

## Zeeman Effect of the Forbidden Lines of Pb I

### II. An Interference Effect in the Mixed Line, $\lambda 7330$

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The transverse Zeeman effect of this line, which contains mixed magnetic dipole and electric quadrupole radiation, is investigated with a Fabry-Perot interferometer, and with magnetic fields up to 4000 gauss. The pattern obtained,  $-2.68, -2.42, (-1.49), (-1.215), (-0.945), -0.255, 0, 0.255, (0.945), (1.215), (1.47), 2.42, 2.68$ , is in substantial agreement with expectations. Quantitative intensity measurements of all components are reported. In the  $\sigma$  pattern, the sum of the intensities of the four outer electric quadrupole components is found to be 0.02 of the three central magnetic dipole components. The relative intensities of the latter, 28, 45 and 27, and of the  $\pi$  components, agree with the theory developed in the following article by E. Gerjuoy, provided that the intensities contributed by the Zeeman components of the hyperfine structure are taken into account. These components are investigated at various magnetic fields, and the development of the Back-Goudsmit effect is verified. For the six  $\pi$  components, in the above order, the measured intensities at 3880 gauss are 22, 23, 7, 7, 18 and 23. Comparison with theory indicates  $2.2 \pm 0.5$  percent of quadrupole radiation, and proves that interference exists between the two types of radiation.

THE preceding part of this paper<sup>1</sup> dealt with the Zeeman effect of three forbidden lines of lead, one of which was pure magnetic dipole radiation, and the others pure electric quadrupole radiation. In this part we wish to report on the effect in the stronger of the two remaining forbidden lines,  $\lambda 7330$  and  $\lambda 9250$ . These are due to transitions from the  $6p^2 \ ^1D_2$  state to  $6p^2 \ ^3P_1$  and  $6p^2 \ ^3P_2$ , respectively. They are permitted by the selection rules for both magnetic dipole and electric quadrupole radiations, and hence may be called "mixed" lines. There are no existing data on the Zeeman effect of such lines, and, since it should have some interesting features, we have made a detailed investigation of the splittings and intensities in  $\lambda 7330$ . A study of  $\lambda 9250$  was not undertaken, because its predicted splitting is only about  $0.03\Delta\nu_n$ , and extremely high fields would be required to separate the components by a distance greater than their Doppler width. It is probable that the pattern of this line cannot be resolved by using the techniques now known.

The interest in these forbidden lines of Pb I arises partly from the fact that they are the

analogs of the forbidden lines coming from transitions within the  $p^2$  configuration of light atoms which give rise to well-known lines in cosmic sources. Thus,  $\lambda 7330$  of Pb is like the strong green nebular line of O III called  $N_2$ , and also like the red nebular line  $\lambda 6548$ . Condon<sup>2</sup> calculated that the  $N_2$  line contains only 0.1 percent of quadrupole radiation, the rest being magnetic dipole. These forbidden lines of light ions would be difficult to observe in the laboratory, but in Pb I they are more intense, thanks to the breakdown of Russell-Saunders coupling. Furthermore, the quadrupole moment is relatively greater in this case, and the proportion of quadrupole radiation to magnetic dipole should be large enough to be measurable. An attempt to detect the pure quadrupole component in the hyperfine structure of  $\lambda 7330$  was made by Mrozowski<sup>3</sup> but was not successful. He estimated an upper limit of 9 percent for the proportion of quadrupole radiation in the line. He also made measurements of the relative intensities of all the observed lines. As is shown in the article by E. Gerjuoy,<sup>4</sup> these intensities lead to a value of slightly less than 5 percent quadrupole radiation. Since this result was unexpectedly low, as ex-

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<sup>1</sup>F. A. Jenkins and S. Mrozowski, *Phys. Rev.* **59**, 808 (1941).

<sup>2</sup>E. U. Condon, *Astrophys. J.* **79**, 217 (1934).

<sup>3</sup>S. Mrozowski, *Phys. Rev.* **58**, 1086 (1940).

<sup>4</sup>E. Gerjuoy, *Phys. Rev.* **60**, 233 (1941), this issue.

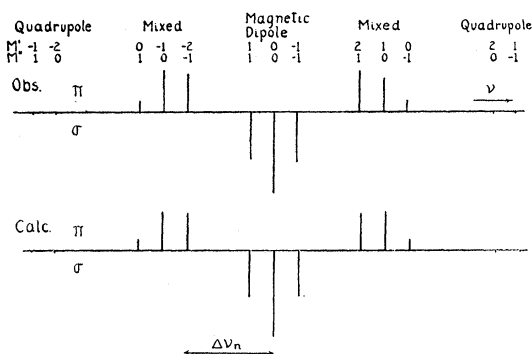


FIG. 1. Theoretical and observed Zeeman patterns for  $\lambda 7330$ . Relative intensities are shown by the heights of the lines, and the calculated values are based on 2.0 percent of quadrupole radiation. The intensities of the two components of the outer pairs of quadrupole components were not measured separately, but only that of the pairs as a whole.

plained by Gerjuoy, we undertook a study of the Zeeman effect, which should provide a more accurate determination. Our final result,  $2.2 \pm 0.5$ , is still lower, and this apparent discrepancy with theory is discussed in Gerjuoy's paper.

