Letters to the Editor

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Communications should not in general exceed 600 words in length.

Half-Life of C^{11}

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MEASUREMENTS of the half-life of C^{11} have been made in connection with biological research in which this element was used as a tracer.[1] The half-life was measured with an alcohol-filled Geiger-Müller counter and a scale-of-16 circuit already described.[1] For these experiments the counting loss at high speeds was determined before and after the measurements and found to be less than 1.7 percent at 10,500 counts per minute, and less than 1.0 percent below 8500 c/min. Half-lives were calculated by the method of Peierls[2] in which the total count of a period of 3 or more half-lives is divided into successive 8-minute intervals. The initial counting rate was below 8500 c/min. except for run 5 in which it was below 10,000 c/min. No corrections were made for this loss. In runs 1 and 2, the experiments terminated when the counting rate was somewhat greater than three times background; in all the other experiments the final rate was greater than eight times background. Background was measured before and after each run, and checks were also made with a uranium standard. Samples were measured as $BaCO_3$. As a check against contamination, in run 3 the C^{11} was synthesized into sodium lactate and measured as such. All the runs gave good straight line plots for logarithm of activity against time. The results are given in Table I.

The half-life of C^{11} calculated from all the runs is 20.42 ± 0.06 minutes. However, in run 1 the background altered by a factor of two during the run; so this result can be discarded. Column 3 in the table gives the ratio of the measured activity of a uranium standard before and after each run. The deviation in runs 2 and 5 seems large, especially since its direction corresponds with the deviation of these runs from the mean. Consequently, the best value is that obtained from the remaining seven runs—20.50 ± 0.6 minutes. This is in good agreement with the value of 20.63 minutes previously obtained graphically[1] as well as the value of 20.35 ± 0.08 minutes given by Smith and Cowie.[3]

Table I. *Data on* C^{11}.

Run	Time Observed (Min.)	Agreement of Standard Activity Before and After Run (%)	Half-Life (Min.)
1	104		20.01
2	120	4.5	19.83
3	88	1.3	20.29
4	88	0.8	20.44
5	72	4.4	20.84
6	96	2.5	20.42
7	96	0.6	20.46
8	96	1.8	20.65
9	88	1.3	20.57
10	72	1.7	20.64

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[1] Conant, Cramer, Hastings, Klemperer, Solomon and Vennesland, J. Biol. Chem. **137**, 557 (1941).
[2] R. Peierls, Proc. Roy. Soc. **A149**, 467 (1935).
[3] J. H. C. Smith and D. B. Cowie, J. App. Phys. **12**, 78 (1941).

On the Slowing Down of Neutrons by Elastic Collisions

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WE consider a homogeneous medium in which Q neutrons of energy E_0 are produced per sec. Let v be the velocity of the neutrons, m their mass, $\lambda(v)$ the free path for scattering, M the mass of the nuclei of the medium, and p the probability of a neutron of initial energy E' and velocity v' being scattered by an elastic collision into the energy interval dE at E. According to the laws of elastic collisions we get for p

$$\begin{aligned} p(E, E') &= \frac{1}{1-\alpha^2}\frac{dE}{E'} \quad \text{if} \quad E'\alpha^2 \leqslant E, \\ p(E, E') &= 0 \quad \text{if} \quad E'\alpha^2 > E, \\ \alpha &= (M-m)/(M+m). \end{aligned} \tag{1}$$

In order to describe p by a uniform analytic expression, we introduce a discontinuous factor of the type of the Dirichlet discontinuous factor. We define $\delta(E, E'\alpha^2)$ by

$$\delta(E, E'\alpha^2) = \begin{cases} 1 & \text{if} \quad E'\alpha^2 \leqslant E \\ 0 & \text{if} \quad E'\alpha^2 > E \end{cases} \tag{2}$$

and p is now given by

$$p(E, E') = \frac{1}{1-\alpha^2}\frac{dE}{E'}\,\delta(E, E'\alpha^2). \tag{3}$$

Let $N(E)dE$ be the number of neutrons of a given energy E present in the stationary state in the system and $M(E)dE$ the number of neutrons per sec. leaving the energy interval dE in consequence of an elastic collision. Then

$$M(E)dE = N(E)(v/\lambda)dE. \tag{4}$$

Hence we have for the stationary state

$$M(E)dE = Qp(E, E_0)dE + \int_E^{E_0} p(E, E')M(E')dE'dE. \tag{5}$$

In this expression $QpdE$ means the number of neutrons of initial energy E_0 thrown by one collision into the energy interval dE at E and the integral gives the sum of the contributions of all the intervals between E_0 and E. By introducing new variables

$$Z=\ln(E_0/E) \tag{6}$$

and defining a new discontinuous factor

$$\delta(x-x';\,a)=\begin{cases}1 & \text{if} \quad x-x'\leqq a\\ 0 & \text{if} \quad x-x'>a,\end{cases} \tag{7}$$

we get for p

$$p(x,x')=\frac{1}{1-\alpha^2}\frac{1}{E_0}\exp(x')\delta(x-x';\,a)dx. \tag{8}$$

The condition that the state is stationary is now given by

$$M(x)=\varphi\frac{1}{1-\alpha^2}\delta(x,a)+\int_0^x\frac{1}{1-\alpha^2}\delta(x-x',a)M(x')dx'. \tag{9}$$

This integral equation is of a particular type, its second member is a convolution (Faltung); therefore, we may solve it by means of a Laplace transformation.

