Letters to the Editor

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Communications should not in general exceed 600 words in length.

On a Theory of Particles with Half-Integral Spin

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FIERZ and Pauli[1] have developed a general theory of particles with arbitrary spin, both integral and half-integral. The quantities describing particles of integral spin are tensors of suitable rank; half-integral spins are described by spinors of multiple order. It is the purpose of this note to suggest an alternative formulation of the theory on half-integral spins which avoids the complicated spinor formalism of Fierz and Pauli. The fundamental quantities of this theory $\Psi_{\mu_1\ldots\mu_k}$, have the mixed transformation properties of a Dirac four-component wave function, and of a symmetric tensor of rank k. (The Dirac index is suppressed.) The equations, which describe a particle of spin $k+\frac{1}{2}$ are:

$$(\gamma_\tau\partial_\tau+\kappa)\Psi_{\mu_1\ldots\mu_k}=0, \quad \gamma_\alpha\Psi_{\alpha\mu_2\ldots\mu_k}=0. \tag{1}$$

The usual supplementary conditions of the integral spin theory:

$$\partial_\alpha\Psi_{\alpha\mu_2\ldots\mu_k}=0, \quad \Psi_{\alpha\alpha\mu_3\ldots\mu_k}=0, \tag{2}$$

appear as consequences of these equations.

The verification that such wave fields are indeed associated with a particle of spin $k+\frac{1}{2}$ may proceed either by demonstrating that the number of independent plane wave solutions of definite energy and momentum is $2(k+\frac{1}{2})+1$, or by a direct proof that the square of the intrinsic angular momentum has the value $(k+\frac{1}{2})(k+\frac{3}{2})$ in the rest system. (It will not have this value in an arbitrary reference system.) The spin multiplicity is an invariant and is easily calculated in the rest system. Since each Ψ obeys the Dirac equation, the "small" components vanish and the two "large" components altogether form $2(k+3)!/k!3!$ quantities. The second equation of (1) provides the information that the $\Psi_{4\mu_2\ldots\mu_k}=0$, which are $2(k+2)!/(k-1)!3!$ in number, and that $\sigma_i\Psi_{i\mu_2\ldots\mu_k}=0$ ($\mu_s\neq4$) which constitute $2(k+1)!/(k-1)!2!$ relations among the Ψ's. The total number of independent components is, therefore, $2(k+1)$, as desired. The operator of total spin consists of the sum of the k infinitesimal rotation operators associated with the tensor indices in addition to the spin operator $\frac{1}{2}\boldsymbol{\sigma}$ of the Dirac theory. The proof that the spin possesses the correct eigenvalue is an elementary consequence of the condition $\sigma_i\Psi_{i\mu_2\ldots\mu_k}=0$.

The special case of spin $\frac{3}{2}$ has been treated in detail by Fierz and Pauli employing a method that necessitated the adjunction of auxiliary spinors in order to produce the proper subsidiary conditions from a variational principle. It is an advantage of the new formalism that a suitable Lagrangian can be constructed without the intervention of additional fields. One of a class of possible Lagrangians is

$$L=\bar{\Psi}_\mu(\gamma_\tau\partial_\tau+\kappa)\Psi_\mu-\tfrac{1}{3}\bar{\Psi}_\mu(\gamma_\mu\partial_\nu+\gamma_\nu\partial_\mu)\Psi_\nu + \tfrac{1}{3}\bar{\Psi}_\mu\gamma_\mu(\gamma_\tau\partial_\tau-\kappa)\gamma_\nu\Psi_\nu. \tag{3}$$

Although the Lagrangian is not unique in the absence of external fields, the form (3) is the only one which permits a relatively simple expression of the equations of motion in the presence of electromagnetic fields. The four-vector of charge and current, obtained by the usual prescription, is

$$\begin{aligned} j_\nu &= \bar{\Psi}_\mu\gamma_\nu\Psi_\mu-\tfrac{1}{3}(\bar{\Psi}_\mu\gamma_\mu)\Psi_\nu-\tfrac{1}{3}\bar{\Psi}_\nu(\gamma_\mu\Psi_\mu) \\ &\quad +\tfrac{1}{3}(\bar{\Psi}_\sigma\gamma_\sigma)\gamma_\nu(\gamma_\tau\Psi_\tau) \\ &= \bar{\Psi}_\mu\gamma_\nu\Psi_\mu \end{aligned} \tag{4}$$

in the absence of external electromagnetic fields. Although the charge density is not a positive definite form, the total charge is necessarily definite since $\Psi_4=0$ in the rest system. In the exceptional case of zero rest mass the wave function admits a gauge transformation,

$$\Psi_\mu'=\Psi_\mu+\partial_\mu\varphi, \quad \gamma_\tau\partial_\tau\varphi=0,$$

which leaves all physical quantities invariant.