A feature of special interest in the Zeeman effect of this mixed line was the predicted interference between electric quadrupole and magnetic dipole radiation, which was first pointed out to us by Dr. L. I. Schiff. Using the theoretical equations derived by Gerjuoy in the accompanying paper, we have been able to prove definitely, for the first time, the existence of this effect. It is, perhaps, of interest to point out that of all forbidden lines now known, only this line  $\lambda 7330$  affords an opportunity of observing the interference effect.

## EXPERIMENTAL

### The spectrograms

A description of the discharge tube and of the spectrographic apparatus has been given in Part I. The special difficulties involved in the study of  $\lambda 7330$ , chief among which is the presence of the strong allowed line  $\lambda 7229$ , were pointed out in Mrozowski's paper<sup>5</sup> on the hyperfine structure. The temperature of the oven surrounding the tube had to be higher for maximum intensity of this line—slightly above  $800^\circ\text{C}$ —than for the other forbidden lines. It could be

<sup>5</sup> Reference 3, p. 1091.

regulated quite accurately to the optimum value by raising it slowly until the helium lines had practically disappeared. In photographing the Zeeman effect of the hyperfine structure, no gain in intensity was obtained by using the ring electrodes around the ends of the tube because the Zeeman splittings of the hyperfine components are rather small, and higher fields had to be used than for the other lines. Eastman I-U plates, hypersensitized for only 30 seconds in a solution of 4 percent ammonia in 40 percent alcohol, were sufficiently sensitive and were quite free of fog. No trouble with non-uniformity of sensitization was experienced in the intensity measurements to be described below. Exposure times of from 25 to 90 minutes were required. The parallel ( $\pi$ ) and perpendicular ( $\sigma$ ) components were photographed separately, by the use of a Nicol prism immediately in front of the spectrograph slit.

The choice of Fabry-Perot etalon spacers and of magnetic fields was governed by several considerations, which will be explained with the aid of the complete Zeeman pattern shown in Fig. 1. The mixed components, which have parallel polarization, are the most interesting, since it is in these that we expect the interference effect. If this effect is present, their relative intensities should be strongly dependent on the proportion of quadrupole radiation present. Calculation showed that with the magnetic field available, 4000 gauss, these  $\pi$  components would not be resolved even by increasing the etalon spacer up to the maximum value possible with-

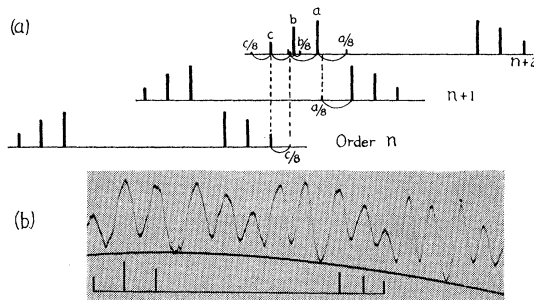


FIG. 2. (a) Diagram of the relative positions of the  $\pi$  components in three successive orders with magnetic field 3880 gauss and etalon spacer 19 mm. The weak components labeled  $a/8$ , etc., are from the hyperfine structure, and are explained later. (b) Microphotometer trace of a plate taken under these conditions. The  $\pi$  components in one order are indicated beneath the trace.

out overlapping of successive orders. This limitation is imposed not by the reflecting power of the etalon surfaces but by the Doppler width of the lines at the high temperature of the source. By further increasing the spacer until the triplets in each order cross over those in the next order, they were found to be well resolved. A diagram of the relative position of the lines under these conditions is given in Fig. 2. To facilitate the intensity measurements, it was found convenient to adjust the magnetic field to a value such that the outermost and weakest component in the pattern of one order was exactly superimposed on that in the next order but one, as indicated by the vertical broken line connecting the components  $c$  in Fig. 2. This yields a pattern consisting of groups of five lines of comparable intensity, a condition favorable for accurate intensity measurements. An enlargement of such a pattern is shown in the right-hand strip of Fig. 3. When the magnetic field was decreased somewhat, it was found that the pattern was still fairly well resolved, and here all six components in each order can be seen, as in the pattern for  $H = 3560$  of Fig. 3.

In photographing the  $\sigma$  components, two combinations of magnetic field and etalon spacer were found convenient. That used in obtaining the pattern in Fig. 3 gave ample resolution of the strong magnetic dipole triplet in the center of the pattern. However, it did not reveal the weak quadrupole components, the total intensity of which is only about 1 percent of the whole pattern. To observe these, we decreased the spacer to 7 mm and the magnetic field to 3025 gauss, so that the quadrupole components of one order were superimposed on those of the next order. This showed the components with sufficient intensity to be measurable, although they were not resolved into two separate lines under these conditions and only their total intensity could be found. Of course, any number of combinations of magnetic field and spacer could have been used to bring about this superposition. The combination given above was chosen because of the presence of a weak line at  $\lambda 7346$ , which was too close to  $\lambda 7330$  to be separated by the auxiliary dispersion of the prism.<sup>6</sup> With the 7-mm

