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The Rest Mass of the Mesotron

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The experimentally found values for the lifetime at rest of the cosmic-ray mesotron, from recent reliable investigations by various authors, are directly compared after recalculating them on the basis of the same assumed rest mass. The thus corrected values for the lifetime still disagree, but are shown to be a function of the average path length of the radiation employed in each experiment. When, instead of the existence of only one possible value for the rest mass for which we have no experimental evidence, a distribution in rest masses is assumed, the described phenomenon can be accounted for regardless of what form the mass distribution may have. The same assumption leads to explanations of other experimental findings, and predicts an actual lifetime τ_0 much smaller than any measured value, strongly indicating that upon correcting the measured results on the basis of the existence of a mass distribution, a value of the order of Yukawa's original prediction $\tau_0 = 5\times10^{-7}$ sec. is obtained.

IT must be admitted that a few cloud-chamber photographs of tracks of slowest observable mesotrons in a strong magnetic field constitute the whole basis of our present knowledge of the rest mass of the mesotron. These records have yielded estimates of the rest mass of from about 100 (Auger[1]) to as high as 540 (Williams and Pickup[2]) times the electron rest mass, values from 150 to 200 times m_e (mass of electron) having been observed most frequently. Thus, one may, at present, merely state that a value of, say, $180m_e$ is *most probable*, and even here we should allow for an error of, say, $\pm 20m_e$.

On the other hand, the determination of the time of decay τ_0 of the mesotron has been subject to numerous and comparatively accurate experimental investigations, although any determination thus made is at least as inaccurate as our knowledge of the rest mass. For in general we have, as regards loss by my mean life considerations alone,

$$-(1/\Delta n)(d\Delta n/dx) = (Mm_e/\tau_0 p), \qquad (1)$$

where Δn is the number of particles per square centimeter per second per unit solid angle falling within a small momentum range dp in the neighborhood of the momentum p appropriate to a group of particles, where x is the path traversed, τ_0 is the lifetime at rest, and M is the ratio of the rest mass of the particle to the electron's rest mass (m_e).

The momentum p is approximately determined by the energy of the particle, so that for a method of measurement which confines itself to a group of particles which, at the top of the atmosphere, have energy lying between E_0, and E_0+dE_0, it is obvious that the value obtained for τ_0 is intimately bound up with the value assumed for M.

[1] P. Auger, Comptes rendus **206**, 346 (1938).

[2] E. J. Williams and E. Pickup, Nature **141**, 684 (1938).

In considering formula (1) as applied to the aforesaid group of particles defined by E_0 and E_0+dE_0, we must suppose that a procedure the equivalent of the following is adopted: For any value of x representative of a distance measured in the atmosphere, the beam of cosmic rays to be measured is suitably defined by a telescopic arrangement and measurements are made below a thickness of lead which, in conjunction with the air path length x, permits passage only of rays of initial energy greater than E_0. Each measurement of this kind is then accompanied by another measurement with an additional thickness of lead such that the total path length in air and lead permits observation of only those rays which initially had energy greater than E_0+dE_0. The difference between corresponding measurements gives the number of rays which initially had an energy between E_0 and E_0+dE_0 and which have survived death by mean life considerations. If a new value of x be now chosen, either by inclining the telescopic system or by altering the altitude, we must suppose that the corresponding lead thicknesses are readjusted to maintain the same equivalent mass absorption thicknesses for combined lead and air path as before.

In work of this kind it has been customary for the most part to assume that the energies concerned are sufficiently high to permit the assumption

$$p=E/c.$$

It has further been customary to neglect the variation of p and so of E along the air path insofar as (1) is concerned, so that with these assumptions, (1) integrates to the form

$$\Delta n_x=\Delta n_0 e^{-kxM}, \qquad (2)$$

where
$$k=cm_e/\tau_0 E_0. \qquad (3)$$
Thus
$$\Delta n_1/\Delta n_2=\exp[-kM(x_1-x_2)]. \qquad (4)$$

From the known data for n_1/n_2, E_0, x_2 and x_1 and an assumption regarding M, τ_0 is determined.

A number of investigators have employed this method generally, although under differing conditions, and with differing apparatus, and these more recent determinations leave no doubt regarding their experimental accuracy. Despite this fact, the disagreement among these individual results exceeds the experimental accuracy.

