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PHYSICAL PHOTOMETRY WITH A THERMOPILE ARTIFICIAL EYE.

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THE idea of a physical photometer, an instrument to indicate luminous intensities by the position of a pointer, is comparatively old.¹ The feasibility of the idea has been demonstrated, notably by Fery,² but it has not heretofore found application in practical form. That this is the case is chiefly because the great field (although not the only one) for the physical photomoter is in the photometry of lights of different color, and this latter is a problem only recently given a satisfactory solution.³ Put differently, we may say that the physical photometer has had to lie dormant because there has been no way to determine definitely whether a given physical photometer reads correctly.

The present paper describes the development and practical use of an artificial eye, consisting of a sensitive thermopile in conjunction with an absorbing medium whose transmission is a copy of the spectral luminosity curve of the average eye. This artificial eye differs from previous essays in the same direction in the accuracy with which it conforms to the latest determinations of the luminosity curve and in the fact that it has been exhaustively tested for its performance upon test color differences whose photometric values have been established by a satisfactory method of colored light photometry.

NECESSARY PHOTOMETRIC AND PHYSICAL PRELIMINARIES.

As a preliminary to the detailed description of the new physical photometer it is desirable to summarize the state of development to which the solution of certain photometric and physical problems must have been brought before physical photometry could be put in practical form.

The Adoption of a Method of Visual Colored Light Photometry.—We may emphasize two features which are practically indispensable in a method

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of visual colored light photometry if it is to serve as a basis for physical photometry. First, the method must conform to the elementary arithmetical axioms that things equal to the same thing are equal to each other, and that the sum of the separately measured parts shall equal the measurement of the whole. The importance of this is best illustrated by an example of a photometric method which is quite unsuitable, namely, "visual acuity." By this method a monochromatic light measures as more effective than a polychromatic one of the same or greater intensity, as measured by other criteria. Obviously any attempt to develop a physical photometer which shall discriminate between monochromatic and other radiation of the same intensity is beset with extreme difficulty. Second, the visual photometric method should be used only at one definite point in the "condition diagram" determined by brightness and field size. In other words, the infinite number of values which are theoretically possible owing to the existence of the Purkinje and allied phenomena, must be reduced to a single one by the adoption of definite standard conditions of illumination and field size. The practical importance of this restriction lies in the obvious difficulty of constructing a radiometer which shall change its relative spectral sensibility with the intensity of total illumination and with the size of the object presented to its view. This latter restriction is not a serious one although it might at first sight so appear. It is not serious because the study of the photometric problem has shown the disturbing phenomena mentioned to be of importance only at very low illuminations. The adoption of standard high illumination conditions is as necessary to visual colored light photometry as to physical. Deviations from these standard, most useful conditions, are to be met by the use where necessary of correcting "quality factors."

The method of visual colored light photometry to which we have worked in this research is that specified by the use of the flicker photometer, with a photometric field of 2 degrees diameter, under an illumination to give the brightness of a white surface under 25 meter candles, a large enough group of observers being used to form an average eye.

The Establishment by the Visual Photometric Method of a Set of Luminosity Values of Representative Color Differences.—The most important and useful color scale is of course that given by the luminosity curve of the normal equal energy spectrum. With this accurately determined by a photometric method conforming to the arithmetical axioms, the physical photometer problem consists solely in the accurate copying of the curve by the spectral sensibility curve of the radiometer. If the different parts of the spectrum measure correctly then all mixtures of these parts must measure rightly.

The luminosity curve of the equal energy spectrum has been determined by substantially the photometric method outlined by Ives and by Nutting.⁴ The curves obtained are shown in Fig. I (dashed and dotand-dashed lines). They are in general similar, but differ somewhat in area and shape of the blue side. These differences will show up as small but detectable differences in the relative values of the complex colors and, pending still more exact determinations of the luminosity curve, the choice of the exact curve to which the physical photometer must conform must be made by a process of trial on other color differences.



Fig. 1.

Luminosity curves of the equal energy spectrum. Dot and dash line, Ives' data. Dashed line, Nutting's data. Full line, Curve called for to secure agreement between visual and physical photometer values on series of test colors (transmission curve of final luminosity curve solution).

It may be remarked in reference to the spectrum luminosity curve that while its exact determination is all that is necessary to provide a color scale, the practical experimental difficulties, due to the necessity of determining the intensity of the radiant energy through the visible spectrum, are great enough to leave the results so far obtained in some doubt where the greatest accuracy is in question.

