Letters to the Editor

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On a Theory of Particles with Half-Integral Spin

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FIERZ and Pauli[1] have developed a general theory of particles with arbitrary spin, both integral and half-integral. The quantities describing particles of integral spin are tensors of suitable rank; half-integral spins are described by spinors of multiple order. It is the purpose of this note to suggest an alternative formulation of the theory on half-integral spins which avoids the complicated spinor formalism of Fierz and Pauli. The fundamental quantities of this theory $\Psi_{\mu_1\ldots\mu_k}$, have the mixed transformation properties of a Dirac four-component wave function, and of a symmetric tensor of rank k. (The Dirac index is suppressed.) The equations, which describe a particle of spin $k+\frac{1}{2}$ are:

$$(\gamma_\tau\partial_\tau+\kappa)\Psi_{\mu_1\ldots\mu_k}=0,\quad \gamma_\alpha\Psi_{\alpha\mu_2\ldots\mu_k}=0. \tag{1}$$

The usual supplementary conditions of the integral spin theory:

$$\partial_\alpha\Psi_{\alpha\mu_2\ldots\mu_k}=0,\quad \Psi_{\alpha\alpha\mu_3\ldots\mu_k}=0, \tag{2}$$

appear as consequences of these equations.

The verification that such wave fields are indeed associated with a particle of spin $k+\frac{1}{2}$ may proceed either by demonstrating that the number of independent plane wave solutions of definite energy and momentum is $2(k+\frac{1}{2})+1$, or by a direct proof that the square of the intrinsic angular momentum has the value $(k+\frac{1}{2})(k+\frac{3}{2})$ in the rest system. (It will not have this value in an arbitrary reference system.) The spin multiplicity is an invariant and is easily calculated in the rest system. Since each Ψ obeys the Dirac equation, the "small" components vanish and the two "large" components altogether form $2(k+3)!/k!3!$ quantities. The second equation of (1) provides the information that the $\Psi_{4\mu_2\ldots\mu_k}=0$, which are $2(k+2)!/(k-1)!3!$ in number, and that $\sigma_i\Psi_{i\mu_2\ldots\mu_k}=0$ $(\mu_s\neq 4)$ which constitute $2(k+1)!/(k-1)!2!$ relations among the Ψ's. The total number of independent components is, therefore, $2(k+1)$, as desired. The operator of total spin consists of the sum of the k infinitesimal rotation operators associated with the tensor indices in addition to the spin operator $\frac{1}{2}\boldsymbol{\sigma}$ of the Dirac theory. The proof that the spin possesses the correct eigenvalue is an elementary consequence of the condition $\sigma_i\Psi_{i\mu_2\ldots\mu_k}=0$.

The special case of spin $\frac{3}{2}$ has been treated in detail by Fierz and Pauli employing a method that necessitated the adjunction of auxiliary spinors in order to produce the proper subsidiary conditions from a variational principle. It is an advantage of the new formalism that a suitable Lagrangian can be constructed without the intervention of additional fields. One of a class of possible Lagrangians is

$$L=\bar{\Psi}_\mu(\gamma_\tau\partial_\tau+\kappa)\Psi_\mu-\tfrac{1}{3}\bar{\Psi}_\mu(\gamma_\mu\partial_\nu+\gamma_\nu\partial_\mu)\Psi_\nu+\tfrac{1}{3}\bar{\Psi}_\mu\gamma_\mu(\gamma_\tau\partial_\tau-\kappa)\gamma_\nu\Psi_\nu. \tag{3}$$

Although the Lagrangian is not unique in the absence of external fields, the form (3) is the only one which permits a relatively simple expression of the equations of motion in the presence of electromagnetic fields. The four-vector of charge and current, obtained by the usual prescription, is

$$\begin{aligned} j_\nu &= \bar{\Psi}_\mu\gamma_\nu\Psi_\mu-\tfrac{1}{3}(\bar{\Psi}_\mu\gamma_\mu)\Psi_\nu-\tfrac{1}{3}\bar{\Psi}_\nu(\gamma_\mu\Psi_\mu)\\ &\quad+\tfrac{1}{3}(\bar{\Psi}_\sigma\gamma_\sigma)\gamma_\nu(\gamma_\tau\Psi_\tau)\\ &=\bar{\Psi}_\mu\gamma_\nu\Psi_\mu \end{aligned} \tag{4}$$

in the absence of external electromagnetic fields. Although the charge density is not a positive definite form, the total charge is necessarily definite since $\Psi_4=0$ in the rest system. In the exceptional case of zero rest mass the wave function admits a gauge transformation,

$$\Psi_\mu'=\Psi_\mu+\partial_\mu\varphi,\quad \gamma_\tau\partial_\tau\varphi=0,$$

which leaves all physical quantities invariant.

The method here presented for developing the theory of spin $\frac{3}{2}$ thus contains many of the features of both the Proca and the Dirac theory. This is particularly evident in the application to β-decay which Mr. Kusaka[2] has discussed in an accompanying letter.

[1] M. Fierz and W. Pauli, Proc. Roy. Soc. **A173**, 211 (1939).
[2] S. Kusaka, Phys. Rev. **60**, 61 (1941), this issue.

β-Decay with Neutrino of Spin $\frac{3}{2}$

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ALL current theories of β-decay assume the neutrino to have the spin $\frac{1}{2}$ although there is no *a priori* reason for this choice, except that of simplicity. Recently Oppenheimer[1] has suggested the possibilities of neutrinos with different masses and spins, and it is of some interest to find what distribution laws, energy-lifetime relations, and selection rules for β-decay are possible with a neutrino of spin $\frac{3}{2}$. The results show that for any coupling involving the neutrino wave function, or its first derivative, the spectrum distribution is predominantly of the Konopinski-Uhlenbeck type, and that, correspondingly, the energy-lifetime relation for high energy is given by the seventh-power law. Further, for neutrino of non-zero mass, both the Fermi and the Gamow-Teller selection rules are possible; while for neutrino of zero mass, only the latter is permitted. Now, since the energy-lifetime relations for the observed β-spectra agree better with the fifth-power