The method here presented for developing the theory of spin $\frac{3}{2}$ thus contains many of the features of both the Proca and the Dirac theory. This is particularly evident in the application to β-decay which Mr. Kusaka[2] has discussed in an accompanying letter.

[1] M. Fierz and W. Pauli, Proc. Roy. Soc. **A173**, 211 (1939).
[2] S. Kusaka, Phys. Rev. **60**, 61 (1941), this issue.

β-Decay with Neutrino of Spin $\frac{3}{2}$

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ALL current theories of β-decay assume the neutrino to have the spin $\frac{1}{2}$ although there is no *a priori* reason for this choice, except that of simplicity. Recently Oppenheimer[1] has suggested the possibilities of neutrinos with different masses and spins, and it is of some interest to find what distribution laws, energy-lifetime relations, and selection rules for β-decay are possible with a neutrino of spin $\frac{3}{2}$. The results show that for any coupling involving the neutrino wave function, or its first derivative, the spectrum distribution is predominantly of the Konopinski-Uhlenbeck type, and that, correspondingly, the energy-lifetime relation for high energy is given by the seventh-power law. Further, for neutrino of non-zero mass, both the Fermi and the Gamow-Teller selection rules are possible; while for neutrino of zero mass, only the latter is permitted. Now, since the energy-lifetime relations for the observed β-spectra agree better with the fifth-power law given by the Fermi distribution rather than the seventh-power law, we have experimental grounds for ruling out the possibility of neutrino with spin $\frac{3}{2}$.

The wave equation for a particle of spin $\frac{3}{2}$ has been developed by Rarita and Schwinger.[2] The wave function ψ_μ consists of a four-vector, each component of which is a Dirac spinor, and satisfies the supplementary conditions $\partial_\mu\psi_\mu=0$ and $\gamma_\mu\psi_\mu=0$. The plane wave solutions can be built up from the polarization vectors

$$\mathbf{e}_1=\frac{1}{\sqrt{2}}(1,\,i,\,0),\quad \mathbf{e}_2=\frac{1}{\sqrt{2}}(1,\,-i,\,0),\quad \mathbf{e}_3=\frac{\mathbf{P}}{p}=(0,\,0,\,1),$$

where the z axis is taken in the direction of the momentum, and the solutions ψ_{+1} and ψ_{-1} of the Dirac equation for the spin parallel and antiparallel to the momentum. If we separate the space and time components of ψ_μ by writing $\mathbf{\Phi}=(\psi_1,\,\psi_2,\,\psi_3)$ and $i\varphi=\psi_4$, the four independent solutions for positive energy are

$$\mathbf{\Phi}^{(1)}=\mathbf{e}_1\psi_{+1},\qquad \varphi^{(1)}=0,$$
$$\mathbf{\Phi}^{(2)}=\mathbf{e}_2\psi_{-1},\qquad \varphi^{(2)}=0,$$
$$\mathbf{\Phi}^{(3)}=\frac{1}{\sqrt{3}}\mathbf{e}_2\psi_{+1}-\sqrt{\tfrac{2}{3}}\frac{E}{\mu}\mathbf{e}_3\psi_{-1},\quad \varphi^{(3)}=-\sqrt{\tfrac{2}{3}}\frac{p}{\mu}\psi_{-1},$$
$$\mathbf{\Phi}^{(4)}=\frac{1}{\sqrt{3}}\mathbf{e}_1\psi_{-1}+\sqrt{\tfrac{2}{3}}\frac{E}{\mu}\mathbf{e}_3\psi_{+1},\quad \varphi^{(4)}=\sqrt{\tfrac{2}{3}}\frac{p}{\mu}\psi_{+1}.$$

They are orthogonal, normalized, and satisfy the supplementary conditions.