The spectral luminosity curve must, therefore, be supplemented by other test colors, whose luminosity values are capable of more exact determination. Such colors are suitable for checking the performance of a luminosity curve solution and for indicating slight changes, but are not suitable for determining it originally. We have had at our disposal for this purpose a series of absorbing solutions, developed for other purposes, whose transmissions have been determined by the standard photometric method by a group of observers representing the average of 61 individuals. These solutions have been described in detail elsewhere. It is only necessary to list them here. They consist of four, as follows:

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I. A solution of potassium dichromate in water,

72 grams to one liter solution.

2. A solution of cupric sulphate in water,

53 grams to one liter solution.

These two solutions, placed in one centimeter thick matched glass tanks, should transmit equal amounts of the light from a standard "4-watt" carbon lamp, as determined by the standard visual photometric method. These solutions we have developed⁵ and used for the selection of groups of observers whose mean visual characteristics shall be the same as our large group of 61. We shall refer to them here as the "test colors."

3. A yellow solution which in varying concentrations reduces the light from an incandescent solid to visual equivalence with that from incandescent solids at lower temperatures.

Composition.				
Cobalt ammonium sulphate	100	g.		
Potassium dichromate	.733	g.		
Nitric acid (1.05 grav.)	10	c.c.		
Mix with distilled water to one liter of solution.				

4. A blue solution which in varying concentrations reduces the light from an incandescent solid to visual equivalence with that from incandescent solids at higher temperatures.

Composition.
Nickel ammonium sulphate
Ammonia (.90 grav.)
Ammonium sulphate10 g.
Mix with distilled water to one liter of solution. Dilute with distilled
water containing 10 g. ammonium sulphate per liter.

These solutions⁶ we shall refer to as "Fabry solutions," the possibility of the development of such media having been pointed out by Fabry.⁷ The transmission of these solutions in thicknesses of one centimeter, at 20° C., against clear water, are accurately given by the equations:

Yellow solution over "4-watt" lamp	log10	T :		 .245	c ^{.9}
Yellow solution over tungsten lamp to reduce to "4-watt" color	\log_{10}	T :	=	368	$c^{1.05}$
Blue solution over "4-watt" lamp	log10	<i>T</i> =	= -	 539	$c^{1.03}$

where "c" (concentration) is expressed in decimal fractions.

The spectral transmissions of these solutions are shown in Fig. 2. It will be seen that solutions I and 2 give a comparatively sharp division of the spectrum (of the carbon lamp) into two complementary parts, while the Fabry solutions give a very gradual change of intensity from end to

end. We have as well in the yellow solution on the test lamp side, and in the blue solution on the carbon or comparison lamp, side, two means of obtaining the same visual color difference, with slight differences in the spectra.

These solutions give us, therefore, two main different types of color difference, and include the most common type now encountered, namely that between the low and high efficiency incandescent lamps. We have worked upon the assumption that a physical photometer which conforms



Spectral transmissions of colored solutions used to test performance of physical photometer. 1. Yellow test color. 2. Green test color. 3. Fabry yellow solution, 35 per cent. conc. 4. Fabry blue solution, 100 per cent. conc.

closely to the determinations of the spectrum luminosity curve and as well measures these color differences accurately is sufficiently practical for immediate needs.

The Existence of a Satisfactory, Highly Sensitive Means of Measuring Radiant Energy.—Luminous flux being simply radiant power which has been passed through an evaluating medium, either the retina or a luminosity curve solution, the physical measurement is necessarily one of radiant power measurement. An essential characteristic of the radiometer to be employed is of course that it shall have adequate sensibility to stand the tremendous reduction of intensity which transformation from radiant power to luminous flux involves with our present light sources (a reduction usually of about 99 per cent.). In addition to this the radiometer should [give a response directly proportional to the stimulus. Further, its spectral sensibility should be fixed and unchangeable in order that the problem of altering its sensibility to that of the eye shall be capable of definite solution. In the preliminary study of the problem of physical photometry¹ the chief types of radiometer were studied with reference to the essential characteristics for this purpose. The most sensitive ones (selenium and the photo-electric cell) were found to be deficient in certain essentials and final choice was made of the thermopile, which has of late been subject to considerable study and development.

PRACTICAL DETAILS.