<sup>6</sup> This line was also observed by Mrozowski, reference 3, and may be seen in his Fig. 2 just below the long wave-

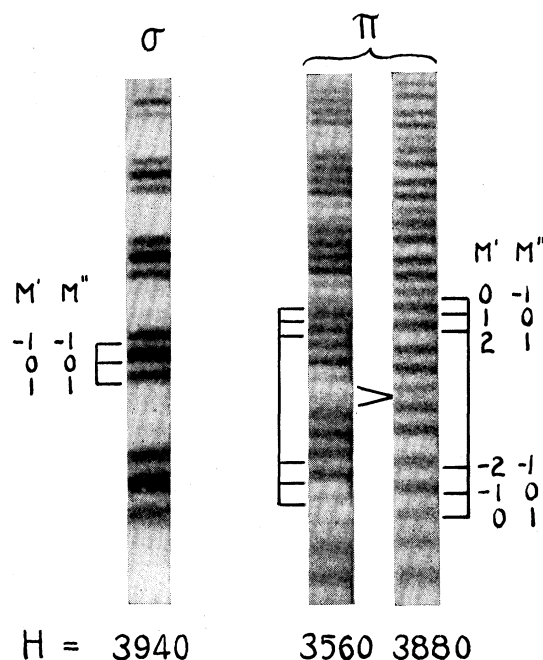


FIG. 3. Fabry-Perot patterns taken with an etalon spacer of 19 mm, and with the magnetic fields indicated. Exposures, 45 minutes each. In the  $\sigma$  components, two weak lines flanking the main triplet may be discernible. These are due to the hyperfine structure, as explained later. The main components belonging to one order are marked in each case.

spacer, the two  $\sigma$  components of this line were well separated from both the quadrupole and magnetic dipole components of  $\lambda 7330$ , and did not interfere with the measurements.

#### Intensity measurements

All plates were calibrated with two sets of density marks, one above and one below the Fabry-Perot pattern. These were impressed by the use of a step-slit in the spectrograph, in the manner previously used by Mrozowski in his measurements of intensities in the hyperfine structure.<sup>3</sup> Photographic densities were determined by means of a Zeiss recording microphotometer, and were converted to intensities by means of a calibration curve for each plate.

Since the relative intensities of the mixed, or  $\pi$  components were of the greatest interest, considerable care was taken to get the best values

length hyperfine component. We first assumed it to be the cadmium line  $6^3S-7^3P_2$ , arising from an impurity. Its Zeeman effect, however, does not confirm this assignment, since it yields an approximately normal triplet. The continued presence of the line after long running of the tube, which should eliminate the cadmium, indicates that it is a faint unclassified line of lead itself.

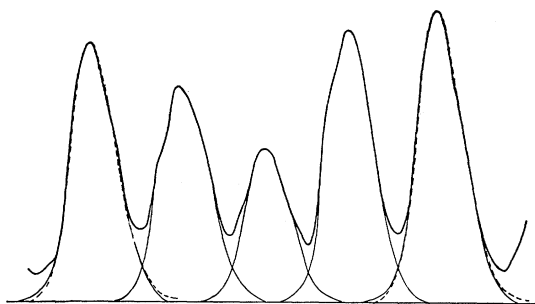


FIG. 4. Analysis of the intensity curves of the  $\pi$  components. The heavy curve is the experimental one, while the light ones give the analysis used. Broken curves on the extreme components are of the theoretical Doppler shape.

of these from the plates showing the "groups of five" referred to above. Two original plates were available, and for each of these two microphotometer curves were made, one for the three higher orders and one for the three lower ones, using a narrower slit in the latter case because the lines in the lower orders are narrower. The peak intensities and the corresponding values of the background, the latter being taken according to the black line on Fig. 2(b), were then evaluated. A curve of the variation of peak intensity with order was then used to find the factor by which the intensities in different orders should be multiplied to render them comparable. The values from the six orders were then averaged. Finally, the whole process was repeated with a new pair of microphotometer plates, and the result averaged with that from the first pair.

The above procedure of measuring the peaks and subtracting the apparent background is open to two objections, one of which requires a correction to the values obtained by its use. This concerns the true background, which might be appreciably lower than that indicated by a curve drawn through the lowest points on the microphotometer trace. A change in the assumed background is rather important for the relative intensities, because it is fairly large, amounting to between 30 and 50 percent of the total intensity at the stronger peaks.<sup>7</sup> The other question is whether the weakest components were exactly superimposed; for, if not, the peak intensity is obviously not a true measure of the sum of the

<sup>7</sup> For a discussion of the origin of this background, see reference 3, p. 1041. It is relatively more important in the present case, because of the fact that the main line is divided into several components.

