PROCEEDINGS

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ON X-RAY WAVE-LENGTHS.[1]

BY WILLIAM DUANE AND FRANKLIN L. HUNT.

AT its Philadelphia meeting (December, 1914) one of us presented a paper to this Society describing experiments on the relation between the wave-lengths of X-Rays and the voltages required to produce them. The X-Rays came from the tungsten target in one of the tubes designed by W. D. Coolidge,[2] and belonged to the general or "white light" X-radiation and not to the characteristic X-radiation. The generator supplying current to the tube produced a *roughly* constant difference of potential, which was measured by means of the voltmeter attached to the generator and also by means of the length of sparks between two metal balls each 2 cm. in diameter. This gave, of course, only an approximate estimate of the voltage. The wave-length λ_e was calculated from measurements of the mass-coefficient of absorption μ/ρ of the X-rays in aluminium, the formula being

$$\frac{\mu}{\rho} = 14.9\,\lambda_e^3 \quad (\lambda_e \text{ expressed in ångströms}).$$

This method gives what may be called the *effective* wave-length of the X-rays. With $V = 43{,}000$ volts difference of potential between the electrodes of the tube the effective wave-length of the X-rays after they had passed through 2 mm. of aluminium was found to be $\lambda_e = .45$ ångström. Calling the ratio of the energy of a cathode-ray particle to the effective frequency of the X-rays h_e, this gives $h_e = 10 \times 10^{-27}$, a quantity of the order of magnitude, but somewhat greater than Planck's radiation constant $h = 6.41 \times 10^{-27}$.

The authors of this paper have repeated the experiments using a high tension storage battery to supply current through the tube. The difference of potential between the electrodes of the tube was measured by means of an excellent electrostatic voltmeter designed by Dr. E. L. Chaffee. We calibrated this instrument by measuring the current flowing from the storage battery through a number of manganin wire coils of known resistance. The total resistance of the coils joined in series amounted to about one million ohms. The measurements appear to be thoroughly reliable.

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 23–24, 1915.

[2] PHYSICAL REVIEW, December, 1913.

On plotting the logarithm of the intensity of the X-rays produced by a given difference of potential against the total mass per square centimeter of the aluminium plates through which the rays had passed, we obtained a line that was somewhat curved near the intensity axis, but which became nearly straight at a short distance from it (Fig. 1). This means that both the coefficient of absorption and the effective wave-length of the X-rays decrease as the rays pass through matter. In other words a *constant* difference of potential such as that of a storage battery does not produce *homogeneous* X-rays. The coefficient of absorption and effective wave-length, however, vary very little after the rays have passed through several millimeters of aluminium.

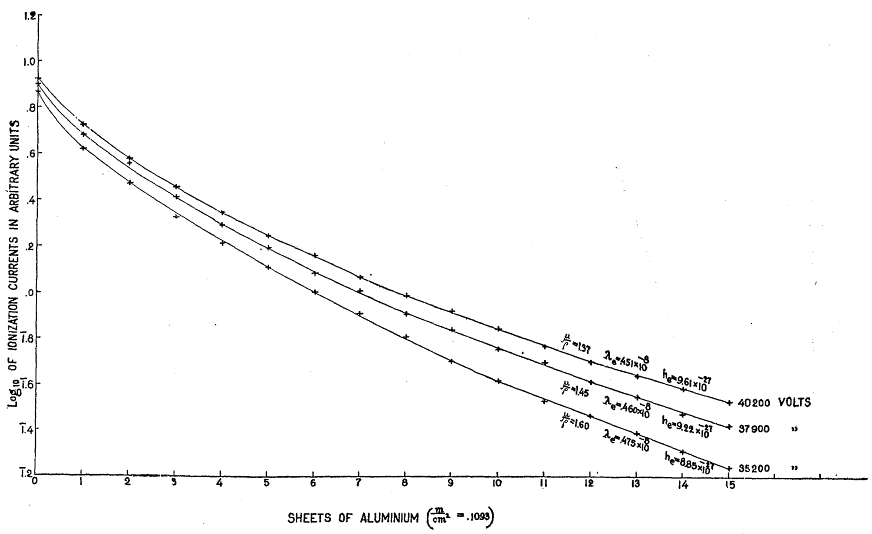

Fig. 1.

