Theory of Internal Friction Introduced by Cold Working

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Experiments indicate that those changes in a cold worked metal which give rise to internal friction are distinct from those changes which give rise to hardening and to the broadening of x-ray lines. It is suggested that this internal friction is due to the inability of certain areas on slip planes to maintain shearing stresses. The theoretical consequences of this suggestion are carried out, and are found to be in agreement with present experimental data.

It is well known that internal friction is introduced into metals by cold working. The changes in the metal which give rise to this internal friction are apparently not related to the changes which give rise to an increase in hardness, and to a broadening of x-ray lines. For this internal friction is removed by annealing at temperatures so low that neither the hardness[1,2] nor the breadth of the x-ray lines[3,4] is affected. Such low temperature annealing does, however, remove to a large extent the residual macroscopic stresses introduced by cold work.[5] Perhaps for this reason, the opinion has frequently been expressed that the internal friction introduced by cold work is caused by these residual stresses.[2,3,6] No mechanism has been suggested, however, whereby residual stresses would give rise to internal friction measured at small strain amplitudes. An alternative explanation has been advanced by the author.[7] In this theory the internal friction is due to the thermal currents flowing between the microscopic stress inhomogeneities introduced by cold work. If this were the case, the internal friction should vary in a characteristic manner with frequency. However, recent measurements[8] have shown this internal friction to be independent of the frequency of measurement over a wide frequency range.

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[2] W. Köster and K. Rosenthal, Zeits. f. Metallkunde **30**, 245 (1938).
[3] J. Norton, Trans. A.I.M.E. **137**, 49 (1940).
[4] S. T. Stephenson and F. Niemann (personal communication).
[5] For aluminum, see L. W. Kempf and K. R. Vanhorn, A.I.M.E. Tech. P. No. 1334 (1941).
[6] W. Köster, Zeits. f. Metallkunde **32**, 282 (1940).
[7] C. Zener, Phys. Rev. **53**, 582 (1938).
[8] C. Zener, C. Clarke, C. S. Smith, A.I.M.E. in press.

The view has been advanced[9–11] that the elastic after-effect may be explained in terms of a two-phase system, a continuous elastic phase and a discontinuous plastic phase. If the proper assumptions are made so that the observed elastic after-effect is reproduced, the associated internal friction automatically becomes independent of the frequency of measurement.[10] This theoretical result, combined with the observed invariance of cold work internal friction with frequency, leads us to suspect that the same physical changes in a cold worked metal are responsible for both the elastic after-effect and internal friction, and that these changes may be described, at least phenomenologically, in terms of a disperse plastic phase.

The essential characteristic of a disperse plastic phase is that it cannot permanently sustain shearing stress. If a constant load is applied, the initial shear energy in the plastic phase is gradually relaxed, resulting in a further elongation of specimen. The theory of the influence of such relaxation centers upon internal friction is developed in §1. In §2 we apply this theory to cold worked metals.

The same problem may, in principle, be attacked from the viewpoint of the dislocations introduced by cold working.[12] However, until we know the laws which govern the movements of the dislocations, and in particular, until we can calculate the consequences of their interaction with each other, more insight is to be gained from the phenomenological viewpoint here adopted.

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[10] R. Becker, Zeits. f. Physik **33**, 185 (1925).
[11] M. F. Sayre, J. Rheology **3**, 206 (1932).
[12] For review, see F. Seitz and T. A. Read, J. App. Phys. **12**, 100, 170 (1941).

§1. Theory of Relaxation Centers

A detailed discussion of the relation between relaxation centers and creep phenomena has been given by Becker.[10] His approach was not, however, well suited for the calculation of internal friction. This calculation was carried out only for one particular type of distribution of relaxation centers. It is thus necessary to give a more general discussion of the relation of relaxation centers to internal friction.

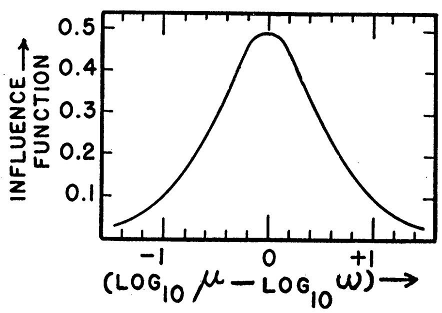

FIG. 1. Influence function $\frac{1}{2}\,\mathrm{sech}(\ln\mu-\ln\omega)$.

The method here adopted will be to calculate first the internal friction due to a single relaxation center. We then assume the centers to be sufficiently far apart so that their contributions to the internal friction are additive.

We consider an isolated unit volume containing a single relaxation center. We denote the macroscopic strain and the corresponding macroscopic stress by e and S, respectively. There are two elastic constants corresponding to the ratio e/S. One, C_∞, is the constant measured so rapidly that no appreciable relaxation takes place. This is the constant measured by very high frequency vibrations. The other, C_0, is the constant measured by a quasi-static method. It is necessarily larger than C_∞.