two lines. Both of these matters were investigated by an analysis of the intensity curves in two different orders. The results for one of these are shown in Fig. 4. The heavy solid line is the experimental curve transformed to a linear scale of frequencies by the use of a parabolic formula. It also contains the correction for the variation of intensity with order. The two extreme components were fitted to empirical curves of the Doppler shape, and the bases of these taken as the true background. By subtraction, the curves for the other three components were then drawn (light solid lines). The areas of the five curves were then measured with a planimeter. The relative values of these areas were the same to within 2 percent as the peak intensities corrected by the new background. This was to be expected, because the half-widths of all the lines were very nearly the same, with an average value of  $0.031 \text{ cm}^{-1}$ . The theoretical Doppler half-width at  $850^\circ\text{C}$  is  $0.023 \text{ cm}^{-1}$  at this wave-length. The results of these analyses thus show that the superposition is sufficiently exact for peak intensities to be used, but that the apparent background must be reduced by 7 percent of the measured intensity of the strongest peaks. This correction was subsequently applied to all the peak measurements. Measurements of the peak intensities were also made on a plate taken with a smaller field ( $H=3560$ , Fig. 3), on which all six components were resolved. The resolution was still sufficient to obviate any correction for overlapping of adjacent lines, and here it could be assumed that the true background was measured by the lowest points of the microphotometer curves, between the weakest components.

The main triplet of the  $\sigma$  components was investigated in a similar manner. The analysis in one order is shown in Fig. 5. Here the fitting

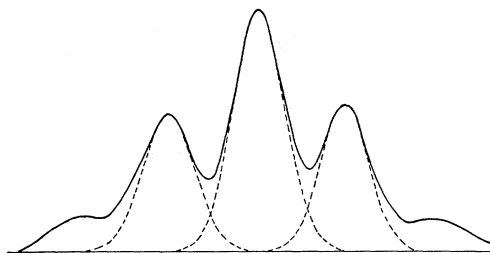


FIG. 5. Analysis of the intensity curves of the  $\sigma$  polarized magnetic dipole components in one order.

TABLE I. Observed and calculated separations in terms of  $\Delta\nu_n = 4.67 \times 10^{-5}H$ .

POLARIZATION	$\sigma$	$\sigma$	$\pi$	$\pi$	$\pi$	$\sigma$	$\sigma$	$\sigma$	$\pi$	$\pi$	$\pi$	$\sigma$	$\sigma$
$M'$	1	2	0	1	2	-1	0	1	-2	-1	0	-2	-1
$M''$	-1	0	-1	0	1	-1	0	1	-1	0	1	0	1
Obs.	2.68	2.42	1.47	1.215	0.945	0.255	0	-0.255	-0.945	-1.215	-1.49	-2.42	-2.68
Calc.	2.73	2.46	1.50	1.23	0.96	0.27	0	-0.27	-0.96	-1.23	-1.50	-2.46	-2.73

by Doppler curves was made difficult by the presence of the weak lines on either side of the triplet. These belong to the hyperfine structure, as explained below. However, the curves show that peak intensities are equivalent to areas, and that no correction need be applied to the apparent background in this case. The results quoted in Table II below were obtained from two original plates, each of which was photometered only once.

The outer  $\sigma$  components were measured on three plates where they were superimposed in adjacent orders, as explained above. Their intensity relative to the central triplet was determined by integration, which here was essential because the triplet was not quite resolved. Since these quadrupole components are very weak, the measured intensity depends rather critically on the choice of the proper background. Nevertheless, the values obtained from the three plates were surprisingly consistent. A point of interest in connection with these plates was the half-width of the lines, which could be found from the unresolved main triplet where the intensities and separations of the lines were already known. The measured half-width,  $0.030 \text{ cm}^{-1}$ , turned out to be the same as on the plates taken with larger spacers. This shows clearly that the observed width is intrinsic in the lines themselves, and is not instrumental. If the latter were the case, the width would be a constant fraction of the order separation, and the present measurement would yield a wave number width almost three times as great as the previous ones.

## RESULTS

### Splitting of the main component

We measured the wave number separations and the magnetic fields by the methods described in Part I. The results are given in Table I, where the separations are expressed in terms of

the normal splitting  $\Delta\nu_n = 4.67 \times 10^{-5}H$ . The table also includes values calculated from the known  $g$  factors of the levels,<sup>8</sup> namely  $g(^1D_2) = 1.230$  and  $g(^3P_1) = 1.269$ . These factors were determined from the Zeeman effect of allowed lines involving the two levels.

As regards our observed values, it should be stated that those for the four inner  $\pi$  components ( $\pm 0.945$ ,  $\pm 1.215$ ) were obtained under the assumption that their splitting is symmetrical about the no-field position. The outer  $\pi$  components, whose total splitting can be accurately found from the field required to overlap them, then appear to be slightly unsymmetrical. This can be seen in the enlargements, Fig. 3. Its probable explanation will be given below under the discussion of the intensities of the  $\pi$  components. A certain amount of arbitrariness is contained in our results for the outer  $\sigma$  components. As stated earlier, these were not observed as resolved pairs, and, in fact, could only be detected by overlapping them in adjacent orders. The results quoted were obtained by taking the observed splitting between the centers of gravity of the unresolved doublets, and by assuming the theoretical value  $0.26 \text{ cm}^{-1}$  for the doublet separation. All of our splittings are systematically a little smaller than those calculated from Back's  $g$  values, a fact which was noted and discussed in connection with the other lines in Part I.<sup>9</sup>

<sup>8</sup> E. Back, Zeits. f. Physik **37**, 193 (1926).

