of the outermost electron of the ion. The dotted line is a relation which follows from one proposed by Bohr [Phys. Rev. **59**, 270 (1941)].

It is equally instructive to make the supposition that the characteristic velocity is equal to the root mean square velocity of the *outermost* electron of the Thomas-Fermi distribution for the ion. This quantity is obtainable from the expression

$$V_e^2 = \frac{4}{5\hbar^3 \pi m^2} \int_{r_2}^{r_0} dr r^2 p_0^5, \qquad (2)$$

where r_2 is so determined that there is one electron in the spherical shell between r_2 and r_0. Results of calculations from (2) are shown plotted in the light curves of Fig. 1.

Equations (1) and (2) represent extremes. In (1) the electron is assumed to have a definite energy as determined by the average potential of the other charges of the ion, but otherwise unaffected by them. In (2) it is supposed that electron collisions are so frequent that states of a definite energy are of little meaning for an individual electron. Then an electron participates

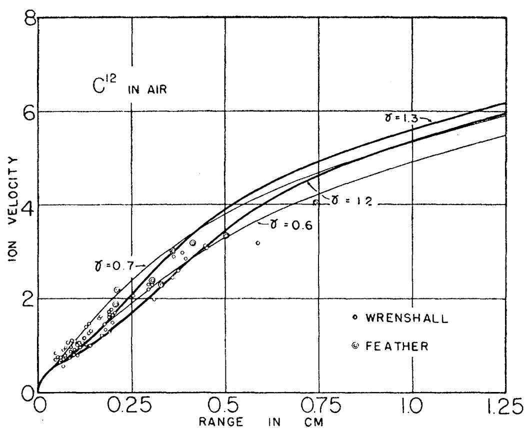

FIG. 2. Range in air of C^{12} ions plotted against the velocity (in units $e^2/\hbar$). The parameter γ, which is the ratio of the characteristic velocity to the particle velocity, is assumed to be constant throughout the range. The two heavy curves are for $\gamma = 1.2$ and 1.3 and are calculated by using (1). The two light curves are for $\gamma = 0.6$ and 0.7 and are calculated from (2). The points are experimental observations of Feather and of Wrenshall on individual tracks in cloud chambers containing various gases. These observations have been reduced to "equivalent air ranges" by comparison with alpha-particle behavior.

in the process of capture and loss whenever it is found near the surface of the ion. At the same time it also becomes the slowest electron in the velocity distribution. Since for the same atomic number velocities from (1) are greater than velocities from (2), the parameter γ will be greater under the first assumption than under the second.

Velocity-range relations have been calculated for C^{12}, Ne^{20}, and the light and heavy fragments of binary fission. The details of method are the same as used in reference 1. Curves are shown in Figs. 2, 3, 4, and 5. The experimental points of Fig. 2 are "reduced air ranges" from observations by Feather and by Wrenshall.[2] The experimental points of Fig. 3 are "reduced air

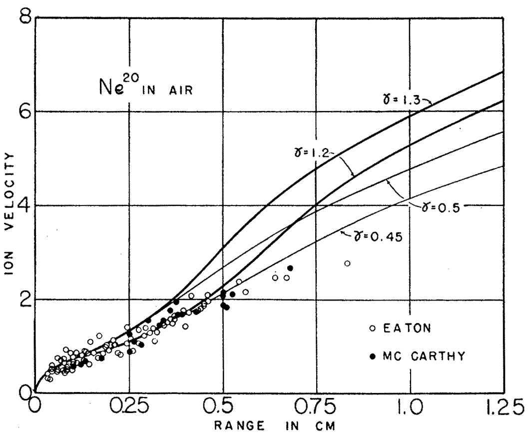

FIG. 3. Range in air of Ne^{20} ions plotted against the velocity (in units $e^2/\hbar$). The two heavy curves are for $\gamma=1.2$ and 1.3 and are calculated from (1). The two light curves are for $\gamma=0.45$ and 0.5 and are calculated from (2). The points are experimental observations of Eaton and of McCarthy.