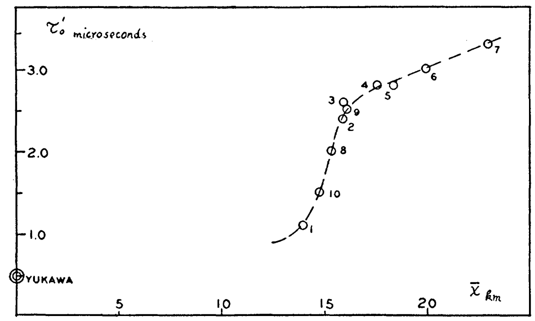

FIG. 1. Relation between measured "pseudo" lifetime at rest for the cosmic-ray mesotrons, by various authors, and the average path length $\bar{x}$ of the radiation in each experiment. (1) P. Auger (two altitudes and angles), see B. Rossi, Rev. Mod. Phys. **11**, 296 (1939). (2) S. de Benedetti (two angles), see B. Rossi, reference 1. (3) W. Kolhoerster and I. Matthes (atmospheric expansion), Physik. Zeits. **40**, 142 (1939). This is the only measurement using the barometric expansion of the atmosphere known to the author to be independent of a knowledge of the pure absorption coefficient, and of any influence of the electron or other component which is not subject to disintegration. The average momentum should be taken for sea level without absorber. (4) Bernardini and Bocciarelli (two angles), see reference 1. (5) H. V. Neher and H. G. Stever (two altitudes, with ionization chambers), Phys. Rev. **58**, 766 (1940). (6) M. A. Pomerantz (two angles), Phys. Rev. **57**, 3 (1940). (7) T. H. Johnson and M. A. Pomerantz (two angles), Phys. Rev. **55**, 104 (1939) see also B. Rossi, reference 1. (8) B. Rossi, N. Hilberry, and J. B. Hoag (two angles and altitudes), Phys. Rev. **57**, 461 (1940). (9) and (10) N. N. Das Gupta and P. C. Bhattacharya, Proc. Nat. Inst. Sci. India **6**, 713 (1940).

On the basis of early and rather inaccurate measurements of the angular distribution of the hard component, it was pointed out[3] that discrepancies arising relative to calculations on the basis of the disintegration theory may be due to our limited knowledge of the mesotron rest mass.

The above-mentioned determinations of various authors were used in a direct comparison by reconsidering the experimental results in each case on the basis of the same assumptions and, in particular, on the assumption of the same rest mass of $160m_e$, used by Rossi in connection with some of these measurements.[4] Upon careful consideration, the values thus obtained for the lifetime are not scattered "at random" but, as is shown in Fig. 1, show a marked relationship to the average path length $\bar{x}$, which was determined from the information of the various investigators on their experimental conditions.

[3] P. Weisz, Phys. Rev. **55**, 1266 (1939).
[4] B. Rossi, Rev. Mod. Phys. **11**, 149 (1939).

In addition to calling attention to the foregoing matters and to elucidating the dependence of *measured* mean life upon average path length in the experiments, the purpose of this paper is to show that the apparent paradox involved in this dependence may be resolved by assuming that for each energy there is a distribution of rest mass among the mesotrons.

Let us suppose that μ is the rest mass of a particle, m_e is the electronic mass, and M the ratio defined by

$$M=\mu/m_e.$$

Let us further suppose that, at the top of the atmosphere, there is, for the energy range E_0 to E_0+dE_0, a spectral distribution formula of the type

$$\delta(\Delta n_0)=\Delta n_0\varphi(M)dM, \tag{5}$$

where Δn_0 corresponds to the range dE_0 at the top of the atmosphere, and the form of $\varphi(M)$ may depend upon the energy, although, for reasons to be given presently, it appears that φ is the same for all energies, If we assume the lowest mass to be the electron mass, $\varphi(M)$, of course, satisfies the condition

$$\int_1^\infty \varphi(M)dM=1. \tag{6}$$

If $\delta(\Delta n)$ is the number of the group $\delta(\Delta n_0)$ which have survived mean life considerations after describing the path length x, we have

$$-\frac{d}{dx}[\delta(\Delta n)]=\frac{Mm_e}{\tau_0 p}[\delta(\Delta n)]. \tag{7}$$

It is possible to express the momentum p in terms of the energy E by the elimination of $\beta(=v/c)$ between the equations

$$E=Mm_ec^2[(1-\beta^2)^{-\frac{1}{2}}-1], \tag{8}$$

$$p=Mm_ec\beta/(1-\beta^2)^{\frac{1}{2}}, \tag{9}$$

where v is the velocity of the particle.