The Laplace transformation of a function is defined by

$$\int_0^\infty e^{-sx}F(x)dx=f(x), \tag{10}$$

where the real part of s is positive. If $F(x)$ is definite and integrable in every finite interval, the inverse transformation exists and f and F are connected by

$$F(x)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}e^{xs}f(s)ds. \tag{11}$$

Here, the integration path has to be chosen in such a manner as to enclose all the singularities of $f(s)$.

Applying the Laplace transformation to our integral equation we get

$$m(s)=\varphi\frac{1}{1-\alpha^2}\frac{1}{s}[1-e^{-as}] \\ +\frac{1}{1-\alpha^2}\int_0^\infty dxe^{-sx}\int_0^x dx'\delta(x-x',a)M(x'), \tag{12}$$

whereas

$$m(s)=\int_0^\infty e^{-sx}M(x)dx\frac{1}{s}[1-e^{-as}] \\ =\int_0^\infty dxe^{-sx}\delta(x,a)=\int_0^a dxe^{-sx}.$$

The last term of Eq. (12) will be simplified by making use of the theorem of composition of the Laplace transformation[1]

$$\int_0^\infty\exp(-s_0x)F_1(x)dx\int_0^\infty\exp(-s_0y)F_2(y)dy \\ =\int_0^\infty\exp(-s_0t)\left\{\int_0^t F_1(t-\tau)F_2(\tau)d\tau\right\}, \tag{13}$$

which holds if

$$\int_0^\infty\exp(-s_0x)F_1(x)dx \quad \text{and} \quad \int_0^\infty\exp(-s_0x)F_2(x)dx$$

exist and are convergent for the same s.

Thus we get for the last term of Eq. (12)

$$\int_0^\infty e^{-sx}dx\int_0^x dx'\delta(x-x',a)M(x') \\ =\int_0^\infty\exp(-sx')M(x')dx'\int_0^\infty e^{-sx}\delta(x;\,a)dx \\ =m(s)\frac{1}{s}[1-e^{-as}]. \tag{14}$$

Now the transformed Eq. (12) runs thus

$$m(s)=\varphi\frac{1}{1-\alpha^2}\frac{1}{s}[1-e^{-as}]+\frac{1}{1-\alpha^2}m(s)\frac{1}{s}[1-e^{-as}], \tag{15}$$

whence

$$m(s)=\varphi\frac{1-e^{-as}}{(1-\alpha^2)s-(1-e^{-as})}\cdot \tag{16}$$

By means of the inverse transformation we get for $M(x)$

$$M(x)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}e^{xs}m(s)ds \\ =\varphi\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}e^{xs}ds\frac{1-e^{-as}}{(1-\alpha^2)s-(1-e^{-as})}\cdot \tag{17}$$

Considering $m(s)$ we find that all the poles $s=s_n$ of $m(s)$ are of the first order; accordingly the residues of $m(s)$ at s_n are

$$c_n=\frac{1-\exp(-as_n)}{1-\alpha^2-a\exp(-as_n)} \tag{18}$$

and we get for $M(x)$

$$M(x)=\varphi\sum_{\text{all poles}}c_n\exp(s_nx). \tag{19}$$

The only pole for which s_n has a positive real part is situated at $s_n=1$. All the other poles of $m(s)$ lie to the left of the imaginary axis. In consequence thereof we get the following asymptotic expression for $M(x)$ for large x, if we replace x by $E=E_0e^{-x}$,

$$M(E)dE=\frac{dE}{E}\frac{1}{1-\alpha^2-\alpha^2\ln(1/\alpha^2)} \quad \text{if} \quad E\ll E_0. \tag{20}$$

For a small M, the other poles were determined too, and we found that all the other terms of the series are completely negligible for $x\sim 5$. Equation (20) is the same expression as that given by Placzek.[2]

[1] Cf., Doetsch, *Laplace Transformation*, p. 161.
[2] G. Placzek, Phys. Rev. **55**, 1130 (1939).

On the Forward Scattering of Neutrons by Paramagnetic Media

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HALPERN and Johnson[1] have concluded that the magnetic interaction between slow neutrons and certain paramagnetic ions should result in an amount of scattering at small angles that is several times the nuclear scattering in this region. This conclusion was arrived at under the assumption that the magnetic moment of the neutron is