If there were no relaxation, the time rate of change of e would be $C_\infty dS/dt$. The relaxation contributes an additional term $\mu(C_0S-e)$, where μ is called the relaxation constant. The general relation between stress and strain is thus given by

$$(d/dt+\mu)e=(C_\infty d/dt+C_0\mu)S. \tag{1}$$

In order to calculate the internal friction associated with a definite angular frequency ω, we substitute into Eq. (1)

$$e=\mathfrak{e}e^{i\omega t},\quad S=\mathfrak{S}e^{i\omega t},$$

where $\mathfrak{e}$ and $\mathfrak{S}$ are complex constants. The ratio $\mathfrak{S}/\mathfrak{e}$ is called the complex modulus. We write it in the form $M+iN$. The ratio $Q^{-1}=N/M$ is a common measure of internal friction.[13] We find

$$Q^{-1}=(C_0-C_\infty)\mu\omega/(C_0\mu^2+C_\infty\omega^2). \tag{2}$$

We now assume, as is usually the case, that C_0 is only slightly larger than C_∞, and so shall replace C_0 by C_∞ in the denominator.

When we add the effects of all the relaxation centers, it will be convenient to have the contribution of each center in the form of a strength factor times an influence function. This is possible if we regard $\ln\mu$ and $\ln\omega$ as variables in place of μ and ω. Thus we may write

$$\mu\omega/(\mu^2+\omega^2)=\tfrac{1}{2}\,\mathrm{sech}(\ln\mu-\ln\omega).$$

Equation (2) may thus be written as

$$Q^{-1}=\{(C_0-C_\infty)/C_\infty\}\cdot\tfrac{1}{2}\,\mathrm{sech}(\ln\mu-\ln\omega).$$

This is the desired form. A plot of the influence function is given in Fig. 1.

We now define a weighting factor $W(\ln\mu)$ as the contribution to $(C_0-C_\infty)/C_\infty$ of all the relaxation centers lying within a unit logarithmic range at $\ln\mu$. The total internal friction is then given by

$$Q^{-1}=\int_{-\infty}^{\infty}W(\ln\mu)\cdot\tfrac{1}{2}\,\mathrm{sech}(\ln\mu-\ln\omega)d\ln\mu. \tag{3}$$

If W is only a slowly varying function of its argument, a useful formula is obtained by expanding W about $\ln\mu=\ln\omega$. This expansion gives the series

$$Q^{-1}=(\pi/2)W(\ln\omega)+(\pi^3/16)d^2W/d\ln\mu^2|_{\mu=\omega}+\cdots. \tag{4}$$

§2. Relaxation Centers in Cold Worked Metals

It is commonly accepted that, in plastic deformation, the individual grains do not deform homogeneously, but that a finite slip occurs

[13] C. Zener, Phys. Rev. 52, 230 (1937).

across certain planes. After slipping the two sides of a plane do not join together perfectly. The crystal structure may be broken down to a depth of several atomic distances. At best there are regions of misfits along the plane. Whatever the imperfections, it is reasonable to suppose that they render it impossible for certain areas permanently to maintain a shearing stress. But the relaxation of shearing stress across an area must also be accompanied by a partial relaxation of stress on either side of the area, and hence throughout a finite volume. Each area which cannot permanently maintain a shearing stress therefore constitutes a relaxation center.

The theory developed in §1 may be applied to these relaxation centers as long as the applied stresses are so small that the linear relation of Eq. (1) between stress and strain is valid. This is the case in those measurements of internal friction which are independent of the amplitude of vibration.

Cold working has a greater effect upon the internal friction of a metal the greater its purity. Thus, for a given elongation, 99.99 percent pure aluminum has an internal friction nearly twice as large as 99.5 percent pure aluminum.[6] The internal friction of the most drastically cold worked 70–30 alpha-brass[8] is, after recovery at room temperature for several days, not more than 10^{-4}. Values for pure copper of more than 10 times this have been reported.[1,2] It is to be expected that, as the homogeneity of a metal increases with increasing purity, the larger will be the areas of the weak places along the slip planes, and hence the larger the effective volume of the centers of relaxation.

In many cases the internal friction has been found to increase rapidly with increasing temperature. This is not the case, however, for that part of the internal friction introduced by cold working.[8] In the case of 70–30 alpha-brass, cold worked and then annealed at 100°C, it even decreased slightly in the range from room temperature to 80°C. According to the theory of relaxation centers, this independence with temperature is correlated with the independence with frequency. If the relaxation centers have the same heat of activation, an increase of temperature will increase the log of all the relaxation constants by the same amount, $\Delta \ln\mu$. By Eq. (3), this has the same effect as a decrease of log angular frequency by the same amount $\Delta \ln\mu$. Hence, if a change of frequency has no effect upon the internal friction, it is to be expected that a change in temperature will have only a slight effect.

Without further information, we cannot predict the frequency dependence of the internal friction. It depends upon the variation of the weighting function $W(\ln\mu)$ in the vicinity of $\ln\mu = \ln\omega$. If, as in alpha-brass, the internal friction is independent of frequency in a certain range, we conclude that the weighting factor is independent of μ for the same range of $\mu/2\pi$.

A decrease of the elastic modulus always accompanies cold working,[6] provided it is not carried so far as to introduce fiber structure. Conversely, low temperature annealing raises the elastic modulus. These effects are, of course, required by the theory of relaxation centers. For the introduction of such centers, as by cold working, must lower the elastic modulus; removal of the centers, as by annealing, must raise the modulus.