Through use of the Bloch formula, it is then possible to express E in terms of E_0 and x, and so realize an appropriate right-hand side for (7) as a function of x.

If, however, we stay within the limits of approximation which have been customary in this work, we may write the results of (8) and (9) in the form $p=E/c$. Moreover, we may write approximately, in place of the Bloch formula

$$E=E_0-\alpha h, \tag{10}$$

where α is a constant and h is the water equivalent of the path x. These approximations still leave the right-hand side of (7) a function of x through h. Thus (7) becomes

$$\frac{-1}{[\delta(\Delta n)]}\frac{d}{dx}[\delta(\Delta n)]=\frac{Mm_ec}{\tau_0(E_0-\alpha h)}. \tag{11}$$

Thus, considering as above any group of particles characterized by E_0 and the range dE_0, and by M and the range dM, we see that since $(E_0-\alpha h)$ is independent of M, the right-hand side of (11) is always greater the greater M. Hence, at each point of the path of the particles of range M to $M+dM$, the percentage loss of particles per unit path is greater the greater M. The result of this is that *the average rest mass of the particles decreases with increase of path traversed.*

Applying these ideas more specifically to the type of experiments which have been performed, we realize from (2) and (3) that the ordinary determination of mean life may be regarded as leading to a quantity which we shall call τ_0' defined by

$$\tau_0'=(x_1-x_2)(m_e\bar{M}c/E_0)/\log_e(\Delta n_1/\Delta n_2), \tag{12}$$

where $\bar{M}$ refers to the value of M used in the calculations, while Δn_1 and Δn_2 refer to the numbers of particles for the distances x_1 and x_2 corresponding to the two experiments, in which it will be recalled that the water equivalent path length for each is made the same by choice of suitable thicknesses of lead.

On the basis of acceptance of a mass distribution implied by (5) and with neglect of energy loss along the path our formula (11) leads to

$$\delta(\Delta n)=\delta(\Delta n_0)e^{-kMx} \tag{13}$$

or, in view of (5),

$$\delta(\Delta n)=\Delta n_0e^{-kMx}\varphi(M)dM, \tag{14}$$

where Δn_0 refers to the top of the atmosphere and k has the value given by (3). Equation (14) leads to

$$\Delta n=\Delta n_0\int_1^\infty \varphi(M)e^{-kMx}dM. \tag{15}$$

It is our purpose to express the pseudo τ_0', which depends upon the average path length of the measurements, in terms of the true τ_0 which is independent of this path length. Writing $\bar{x}$ as the average path length, and Δx as the difference in path length for the two experiments, we have, from (12) and (15)

$$\tau_0' = \frac{m_e \bar{M} c \Delta x}{E_0} \Bigg/ \log_e \left[\frac{\int_1^\infty \varphi(M) \exp[-kM(\bar{x}-\Delta x)] dM}{\int_1^\infty \varphi(M) \exp[-kM(\bar{x}+\Delta x)] dM} \right], \tag{16}$$

so that

$$\frac{1}{(\tau_0')^2} \frac{d\tau_0'}{d\bar{x}} = \frac{1}{\tau_0 \bar{M} \Delta x} \left[\frac{\int_1^\infty M\varphi(M) \exp[-kM(\bar{x}-\Delta x)] dM}{\int_1^\infty \varphi(M) \exp[-kM(\bar{x}-\Delta x)] dM} - \frac{\int_1^\infty M\varphi(M) \exp[-kM(\bar{x}-\Delta x)] dM}{\int_1^\infty \varphi(M) \exp[-kM(\bar{x}+\Delta x)] dM} \right]. \tag{17}$$

In other words, having due regard to the meanings of the integrals, we see that $d\tau_0'/d\bar{x}$ is proportional to

(average mass at $\bar{x}-\Delta x$)
$-$(average mass at $\bar{x}+\Delta x$).

Since, as already shown, the average mass decreases with $\bar{x}$, we conclude that $d\tau_0'/d\bar{x}$ is positive. In other words, it is to be expected that the pseudo lifetime, calculated on the basis of neglect of the mass distribution law, should increase with average path length. This is exactly the conclusion represented by the experimental data shown in Fig. 1, which are consequently in harmony with the idea here proposed as to the existence of a spectral distribution in the rest masses of the mesotrons.