The following table contains the mass coefficients of absorption μ/ρ and the effective wave-lengths λ_e, frequencies γ_e and radiation constants h_e after the rays had passed through three millimeters of aluminium for voltages in the neighborhood of 35,000–40,000 volts.

Voltage.	Coefficient of Absorption, μ/ρ.	Effective Wave-Length, λ_e.	Effective Frequency, ν_e.	Effective Radiation Constant, h_e.
35,200	1.60	$.475\times10^{-8}$	6.32×10^{18}	8.85×10^{-27}
37,900	1.45	$.460\times10^{-8}$	6.52×10^{18}	9.22×10^{-27}
40,200	1.37	$.451\times10^{-8}$	6.65×10^{18}	9.61×10^{-27}

The value of h_e is in good agreement with that previously reported. It increases slightly as the voltage rises.

As stated above a constant voltage applied to a tube does not produce X-rays of a single wave-length; and we therefore set ourselves the problem of determining the minimum wave-length that can be produced by a given difference of potential. For this purpose we used an X-ray spectrometer, kindly loaned to us by Dr. D. L. Webster. The X-rays were reflected from the 100 planes of calcite ($CaCO_3$) into a small ionization chamber containing air and ethylbromide (C_2H_5Br). The formula

$$\lambda = 2a \sin \theta$$

gives the wave-length, where θ is the glancing angle of reflection, and $a = 3.04 \times 10^{-8}$ according to Bragg.

Two methods of procedure are open to us: either we may maintain the voltage at a constant value and measure the ionization currents due to the X-rays reflected at different angles, or we may set the spectrometer for a given angle and change the voltage. The latter appears to be the better method, for

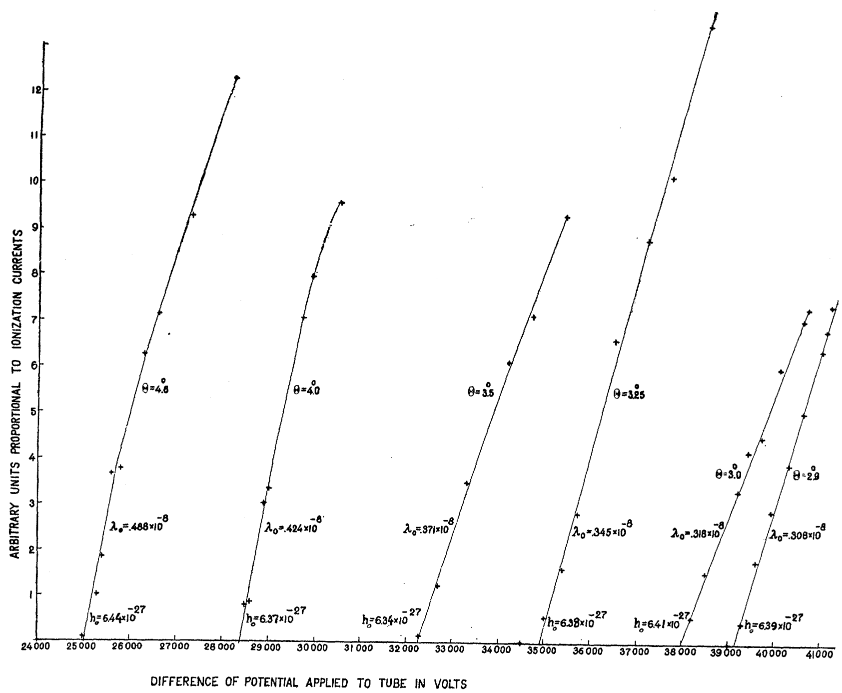

Fig. 2.

the ionization currents vary in a complicated way with the coefficients of absorption in the crystal and ethylbromide, etc., and with the coefficient of reflection from the crystal, and all of these coefficients vary with the wave-length and glancing angle.

We have made six series of measurements at six different glancing angles corresponding respectively to six different wave-lengths. The quantities measured were the voltage applied to the tube, the ionization current and the current passing through the tube. The last furnished a correction to be applied to the ionization current, for, other things being the same, the two currents are approximately proportional to each other.