<sup>9</sup> At the time of writing Part I, we were unaware of the more recent work on the mercury lines by Green and Loring, Phys. Rev. **46**, 888 (1934). The  $g$  value 1.013 given by them for our calibrating line  $\lambda 4916$  agrees, within experimental error, with the one we adopted and requires no change in our field calibration. However, if one adopts their mean  $g$  value for  $^1P_1$  from the two lines  $\lambda 4916$  and  $5770$ , namely 1.019, perfect agreement would be obtained with the splittings predicted from Back's values. It is to be noted that the value 1.019 is in close agreement with Houston's equations and with some measurements on  $\lambda 4916$  made by Gehrcke. The anomaly in  $\lambda 5790$ , mentioned in our footnote 8, Part I, is completely explained by Green and Loring.

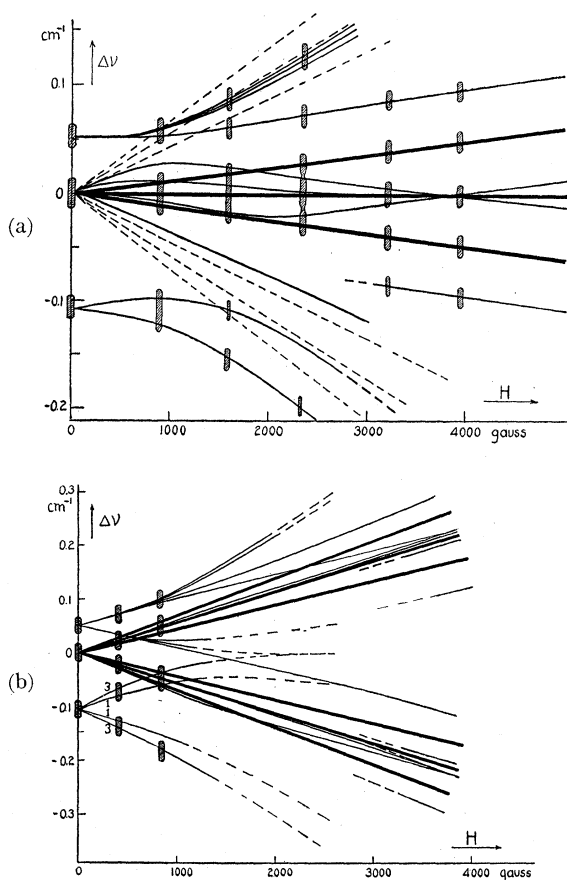


FIG. 6. Predicted and observed patterns for the Zeeman effect of the hyperfine structure. The heights and widths of the shaded marks indicate the approximate widths and intensities of the observed lines. (a)  $\sigma$  components. The three heavy lines show the positions of the magnetic dipole components of the main line. Broken lines show the positions of the main  $\pi$  components. (b)  $\pi$  components. The figure is extended to cover fields up to 4000 gauss although no direct observations of h.f.s. components were made above 1000 gauss, in order to show the origin of the lines which necessitate corrections to the intensities of the main components at high fields.

### Zeeman effect of the hyperfine structure

Before we take up the important question of the intensities in the main Zeeman pattern, we must describe our results on the splitting of the three hyperfine-structure components, because these have an appreciable influence on the intensities in the main pattern. Figures 6(a) and (b) show, in the same manner as was used in Part I, the results on the  $\sigma$  and  $\pi$  components respectively of the hyperfine structure. In computing the predicted patterns, we used for the lower state the splittings of the line  $\lambda 4618$ , de-

scribed in Part I. For the upper state,  $^1D_2$ , a graphical interpolation between weak and strong fields was made. This is sufficiently accurate here because the hyperfine structure of the  $^1D_2$  level is very narrow, and the Back-Goudsmit effect is almost complete at fields as low as 1000 gauss. The splittings of the lines were deduced from these two sets of levels. In the  $\sigma$  components of Fig. 6(a), only the magnetic dipole part has been included. The electric quadrupole part,  $\Delta M = \pm 2$ , would be exceedingly weak for these hyperfine components. In Fig. 6(b) the intensities for the weak field pattern of the low frequency h.f.s. component are given to indicate why we were not able to observe one of the components at  $H \sim 900$  gauss. In the  $\sigma$  pattern, the weak field intensities do not play any role because, for the fields at which the lines are separated, the intensities are already distorted by the beginning of the Back-Goudsmit effect. In general, both splittings and intensities agree satisfactorily with theory. From the plots, one can see that at the fields for which we measured the intensities of the main components ( $\sim 3900$  gauss) the Back-Goudsmit effect of the hyperfine structure is practically complete. The h.f.s. components on either side of the magnetic dipole triplet may be seen in the curve of Fig. 5.

### Intensities of the $\sigma$ components

In the absence of hyperfine structure, the strong magnetic dipole triplet should have the intensities 3 : 4 : 3, as calculated from the usual electric dipole formulas (see Eqs. (7) of the article by Gerjuoy<sup>4</sup>). The observed values, given in Table II, show that the central component is relatively stronger than this. The reason is

TABLE II. Intensities of magnetic dipole and electric quadrupole components.