Following the usual recipe, the interaction between the heavy and light particles can be written down. Because of the supplementary conditions, there is no scalar coupling term. The vector coupling term is

$$i\psi_P{}^\dagger\gamma_\mu\psi_N\psi_e{}^\dagger\psi_\mu,$$

which gives in the non-relativistic limit for the heavy particle

$$\psi_P{}^*\psi_N\psi_e{}^*\beta\varphi.$$

The tensor and the pseudo-vector coupling terms can similarly be written down.

Since the plane wave solutions have terms with the factor E/μ or p/μ, the square of the matrix elements contains a factor of the form $E+a\mu$ where a is a numerical constant depending on the type of coupling. Hence, for high energy, this theory gives the seventh-power law for the relation between the lifetime and the energy. It has actually been verified, by calculating the transition probability for the most general type of interaction, that there is no possible choice of the constants which would lead to a cancellation of the energy and momentum terms and so give the Fermi law.

When the mass of the neutrino is taken to be zero, the wave function admits a gauge transformation. Now it is easily seen that any interaction involving the neutrino wave function is not gauge invariant: in fact the only interactions satisfying this condition are those involving the curl of the wave function. Thus only the tensor and the pseudo-vector couplings are possible, and hence only the Gamow-Teller selection rule is permitted. However the energy-lifetime relation is still given by the seventh-power law since there remains only the first two of the plane wave solutions given above,[3] and the differentiation introduces the momentum factor.

In conclusion, the author wishes to thank Professor J. R. Oppenheimer and Dr. Julian Schwinger for helpful discussions on this subject.

[1] J. R. Oppenheimer, Phys. Rev. **59**, 908 (1941).
[2] W. Rarita and J. Schwinger, Phys. Rev. **60**, 61 (1941), this issue. The author is greatly indebted to Drs. Rarita and Schwinger for letting him make use of their theory before publication.
[3] This is a special case of the general theorem due to M. Fierz, Helv. Phys. Acta **13**, 45 (1940), that for a particle of zero mass and any spin, there are only two independent, non-trivial solutions.

Observations Bearing on Stellar Collapse

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THE recent publication, by G. Gamow and M. Schoenberg,[1] of an ingenious theory of the cause of nova outbursts makes it of interest to examine the observational facts concerning pre-nova and post-nova stars.

About pre-nova stars, we know only that their *photographic* luminosities are comparable with that of the sun, and that one and only one of them had its spectrum recorded feebly before outburst. The faint record of Nova Aquilae 1918 was classified by Miss Cannon on the basis of violet extent of the spectrum, and not from spectral lines, as "resembling classes *B* and *A*."[2] Actually class *O* would also be a possibility, since the violet extents of *O* and *B* spectra look alike when photographed with *glass* prisms.

Some years after outburst, all stars which have been studied (sixteen in all) show continuous spectra with great violet extent and no absorption lines. Some have bright lines of hydrogen, HeII, CIII, and other elements which indicate temperatures similar to those of class *O* stars.[3] Of special importance is the fact that the *photographic* luminosities are equal to those of the pre-nova objects. No observational fact clearly contradicts the working hypothesis that the physical conditions of pre- and post-nova stars are nearly identical. The burden of proof thus rests with those who postulate that stars change from normal to degenerate configurations at the time of outburst.

The collapse theory encounters grave difficulties in the attempt to reconcile radical changes of radius and temperature with the observed equality of pre- and post-nova brightness in a narrow range of wave-lengths, the common photographic region.

To fix ideas, suppose a normal main sequence star collapses to a sub-dwarf or white dwarf configuration such that the *bolometric* luminosity is unchanged. Then, assuming blackbody radiation, we must have

$$R_0{}^2T_0{}^4=R_c{}^2T_c{}^4,$$

where R_0 and T_0 are the original radius and temperature, and R_c and T_c refer to the collapsed star. If we take $T_0=6000°\mathrm{K}$ (equal to that of the sun) and $T_c=50{,}000°\mathrm{K}$ (estimated from spectra of old novae), we find $R_c=0.014R_0$, and the density would obviously be increased by a factor of 3×10^5. These figures are characteristic of the known white dwarfs. Of more direct application here is the result