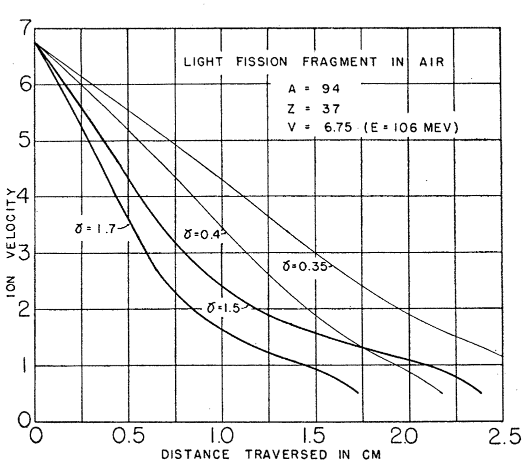

FIG. 4. Velocity (in units $e^2/\hbar$) of the light fragment produced in binary fission plotted against the distance traversed in air, measured from the point where fission took place. The two heavy curves are for $\gamma=1.5$ and 1.7 and are calculated by use of (1a). The two light curves are for $\gamma=0.35$ and 0.4 and are calculated from (2). The range of the light fission particle is observed to be about 2.5 cm.

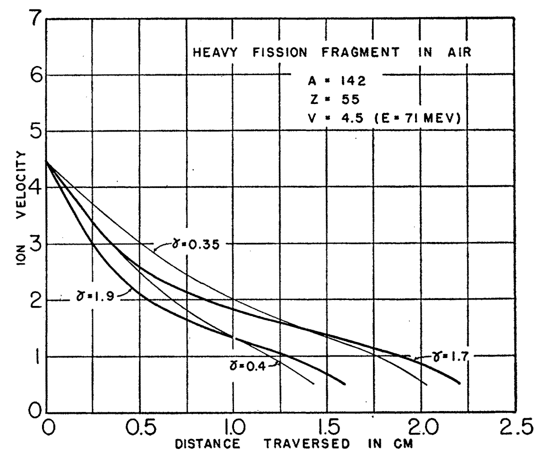

FIG. 5. Velocity (in units $e^2/\hbar$) of the heavy fragment produced in binary fission plotted against the distance traversed in air, measured from the point where fission took place. The two heavy curves are for $\gamma=1.7$ and 1.9 and are calculated by use of (1a). The two light curves are for $\gamma=0.35$ and 0.4 and are calculated by using (2). The range of the heavy fission particle is observed to be about 2 cm.

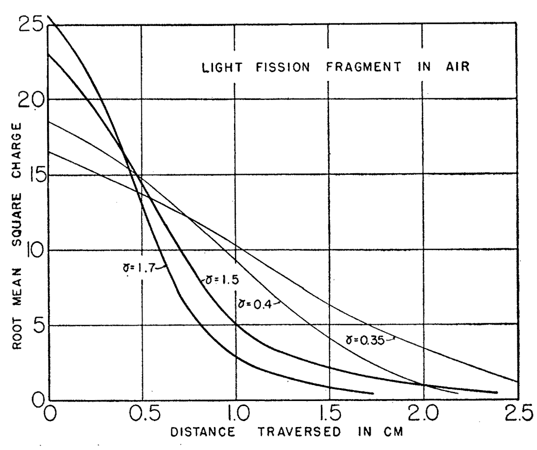

FIG. 6. Root mean square charge of the light fission fragment plotted against the distance traversed in air for the values of the parameter used in Fig. 4.

[2] N. Feather, Proc. Roy. Soc. **141**, 194 (1933); recoils from alphas in a mixture of 80 percent helium and 20 percent carbon tetrafluoride. G. A. Wrenshall, Phys. Rev. **57**, 1095 (1940); recoils from alphas in a mixture containing 52 percent methyl-chloride, 26 percent helium, 16 percent ethyl alcohol and 6 percent water vapor and also in a mixture containing 49 percent acetylene, 40 percent helium, 8 percent ethyl alcohol and 3 percent water vapor.

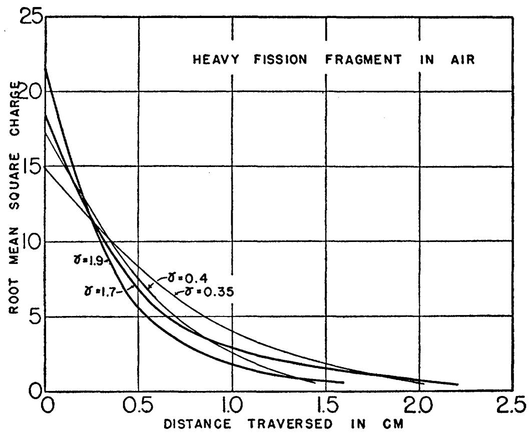

FIG. 7. Root mean square charge of the heavy fission fragment plotted against the distance traversed in air for the values of the parameter used in Fig. 5.