M.D. COMPONENTS	OBS.		CALC. No h.f.s.	CORRECTIONS h.f.s. E.Q.		CALC. CORRECTED	
	Pl. 6	Pl. 7					
-1 $\rightarrow$ -1	28	28	30	0	1	31	26
0 $\rightarrow$ 0	46	45	40	17	0	57	48
1 $\rightarrow$ 1	26	27	30	0	1	31	26
E.Q. COMPONENTS	Pl. 8	Pl. 9	Pl. 10				
1 $\rightarrow$ -1	1.8	1.9	1.5	2.0		2.0	
2 $\rightarrow$ 0						1.7	
-2 $\rightarrow$ 0							
-1 $\rightarrow$ -1							

TABLE III. Intensities of the  $\pi$  components.

1.	2. 3.		4. 5.		6.	7. 8.		9. 10.		11.	12.	13. 14.		15. 16.		
	OBS. ( $H=3880$ )		CALC. 2% E.Q.			CALC. (WITH CORR.)		CALC. (REDUCED)				OBS. ( $H=3560$ )	CALC. (WITH CORR.)		CALC. (REDUCED)	
	Pl. 3	Pl. 4	INTERF.	No INTERF.		h.f.s. CORR.	INTERF.	No INTERF.	INTERF.			No INTERF.	Pl. 5	h.f.s. CORR.	INTERF.	No INTERF.
-2 → -1	22	21	24	30	3	27	34	22	27	22	0	24	30	20	26	
-1 → 0	23	23	18	15	9	27	24	22	19	24	8	26	23	22	19	
0 → 1										7	1	9	6	8	5	
0 → -1	14	14	16	10	0	16	10	13	8	7	1	9	6	8	5	
1 → 0	18	19	18	15	9	27	24	22	19	18	8	26	23	22	19	
2 → 1	23	23	24	30	3	27	34	22	27	22	0	24	30	20	26	

apparent from Fig. 6(a). At the magnetic field used, 3940 gauss, two lines of the hyperfine structure cross the central component, while the h.f.s. contributes nothing to the side components. The calculated values may be easily corrected for this, since we know the abundance of the 207 isotope (~20 percent). Using the dipole formulas for the intensities in the complete Back-Goudsmit effect, one obtains a 17 percent increase in the central component, as shown in Table II. There is also a slight correction to be made because of the overlapping of the outer components of the triplet by the weak electric quadrupole components belonging to the patterns of the main line two orders away on either side. The intensities of these are discussed in the following paragraph. When the two corrections are applied, and the resulting intensities reduced to a total of 100, the final calculated values compare favorably with the observations, as shown in the table. If anything, the correction for h.f.s. appears to be a little too large. The observed central component is not quite as strong as it should be, although much stronger than predicted without h.f.s. Possibly this is due to the Back-Goudsmit effect not being quite as complete as we have assumed it, although it must be nearly so because the symmetry of the five h.f.s. components is complete, as far as can be seen.

Table II also contains the results of three measurements of the electric quadrupole components from the plates where the two pairs from adjacent orders are superimposed. Since these four components together represent one-

half of the total quadrupole radiation (the other half being in the mixed  $\pi$  components), the ratio of their total intensity to that of the magnetic dipole triplet, which contains half the magnetic dipole radiation, gives directly the proportion of quadrupole radiation in the line. If we assume 2.0 percent for this quantity, the observed ratio should be somewhat less, owing to the blending of the h.f.s. components with the magnetic dipole triplet. Thus we find 1.7 percent as a predicted value, and since this agrees exactly with the average observed value, these measurements indicate 2 percent of quadrupole radiation. This result is only approximate, for reasons discussed above under intensity measurements.

**Intensities of the  $\pi$  components**

These should give a more exact determination of the proportion of quadrupole radiation. Also of interest is the probable existence of the interference effect in such mixed components. The observed intensities are given in Table III, columns 2, 3, and 11, and shown graphically in Fig. 7, for the two cases  $H=3880$  (groups of five) and  $H=3560$  (groups of six). The calculated intensities, without correction for hyperfine structure, were obtained from Eqs. (7) of Gerjuoy's article. They are given both for the case where interference is assumed to occur, and for that where it is assumed absent. For the latter, Eqs. (7) are also used, merely omitting the terms linear in  $s_2$ . In Table III the value 2 percent of quadrupole radiation obtained above has been assumed, while Fig. 7 also includes the intensities for 5 percent quadrupole, the value

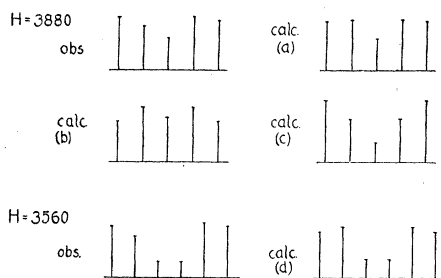


FIG. 7. Observed and computed intensities of the  $\pi$  components. For convenience, the lines are shown as they actually occur in the Fabry-Perot pattern, i.e., three components from each of two different orders. The observed intensities are shown with the weaker components superposed ( $H=3880$ ) and separate ( $H=3560$ ). The calculated values contain the correction for h.f.s. in all cases, and are obtained from the following assumptions: (a) 2 percent electric quadrupole radiation, with interference.  $H=3880$ . (b) 5 percent E.Q., with interference. (c) 2 percent E.Q., without interference. (d) 2 percent E.Q., with interference.  $H=3560$ .

obtained by Gerjuoy from Mrozowski's intensity measurements.