The departure of τ_0' from τ_0 is obviously a function of the form of $\varphi(M)$. Now Pomerantz,[5] in the course of very accurate measurements, has recently shown that the lifetime measured under the same conditions, but for different absorber thicknesses, i.e., different momenta, is the same, as the slight discrepancy which existed could be accounted for quantitatively as well as in character by transition effects and scattering, according to Pomerantz and Johnson.[6] Also, using Bruins' data and method,[7] i.e., the comparison of the vertical intensity (counter arrangement) with the "all-around" intensities (ionization chamber) underground one finds the same values of the lifetime at different thicknesses of dense absorber

If there existed an essential variation of the mass distribution $\varphi(M)$ with energy, this would result in the determination of different values for τ_0 if in the same kind of experiment, i.e., the same average path length, radiation of different initial energy E_0 is dealt with, and if the variation in $\varphi(M)$ stays unconsidered, as is the case when a constant value is assumed.

Therefore, we may conclude from the quoted measurements[5–7] that there exists no essential variation of the form of $\varphi(M)$, at least not in the region of high energies where these experiments were performed. Hence, there should be no objection to comparing directly the values for the lifetime as measured by different authors for components of the cosmic radiation which had different initial energies in the various experiments.

It seems worth mentioning that on account of the phenomenon of decreasing average rest mass with path length, the real lifetime τ_0 is smaller than any measured pseudo lifetime, unless we had some means of making the measurement at or very close to the origin of the radiation ($x=0$). On roughly extrapolating the relation in Fig. 1, found for the pseudo lifetime τ_0' with path length $\bar{x}$ towards $\bar{x}=0$, one may actually obtain a value of τ_0 corresponding to a value of about

[5] M. A. Pomerantz, Phys. Rev. **57**, 3 (1940).
[6] M. A. Pomerantz and T. H. Johnson, Phys. Rev. **59**, 143 (1941).
[7] E. M. Bruins, Proc. Ned. Akad. Wet. **18**, 75 (1940).

$\tau_0 = 5 \times 10^{-7}$ sec., which is Yukawa's original estimate of the mesotron's lifetime.

There are a great number of consequences, of course, which follow from the assumption of a distribution in rest mass, a few of which will be discussed in brief.

Recent investigations[8] have revealed that cloud-chamber photographs of highly ionizing particles are obtained more frequently at high than at low altitudes in the atmosphere, contradictory to the present knowledge concerning the number of slow mesotrons in the atmosphere. As has already been explained, the average mass of all mesotrons must decrease with path length, i.e., it will increase with altitude in the atmosphere, and this could explain the increasing frequency of heavy ionized tracks. For the rest mass goes in as a parameter in the Bloch formula, where it becomes of great importance at low energies.

This expresses itself in measurements of the energy loss of low energy particles ($<10^9$ ev electron energy) when traversing an absorber, such as Anderson and Neddermeyer have carried out, and which were used as part of their original evidence for the existence of a rest mass greater than that of the electron. Their improved experimental data,[9] indeed show a considerable parameter "scattering" of the experimental points attributed to the "heavy particles." For we can estimate the natural fluctuations including experimental error, statistics, etc., from their data for ordinary electrons to be smaller than 100 Mev while the heavy particle data are scattered over a parameter of as much as 300 Mev and more.

In all phenomena described above, the particular shape of the distribution function $\varphi(M)$ is essentially unimportant. However, there are a number of other phenomena for which this will be of particular interest. In this connection, W. F. G. Swann[10] has pointed out the possibility of an explanation of the secondary peak in the Rossi curve on the basis of different mesotron rest masses. Other effects, as for example a fine structure of the angular distribution, may find their explanation on the same grounds.

It is a pleasure to me to take this opportunity to thank Dr. W. F. G. Swann for his interest and his steady guidance in the mathematical formulation of the problem.

Note added in proof: The variation of energy along the path of the radiation due to ionization loss was neglected when showing theoretically that a distribution in rest masses tends to increase the measured lifetime at rest with increasing average path traversed. It was not clearly expressed, however, that the values for the measured lifetime at rest at various average path lengths from determinations of different authors were calculated, and *the increase of the measured pseudo lifetime with average path length (presented in Fig. 1) was thus found due consideration being given to the variation of energy along the path of the mesotrons.*

[8] G. Herzog, Phys. Rev. **59**, 117 (1941); G. Herzog and W. H. Bostick, Phys. Rev. **59**, 122 (1941).

[9] S. H. Neddermeyer and C. D. Anderson, Rev. Mod. Phys. **11**, 196 (1939).

[10] W. F. G. Swann, paper read before the American Philosophical Society at Philadelphia, on November 22, 1940.