The Thermopile and Galvanometer.—The thermopile used in the majority of the work was a linear one of bismuth-silver, Coblentz design, of 18 junctions, having a total resistance of 26 ohms, the junctions being in series. The galvanometer was a d'Arsonval, constructed by Leeds & Northrup. Its period was 6.5 seconds; its internal resistance 12.5 ohms; its critical damping resistance 32.5 ohms and its sensibility 33 mm. per microvolt.

The Luminosity Curve Solution.—The luminosity curve solution was worked out by trial with the spectrophotometer, to conform quite closely to the luminosity curve as determined by Ives. At the same time the number and character of the constituents was such that by altering their relative proportions the transmission curve could be altered to correspond quite closely to the characteristics of the Nutting curve, or toward the characteristics of individual curves in the groups of which the ones quoted are average. This first approximation to the working solution was composed as follows:



Fig. 3.

Spectral transmissions of constituents of luminosity curve solution. A. Cupric chloride. B. Potassium chromate. C. Cobalt ammonium sulphate.

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The chromate should be dissolved separately and the acid added to it. These constituents have been carefully selected for permanence and other practical qualities.

In Fig. 3 are shown the transmission of the constituents separately. This solution has a transmission closely that of the Ives curve and it will be easily seen that it should be possible by altering the amount of the cobalt constituent to remove the chief difference between that and the other experimental luminosity curve obtained by Nutting and by changing the copper or potassium chromate to shift the curve bodily toward red or blue. The Nutting curve we find is very accurately copied by the following composition.

Cupric chloride	grams.
Cobalt ammonium sulphate14	grams.
Potassium chromate19	grams.
Nitric acid (1.05 gr.)15	c.c.
Water to 1	liter.

The Arrangement of Apparatus and Method of Reading.—Fig. 4 shows the arrangement of apparatus to scale, as set up in our physical photom-



Diagram of thermopile artificial eye. i, thermopile; l, luminosity curve solution; g, galvanometer; j, lamp; s', screens; k, protective water tank; s, shutter; d, sector disc; b, ledge holding absorbing media to be measured; m, mirror.

eter room, a basement room 10 \times 10 feet square, where the temperature is reasonably constant.

The thermopile t, the luminosity curve solution in a one centimeter thick glass tank l, and the galvanometer g, are all placed close together on the top of a concrete pier in an enclosing box. The box has a small aperture a, facing toward the 100-candle-power stereopticon lamp j, 25 cm. distant. The radiation from the latter passes through openings in two tin screens s^1 and through the water tank k. This latter was adopted originally for the purpose of keeping the surroundings of the thermopile from unnecessary heating, and is, we find, essential but for this purpose and to completely obstruct infra-red radiation. A shutter s and a sector disc d are interposed between the luminosity curve solution and the water tank. The absorbing solutions to be measured are placed on a ledge b. m is a concave mirror which may be placed behind the lamp to increase the size of the deflections when desired.

The apparatus as shown in the diagram is arranged for the measurement of absorptions. It can, however, be just as well utilized for measuring the intensities of light sources. With the point source carbon lamp, which gives approximately 45 candle power at "4-watt" color, we obtain deflections of 7 centimeters without the concave reflector, or as much as 20 with it. For the measurement of absorption the sensibility may be greatly increased by the use of a convex lens between light source and thermopile. Smaller light sources could be brought considerably nearer the thermopile. Actually we find no advantage in going over five centimeters with the deflections, so that we believe the sensibility of the apparatus in its present form is sufficient for all ordinary purposes. Candle-powers as low as 8 can be measured accurately.

The chief impediments to accuracy in physical photometry we have found to be mechanical disturbance, drift and lack of exact proportionality between stimulus and response in the galvanometer. Mechanical disturbances depend for their magnitude upon the location of the laboratory and the location of the galvanometer in the laboratory. With the d'Arsonval galvanometer magnetic disturbances are not to be feared under usual conditions; purely mechanical ones are slight in the ordinary basement and, where heavy machinery is near, we find the Julius suspension very effective. In short, the arrangement we have adopted is entirely practical from this standpoint for the average laboratory.