After correcting for variations in the current flowing through the tube and for the natural leak in the ionization chamber we plotted six curves (Fig. 2) each representing the ionization current as a function of the voltage for a constant wave-length. Near the axis of zero ionization currents the curves are almost straight and meet this axis at certain definite points making apparently finite angles (less than 90°) with it. These points represent in each case the minimum voltage V_0 required to produce X-rays of the corresponding wave-length λ_0. If the voltage is less than V_0 no rays of the given wave-length λ_0 are produced, but if the voltage exceeds V_0 X-rays of that wave-length are always produced, no matter how high the voltage rises within the limits of our battery, and there is no reason to suppose that higher voltages would not also produce them. Our results show that the minimum voltage V_0 required to produce X-rays of wave-length λ_0 is given by the energy equation

$$V_0 e = h\nu_0 = \frac{hc}{\lambda_0},$$

where e = electron charge, c = velocity of light, and also that the energy radiated per second *at* this wave-length λ_0 is infinitely small.

The following table contains the experimental data and the values of h calculated from them.

V_0 (Volts).	θ_0	λ_0	ν_0	h
39,150	2.90	$.307\times10^{-8}$	9.77×10^{18}	6.39×10^{-27}
37,950	3.00	$.318\times10^{-8}$	9.44×10^{18}	6.41×10^{-27}
34,900	3.25	$.345\times10^{-8}$	8.69×10^{18}	6.38×10^{-27}
32,250	3.50	$.371\times10^{-8}$	8.08×10^{18}	6.34×10^{-27}
28,400	4.00	$.425\times10^{-8}$	7.06×10^{18}	6.37×10^{-27}
25,000	4.60	$.486\times10^{-8}$	6.18×10^{18}	6.44×10^{-27}

Corrections for the width of the slit and of the source of the rays amounting to an increase of $2'.5$ in the value of θ_0 have been added in calculating the values of h. The average value of h is $h = 6.39 \times 10^{-27}$, in very close agreement with the value given by Planck in the last edition of his book, namely 6.41×10^{-27}. We used an average value of $e = 4.71 \times 10^{-10}$ in calculating the energy of the cathode particles. Any error in estimating the grating constant a of the crystal enters, of course, into our measurement of λ_0. Our results furnish strong evidence in favor of the fundamental principle of the quantum hypothesis, for they show that rays of frequency ν are not produced unless the energy available equals $h\nu$, and also, incidentally, they prove the general correctness of W. L. Bragg's estimates of the distances between the atoms of crystals.

In attacking the problem by the other method mentioned above we kept the voltage as nearly constant as possible by adding cells to the battery as it gradually ran down, and measured the ionization currents with the spectrometer set for different angles corresponding to different X-ray wave-lengths. This work was done in collaboration with Dr. D. L. Webster. The curves (Fig. 3) representing the ionization currents (corrected as before for the natural

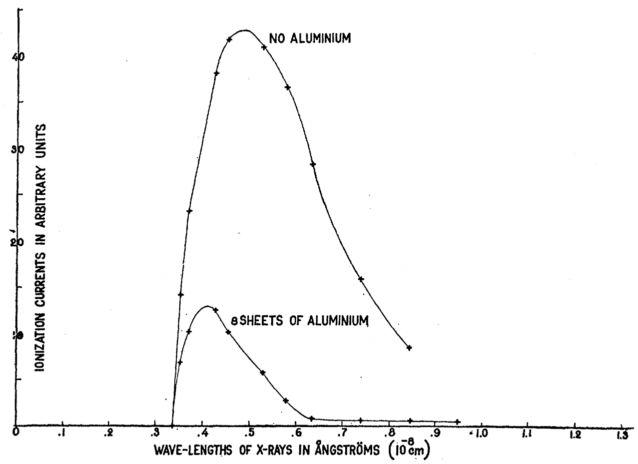

Fig. 3.

leak of the ionization chamber and variations in the current through the tube) as a function of the wave-length at constant voltage show that no energy is radiated at wave-lengths shorter than λ_0 given by Planck's radiation constant. As the wave-length increases beyond λ_0 the ionization current rises, reaches a maximum and then rapidly falls. As stated above these ionization currents do not represent accurately the energy radiated. There can be no doubt, however, but that the energy radiated between the wave-lengths λ and $\lambda + d\lambda$ reaches a maximum value at some wave-length λ_m.

A second curve (Fig. 3) represents the X-radiation that has passed through 3 mm. of aluminium in addition to the walls of the tube as a function of the wave-length. The maximum in the curve is much lower than before and has shifted toward the smaller wave-lengths, without, however, changing the value of λ_0.