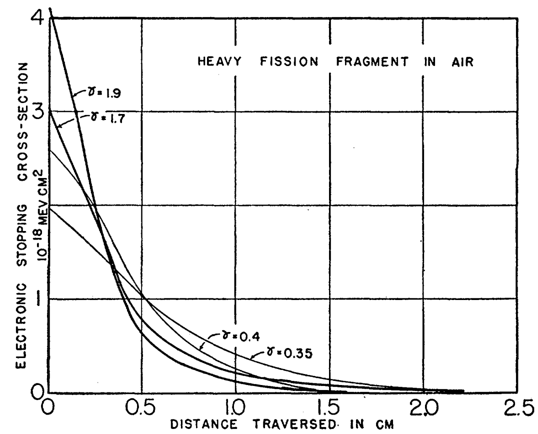

FIG. 9. Electronic stopping cross section of the heavy fission fragment plotted against the distance traversed in air for the values of the parameter used in Fig. 5.

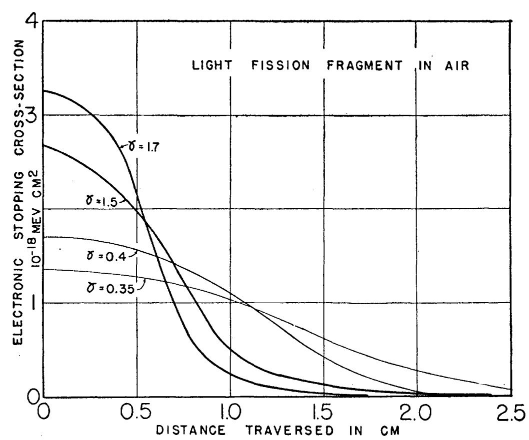

FIG. 8. Electronic stopping cross section of the light fission fragment plotted against the distance traversed in air for the values of the parameter used in Fig. 4..

ranges" from observations by Eaton and by McCarthy.[3]

If we assume (1), the best values of the parameter seem to be: C^{12}, $\gamma=1.3$ (below an ion velocity of $2e^2/\hbar$); Ne^{20}, $\gamma=1.2$ (below an ion velocity of $2e^2/\hbar$); light fission fragment ($Z=37$), $\gamma=1.5$; heavy fission fragment ($Z=55$), $\gamma=1.8$. The experimental data for intermediate ions seem to indicate that the parameter decreases with increasing ion velocity. The parameter is smaller for more tightly bound electrons.

If we assume (2), the best values seem to be: C^{12}, $\gamma=0.6$; Ne^{20}, $\gamma=0.45$; light fission fragment, $\gamma=0.37$; heavy fission fragment, $\gamma=0.35$.

The parameter tends to increase with increasing atomic number for (1) and to decrease for (2). The fact that both extremes seem to give fairly reasonable results shows that there is in the method an uncertainty which is the uncertainty in the characteristic velocities and the parameters as calculated by the two methods.[4]

The root mean square charge of the fission fragments is plotted in Figs. 6 and 7 as a function of the distance traversed. The electronic stopping cross section, which is roughly proportional to the ionization produced, is plotted in Figs. 8 and 9. Experimental data on these quantities will tend to clear up the uncertainty in the methods of calculation used.

[3] W. W. Eaton, Phys. Rev. **48**, 921 (1935); recoils from alphas, some in almost pure neon and others in a mixture of about 85 percent neon, 10 percent air and 5 percent hydrogen. J. T. McCarthy, Phys. Rev. **53**, 30 (1938); recoils from alphas in a mixture of about equal parts of neon and deuterium.

[4] The procedure proposed by Bohr (see Fig. 1) is, over a considerable range, intermediate between our extreme cases (1) and (2). In fact Bohr's calculations would seem to give fairly satisfactory results assuming $\gamma=1$.