Drift of the galvanometer-thermopile system we find by far the most serious trouble we have to contend with. Even in a room at unusually constant temperature there is apt to be a slow creep of the zero, amounting sometimes to several per cent. of the reading during the time taken for a deflection and the return to zero. We believe that much of this might be eliminated by enclosing the galvanometer and thermopile in a thermostat, as could be done easily with the whole system occupying the small space it does here. Experiments along this line are now in progress in connection with another problem. Meanwhile we have found it easy

to meet this difficulty by making the manner of reading such as to automatically eliminate the effect of drift. To do this we make our readings on a timed basis. We have found with our system that by allowing exactly the same time interval between the first zero reading, the deflection and the final zero reading, that the true zero may be taken as the mean of the first and last readings. We find 30 seconds a convenient and sufficient period of exposure. In addition to this procedure we so arrange that all our deflections are of approximately the same size, securing this condition by the use of the sector discs (operated by a distant motor to avoid magnetic disturbances).

Any lack of exact proportionality between stimulus and deflection such as is caused by an off-center suspension of the galvanometer coil, is also neutralized by the use of the sector disc to maintain the deflections of the same size. Another source of error, which falls under the same head, is a slow change in the sensibility of the system, which we notice at times. The exact cause of this we have not determined. It may lie in the gradual heating up of the entire thermopile, or it may lie entirely in the galvanometer. We find that this effect may usually be reduced by exposing the thermopile steadily for some minutes before beginning regular work. It is, however, our practice, in addition, to alternate the readings on the standard and test lights so that any effect on the result is eliminated. We have good reason to believe that most of these difficulties are to be ascribed to the very severe conditions with respect to acid atmosphere and dust peculiar to our laboratory, situated in the midst of gas works.

By following these precautions we have no difficulty in securing a precision of better than one per cent.

We may remark here parenthetically that the attainment of this degree of precision with a degree of sensibility adequate to measure the usual light sources suggests the desirability of applying this physical method to precision photometry even where there is no problem of color difference. For instance the intercomparison of standard lamps. Under good laboratory conditions a precision three or four times as good as we attain should be easily possible.

Since the greater part of the work here reported was completed we have made a trial of a vacuum thermocouple with very promising results. This, constructed by Dr. E. Karrer and in general agreement with the design of Dr. A. H. Pfund,⁸ consists of a single junction (with a second similar junction for compensation) of BiSn, BiSb alloys, in a glass container exhausted with the aid of a charcoal evacuator. This junction has a very small mass, but is connected to a rather large mass of metal intended to act as a heat reservoir.

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We find this junction to be more definite in action and freer from drift and changing sensibility than the exposed type. Where it is possible to make or obtain thermocouples of this type we believe they will give great satisfaction.

We mention the trial of this extra junction in order to suggest that a more systematic study of the galvanometer-thermopile system than we have made may be expected to greatly improve on the precision obtained by us.

The Changes Found Necessary in the Original Luminosity Curve Solution.—With the luminosity curve solution matching the Ives curve as described, the first set of color differences measured were those given by the yellow Fabry solution. It was found that over the range represented by the commoner illuminants and standards, namely, from the Hefner to the "I watt per candle" tungsten, the agreement between visually determined and physically determined values was practically perfect.

Beyond this range of color difference, however, a progressive divergence appeared, greater where the blue end of the luminosity curve was in question than where the red end was most used. Thus on the "comparison lamp" side⁹ the difference between visual and physical values approached one per cent., at the end of the observed curve, on the "test lamp" side, where the color difference amounted to that between a "4watt" and a "half-watt" lamp, the difference was as much as two per cent. Now, although these differences fall well within the scattering points obtained by a group of selected observers, we considered it undesirable to stop short of exact agreement and, therefore, undertook a cut and try modification of the luminosity curve solution.

As the first step in this we measured the two "test" colors. These we found to measure two per cent. off the visual value, the yellow solution measuring too small. We had then at our disposal several means of effecting a correction. We could decrease the concentration of the cupric chloride, thereby making the system more red sensitive. Or we could increase the concentration of either the cobalt or the chromate. Bearing in mind that the chief difference between the Nutting and Ives curves lies in the blue-green, the correction decided upon was an increase in the concentration of the cobalt. Such an increase was found by trial as would make the two test colors read alike.

This new solution was then tried out on the Fabry yellow, with the result that the discrepancy at the test lamp end was found little affected. It was soon evident why this was the case. The spectral transmission of the green test color is such that a change in the concentration of the cobalt constituent is very effective on it. The color difference intro-

duced by the tungsten lamp is however relatively much less sensitive to this change than it would be to a change farther from the center of the spectrum. This gave the clue to the next change, namely, an increase in the concentration of the potassium chromate. After several trials a balance was hit which behaved equally well with all the test color differences. The final solution has the composition:

Cupric chloride	grams.
Cobalt ammonium sulphate14.5 g	grams.
Potassium chromate 1.9 g	grams.
Nitric acid (1.05 gr.)	c.c.
Water to	iter.