To sum up: a *constant* difference of potential applied to an X-ray tube produces X-rays of a great variety of wave-lengths. The radiation is not homogeneous. We recognize three important wave-lengths and corresponding frequencies associated with the constant voltage. (a) The maximum limit of the frequencies ν_0, which multiplied by Planck's radiation constant equals the energy of a cathode particle as given by the product of its charge into the constant voltage applied to the tube. The energy radiated per second between the frequencies ν_0 and $\nu_0 - d\nu$ is, however, an infinitesimal of higher order than $d\nu$, apparently of the second, if $d\nu$ is of the first order. (b) The effective wave-length λ_e and frequency ν_e, which are the wave-length and frequency of

X-rays having the same coefficient of absorption as the whole radiation. λ_e decreases and ν_e increases as the rays pass through matter. The ratio of λ_e to λ_0 and of ν_0 to ν_e is about 1.50 at 40,000 volts and after the rays have passed through 3 mm. of aluminium. This ratio increases as the voltage increases and consequently the effective frequency is not proportional to the voltage. It increases less rapidly than the voltage. (*c*) The wave-length λ_m and frequency ν_m corresponding to the maximum energy radiation. These we have not yet determined accurately.

We have found no evidence that an appreciable number of electrical vibrators in the target store up energy from two or more cathode particles and then radiate it in the form of a quantum $h\nu$ of frequency higher than that corresponding to the energy of any one of them.

Harvard University,
Boston, Mass.

Magnetization by Rotation.[1]

By S. J. Barnett.

1. On the electron theory it is shown[2] that by a sort of molecular gyroscopic action a body of any magnetic substance becomes magnetized when set into rotation, the intrinsic magnetic intensity being uniform, parallel to the axis of rotation, proportional to the angular velocity, and (like the magnetization of the earth) directed oppositely to the intensity which would be produced by an electric current circulating around the body in the direction of rotation. It is shown further that a non-magnetic substance does not become magnetized. The theory of the converse effect is developed in a very simple manner. An experimental investigation of this effect is now in progress.

2. There is described a series of experiments, beginning in 1909, on the magnetization of iron rods by rotation. After the elimination of all suspected sources of systematic error there has been found and measured at different speeds an effect precisely similar to that required by the above theory, and inexplicable on any other theory hitherto proposed. The method of electromagnetic induction was used, a fluxmeter whose deflections were read to 0.1 mm. at the scale distance 8 m. being the chief measuring instrument. The intrinsic magnetic intensity of rotation, and the change of flux-density, per unit speed were found to be $3.1 \times 10^{-7} \frac{\text{gauss}}{\text{r.p.s.}}$ and $1.9 \times 10^{-5}\left(\frac{\text{maxwell}}{\text{cm.}^2}\right)$ per r.p.s., respectively. Experiments made for a different purpose by Lebedew[3] in 1912 show that this effect does not exist in the five non-magnetic substances investigated.

[1] Abstract of papers presented to the American Physical Society in December, 1914, and April, 1915.

[2] See also S. J. Barnett, Science, 30, 1909, p. 413, and A. Schuster, Proc. Phys. Soc. London, 24, 1911–12, p. 121.

[3] P. Lebedew, Ann. d. Phys., 39, 1912, p. 840.

3. Together with the change of (residual) flux proportional to the angular velocity, another change proportional to its square has been found and proved to be due to the radial expansion of the rod produced by the rotation.

4. The intensity of magnetization produced in the rotating iron per unit speed was about 1.5×10^{-6} c.g.s. unit per r.p.s. If the rod had been rotated at the speed of the earth its intensity of magnetization would have been about 2×10^{-10} that of the earth, and still less if the form had been spherical. This, however, does not prove that the earth's magnetization is not produced by the effect in question, as we are entirely ignorant of the magnetic properties of all substances under the conditions prevailing within almost the whole of the earth. Other causes are mentioned as probably accounting for at least part of the earth's magnetism. Schuster has pointed out that an effect of the kind investigated here may explain the secular variation as well as the mean magnetization of the earth.

5. Experiments, incidental to the work, on the use of a fluxmeter when direct and alternating flux changes occur separately or superposed are described.

OHIO STATE UNIVERSITY,
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