Before the calculated intensities in Table III, columns 4 and 5, can be compared with the observations, they must be corrected for the underlying components of the hyperfine structure. If we assume a complete Back-Goudsmit effect of the latter, each  $\pi$  component of the main pattern is accompanied by two hyperfine components at equal distances on either side. The sum of their intensities should be  $\frac{1}{4}$  that of the main line, since the odd isotope 207 has an abundance of 20 percent. Let us call the three main components on either side of the center of the pattern, which should be symmetrical,  $a$ ,  $b$ , and  $c$  in the order of decreasing intensity (Fig. 2). Using the known h.f.s. splittings in zero field,<sup>3</sup> we find that the satellites of  $a$  should be at a distance  $0.061 \text{ cm}^{-1}$ , of  $b$  at  $0.0105 \text{ cm}^{-1}$ , and of  $c$  at  $0.040 \text{ cm}^{-1}$ . For the case  $H=3880$  this places them in the relative positions shown in the diagram of Fig. 2. From this diagram, one sees that the observed intensities for this case should be in the ratio

$$(a+a/8) : (b+a/8+2b/8+2c/8) : (2c).$$

The factor  $\frac{1}{8}$  arises upon dividing the abundance ratio by two, for the two h.f.s. components. These corrections are listed in column 6 of Table III. Columns 7 and 8 give the corrected theoretical ratios, and in columns 9 and 10 they

are reduced to a scale of 100 for comparison with the observed values in columns 2 and 3.

It will be seen that the agreement with column 9 is good, except for the component  $1 \rightarrow 0$  which appears to be too weak. This observed asymmetry in the intensities,  $1 \rightarrow 0$  being consistently weaker than  $-1 \rightarrow 0$ , cannot be explained by the h.f.s. if the Back-Goudsmit effect is taken to be complete, because then one must obtain a symmetrical pattern. If it is not quite complete, the component of intensity  $a/8$  to the left of  $a$  in Fig. 2 would not be fused with  $b$  but would lie between  $b$  and  $c$ . In fact, there are definite indications of an extra line between these two components, as may be seen in the upper or high frequency triplet of Fig. 3 and in the microphotometer curves. It is probably responsible for the slight asymmetry of the measured splittings of components  $c$  that was mentioned above. By omitting the correction  $a/8$  for the  $1 \rightarrow 0$  component, its calculated intensity is changed from 22 to 19, and brought into agreement with experiment.

On the other hand, the values in column 10, calculated by assuming no interference, are in definite disagreement with the observations. Nor can they be made to agree by changing the assumed proportion of quadrupole radiation. For any proportion whatever, this theory would give intensities of the two stronger components in the ratio 2 : 1 if h.f.s. is neglected, and about 3 : 2 if h.f.s. is included. Our evidence for the existence of the interference effect is brought out most clearly in Fig. 7 by comparison of the observed intensities for  $H=3880$  with those calculated in (a) with interference, and in (c) without it.

The results from the plate taken at the smaller magnetic field, Table III, columns 11 to 16, are of less accuracy because the resolution was not as complete. Here the corrections for h.f.s. come in somewhat differently, but were worked out by the method outlined above. The results bear out those from the other plates. It is especially gratifying that the components  $c$ , which are here measured separately, (column 11), are equal in intensity and are each equal to half the combined intensity previously measured (columns 2 and 3). This proves that there can be no serious error in the assumed value of the background.



Some discussion is necessary concerning the accuracy of our assumed value 2.0 for the percentage of quadrupole radiation. Gerjuoy's calculations using the earlier measurements of the relative intensities of the various forbidden lines had indicated a proportion of about 5 percent. This would give for  $H=3880$  a group of five lines of comparable intensity, as in Fig. 7(b). The observed intensities require a figure definitely lower than this. A consideration of the way in which the intensities of these  $\pi$  components change with the percentage of quadrupole radiation assumed, and of the accuracy of the measured intensities, led us to conclude that the figure could not be altered by more than 0.3 percent without definite disagreement with experiment.<sup>10</sup> There is another possible source of error, however, in the presence of the weak line  $\lambda 7346$ . In the parallel effect this gives a single component which should lie almost symmetrically in the gap between the "groups of five." The gap is not wide enough to see the line as separate, so

<sup>10</sup> This was the limit of error given in our preliminary report of the present work, Phys. Rev. **59**, 915A (1941).

probably it merely increases the observed background in this region. It is difficult to estimate the uncertainty due to this cause, since the relative intensity of the line changes rapidly with the temperature of the source, decreasing at higher temperatures. Measurements on one plate of the  $\sigma$  components at  $H=3025$  gave an intensity relative to  $\lambda 7330$  as high as 7 percent, but it was probably less than this on the plates of the  $\pi$  components. If we decrease the assumed background by this amount, the lines become more nearly equal in intensity, but the estimated percentage of quadrupole is raised by only 0.4 percent. It appears necessary to take this uncertainty into account, and to raise our final estimate and its limit of error to  $2.2 \pm 0.5$  percent. As will be shown in the article by E. Gerjuoy, this result is unexpectedly low. Nevertheless, the internal consistency of our results convinces us that there is no large error involved. Further discussion of our results in connection with the theoretical predictions will be found in Gerjuoy's article.