[An essential part of the absorbing means is the protective water layer of at least 2 cm. thickness.]

The spectral transmission is shown in Fig. 1 (full line), plotted to a maximum value of unity. This curve differs from the experimentally determined ones in being shifted somewhat toward the red end of the spectrum, a point which will be discussed presently. The transmission of this solution is represented very accurately by the following equation:⁹

$$L_{\lambda} = A \left(\frac{R_1}{\lambda} e^{1 - (R_1/\lambda)}\right)^{\alpha} + B \left(\frac{R_2}{\lambda} e^{1 - (R_2/\lambda)}\right)^{\beta} + C \left(\frac{R_3}{\lambda} e^{1 - (R_3/\lambda)}\right)^{\gamma}$$

where
$$A = .999, \quad R_1 = .556, \quad \alpha = 200,$$
$$B = .04, \quad R_2 = .465, \quad \beta = 400,$$
$$C = .095, \quad R_3 = .610, \quad \gamma = 1,000.$$

Performance.—The final physical photometer measures the two test colors equal to within one half per cent. On the Fabry solutions the agreement between visual and physical values is everywhere well within one per cent. and the calculated value drawn through the visual points lies nowhere more than one half per cent. away from the physical values. This agreement we feel justified in calling complete, and, as stated above, we believe that since these test color differences are measured correctly, all other ordinary color differences of the kind met with in practical photometry (which excludes monochromatic lights) will also be measured satisfactorily. The visual, calculated and physical values for the Fabry solutions are shown in Figs. 5 and 6. These data indicate of course the accuracy to be expected in the measurement of the various light sources which the solutions counterfeit in color.

DISCUSSION.

It is not out of place to mention that the physical photometer is extremely simple and convenient in operation. It has been so arranged

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Visual and physical photometer values of Fabry yellow solution compared. Circles, visual points; full line, calculated curve; dots, physical photometer points. The color range represented is from the Hefner, at the right, to the "half watt" tungsten, at the left.





Visual and physical photometer values of Fabry blue solution compared. Circles, visual points; full line, calculated curve; dots, physical photometer points. The color range represented is from the standard carbon lamp, at the left, to the "half watt" tungsten, at the right.

that a single person can operate it perfectly. What has heretofore been a task for a group of five or more men, with a complete photometric track and accessories, can now be done by one man in a tenth the time.

The point perhaps of chief importance for discussion is the character of the luminosity curve demanded to secure the agreement between the visual and physical values. Why is it shifted toward the red end of the spectrum as compared with the directly determined ones? The explanation may lie, as already suggested, in the inherent difficulties of the direct determination. Or again it may be that our large group of observers on whom the visual values depend have a somewhat different average from the much smaller groups used in the luminosity curve determinations.

Another question of particular interest to the writers is why in our recent determination of the mechanical equivalent of light¹⁰ the evidence was in favor of a curve shifted more toward the blue than that now indicated. The decision in favor of the bluer curve was made on the basis of the apparent better agreement, when such a curve was used, between the two methods of attack, in one of which (B) we measured the radiation transmitted by the luminosity curve solution, in the other (C) the intensity of a monochromatic green radiation.

With a view to tracing out the cause of the discrepancy which would exist were the luminosity curve as here determined employed in that investigation, we have measured on the physical photometer the transmission of the monochromatic green solution there used. We have performed the measurement both with the Nutting curve and with the new one. With the Nutting curve solution the transmission measured .0463, with the new curve .0441, the visual determined value being .0437. Now bearing in mind that the physical photometer cannot in its present form be expected to measure monochromatic light exactly right, this agreement of better than one per cent. using the new luminosity curve, shows conclusively that our visual and physical photometric work is throughout very satisfactorily consistent.

But more than that it shows that the discrepancy between the mechanical equivalent values must of necessity lie in the experimental apparatus and manipulation. No question of photometric method or of physiological or other obscure factors is at issue. Since the luminosity curve solution gives the ratio of the green light to the "white" exactly as used the new physical photometer might have been employed in the mechanical equivalent study instead of the eye and the two methods would become identical and of necessity would agree.

The answer to this riddle has fortunately been found through the accidental omission of the protective water tank in some measurements

undertaken with the physical photometer. These indicated at once what has since been verified, that there must be an infra-red transmission band in the luminosity curve solution as used in the mechanical equivalent investigation. The effect of this is to give too large a value to the energy measured by that method called (B).

Fortunately, as was emphasized in the account of that study, the data were obtained in such a way that they are available for any re-calculation that a change in the units or luminosity curve might later render necessary. It has therefore been possible to correct the mechanical equivalent value as the result of making one comparatively simple measurement. That measurement is of the radiant luminous efficiency of the "4-watt" carbon lamp. As previously made with the unprotected "Ives" solution (with which the actual measurements were made, and then corrected to Nutting curve values) its value was .0049. As now determined with the protective tank of water and the new luminosity curve solution, its value is .0043. This means that the value for the lumens per watt of the most efficient light, as determined with the "Ives" solution is to be increased by 12 per cent. The corrected (Ives curve) figure by method (B) thus becomes $563.7 \times 1.12 = 631$.

The use of the new luminosity curve also affects the value by method (C), since the luminous efficiency of the green mercury radiation is now reduced to 97.5. Applying this to the quantity tabulated as "m" in the mechanical equivalent work, we obtain the figure 630. In short, the effect of the information derived from the development of the physical photometer is to rather firmly substantiate a luminosity curve shifted somewhat toward the red as compared with the experimentally obtained ones, and to harmonize all our previous data on the mechanical equivalent of light. As an additional result of our work we therefore announce as the corrected value of the mechanical equivalent of light

I lumen = .00159 watt of luminous flux.

(The new value is 2 per cent. lower than the old.)

Satisfactory as is therefore the final result of the investigation of physical photometry, it must not be overlooked that the actual luminosity curve which is worked to is only determined by the transmission of a certain inorganic salt solution. We would therefore conclude this discussion by emphasizing the importance, both from the standpoint of developing the physical photometer and for the more accurate fixing of the mechanical equivalent of light, of securing a more accurately determined luminosity curve directly, using a very large number of observers.

This is a task which could be undertaken with especial fitness by such

an organization as the Bureau of Standards. Until the luminosity curve of the spectrum is completely determined and exactly matched in the sensibility curve of a radiometer the physical photometer can be used with entire safety only for those classes of color difference for which it has been proved to agree with the visual scale. It is in fact to a certain extent empirical in its present form. Where there is any question it will be advisable to work over rather small ranges of color difference from standards calibrated by the visual method. Obviously monochromatic radiations can hardly be expected to measure correctly with any except an exact copy of the luminosity curve.

We emphasize these limitations of the physical photometer here described in order to show in which direction lies the research work which shall keep the physical photometer abreast of the probably more exacting needs of the future. As far as the present is concerned the emphasis is to be placed on the fact that we have here in practical use a physical average eye, proven capable of measuring the more usual color differences met with in practical photometry, with the same results as are given by the method of visual colored light photometry used by us.

SUMMARY.

I. A practical physical photometer has been developed suitable for colored light photometry in the standardizing laboratory.

2. A spectral luminosity curve has been determined slightly different from one previously published.

3. A corrected value is obtained for the mechanical equivalent of light.

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The United Gas Improvement Company, Philadelphia, April 20, 1915.

 $^{\rm 1}$ See Ives, Physical Photometry, Trans. Illuminating Engineering Society, No. 1, 1915, p. 101.

² Fery, Journal de Physique, August, 1908.

⁸ Ives, Photometry of Lights of Different Color, Phil. Mag., July, September, November, December, 1912.

⁴ Nutting, Visibility of Radiation, Trans. I. E. S., No. 7, 1914, p. 633.

⁵ Ives and Kingsbury, On the Choice of a Group of Observers for Heterochromatic Measurements, Trans. I. E. S., 1915.

⁶ Ives and Kingsbury, Experiments on Colored Absorbing Solutions for use in Heterochromatic Photometry, Trans. I. E. S., No. 1914.

Ives and Kingsbury, Additional Experiments on Colored Absorbing Solutions for use in Heterochromatic Photometry, Trans. I. E. S., 1915.

⁷ Fabry, A Practical Solution of the Problem of Heterochromatic Photometry, Trans. I. E. S., June, 1913, p. 302.

8 Pfund, PHys. Rev., XXXIV., No. 3, p. 228.

⁹ The derivation of this equation will be treated in a separate paper.

¹⁰ Ives, Coblentz and Kingsbury, The Mechanical Equivalent of Light, PHVS. REV., April, 1915, p. 269.