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## Interference in the Zeeman Effect of Forbidden Lines

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The Zeeman effect of forbidden lines involving simultaneous electric quadrupole and magnetic dipole radiation will exhibit interference between these two different modes of radiation. The predicted intensity of any Zeeman component includes, in general, a term dependent on the fact that both modes of radiation are possible, as well as the usual transition probabilities for independent electric quadrupole and magnetic dipole radiation. Such additional interference terms appear only in the Zeeman effect and not in total line intensities. Proofs of these assertions and formulas for the Zeeman intensities are developed. The latter are compared with observations of the Zeeman effect of the forbidden lines of the  $6p^2$  configuration of Pb I, by Jenkins and Mrozowski. Good agreement with experiment is obtained only if interference is taken into

account. Their observations on the Zeeman effect correspond to a somewhat smaller value of the quadrupole moment of the transition electron in Pb I than is computed from comparison of total line intensities. Both these values of the quadrupole moment of the transition electron are 100 percent or more smaller than that given by a rough estimate from screening constant data using hydrogenic wave functions and the known positions of the levels to evaluate the effective nuclear charge. In the absence of Hartree wave functions for Pb I, no better estimate seems possible. A brief discussion of the hyperfine structure of forbidden lines is included, in which it is shown that the rules for determining the relative intensities of electric dipole hyperfine multiplets also give the intensity ratios in the hyperfine structure of magnetic dipole lines.

**T**HE forbidden lines resulting from transitions between levels of the  $6p^2$  configuration of Pb I have recently been investigated by

Mrozowski and Jenkins. Mrozowski<sup>1</sup> studied the hyperfine structure and relative intensities of the

<sup>1</sup> S. Mrozowski, Phys. Rev. **58**, 1086 (1940).

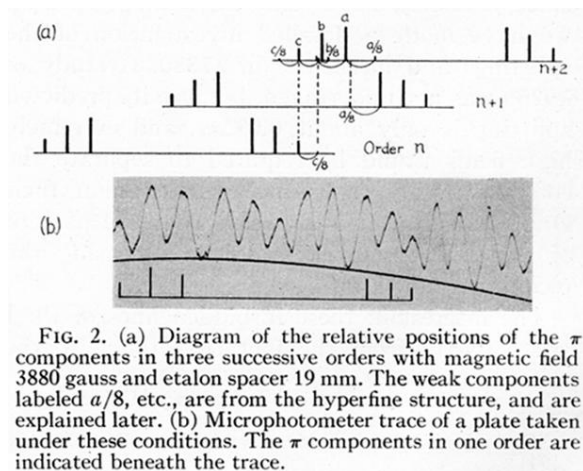


FIG. 2. (a) Diagram of the relative positions of the  $\pi$  components in three successive orders with magnetic field 3880 gauss and etalon spacer 19 mm. The weak components labeled  $a/8$ , etc., are from the hyperfine structure, and are explained later. (b) Microphotometer trace of a plate taken under these conditions. The  $\pi$  components in one order are indicated beneath the trace.

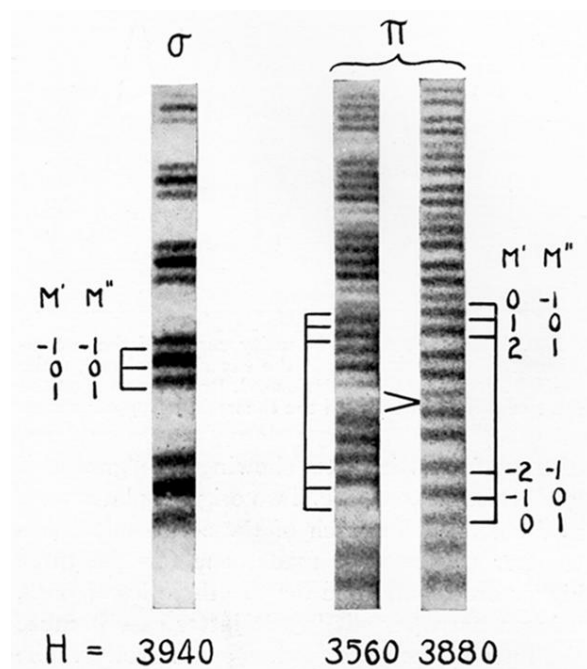


FIG. 3. Fabry-Perot patterns taken with an etalon spacer of 19 mm, and with the magnetic fields indicated. Exposures, 45 minutes each. In the  $\sigma$  components, two weak lines flanking the main triplet may be discernible. These are due to the hyperfine structure, as explained later. The main components belonging to one order are marked in each case.