exciting the vibrations. Thus the external field is not zero, and this produces another shift in the fundamental frequency, of the same order of magnitude but opposite to that contemplated here. However, Cady has shown that this correction decreases as $1/n^2$ and since n is very large in the experiments under review, this correction is negligible.

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Note on the Structure of Liquid Helium

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1. The existence of surface transfer films of liquid helium II of the order of 100 atoms in thickness[1] has sometimes seemed difficult to understand, although actually this thickness is in accord with the usual theory of interatomic forces. The difference between the $1/r^6$ van der Waals interactions of two helium atoms and of a helium atom and an atom of the solid container yields a $1/a^3$ dependence on the distance from the surface. In the case of a metallic container, there is also a comparable $1/a^3$ energy arising from interaction with the electrons of the metal.[2] If a solid surface projects out of a reservoir of liquid helium II, the thickness a_h of the film at a height h cm above the free surface of the liquid is obtained by equating the total interaction energy to the gravitational potential energy of a helium atom. This gives for copper: $a_h = 4.3 \times 10^{-6}/h^{\frac{1}{3}}$ cm, which is in excellent agreement with the rather uncertain experimental value of 3.5×10^{-6} cm for a sheet extending a few cm above the liquid surface.[1] The numerical value 4.3 for copper is replaced by 4.7 for silver, ~ 4 for glass, and 2.2 for rocksalt (100 plane exposed). Although such films probably exist in general, this simple calculation is valid only when the fluid under consideration is sufficiently nonviscous so that it can come into equilibrium under these forces before it evaporates; this explains the failure to observe such films with helium I.[1] Accurate measurements of film thickness would give useful information about long range interatomic forces.

2. The phase transition in liquid helium appears to be a continuous one in the sense that all properties of the state that do not involve differentiation with respect to the temperature are continuous.[3] It is therefore plausible to assume that helium I consists of a single phase A and helium II of a mixture of two phases A and B, phase B being absent at the λ-point and growing continuously at the expense of phase A as the temperature decreases. This point of view was proposed by Tisza[4] and developed by H. London;[5] they showed that the experiments could be understood, qualitatively at least, by assuming that A is disordered and has a normal viscosity, while B is ordered and has little or no viscosity. However, there appears to be no need to assume, as has sometimes been done,[5, 6]

that the liquid in the surface layer is qualitatively different from that in the volume of the fluid, since the capillary experiments can be understood equally well in terms of the viscous drag exerted by a nearby wall on phase A. Moreover, a different structure of the surface layer could be expected only if the interaction of an atom with the wall were comparable with the interaction with its neighbors. The calculations of paragraph 1 indicate that this is true only for the first 2 or 3 atom layers, while the thermomechanical effect is appreciable for much larger channels.[7]

3. The phase A referred to in paragraph 2 can be understood in terms of an Einstein-Bose gas model, in which each helium atom moves in the periodic field that approximately represents the effect of correlations between the positions of the atoms. This model explains the x-ray data, the position of the λ-point[8] and the positive temperature coefficient of viscosity of helium I.[3, 4] Its application to phase B does not, however, explain the observed specific heat anomaly,[9] and the failure of a more complete treatment[10] to accomplish this makes it seem likely that no gas model will succeed here. It is natural then to assume a more typically liquid model for phase B, in which there exists a discrete energy level for each atom. Because of the weak attractive forces and large zero-point kinetic energy of helium atoms there will not be more than one such state per atom, if any, and it would probably lie $\sim 10^{-4}$ ev below the continuum of phase A states. These "bound" atoms would diffract over several atom distances, so that the aggregation would not possess the rigidity of a lattice. The energy of a given bound atom could depend on the number of its neighbors that are similarly bound; the inclusion of an "ordering energy" of this type would favor the transition from A to B and sharpen the specific heat curve. Further, volume-conserving deformations would not alter the energy of phase B, and the adiabatic nature of laboratory manipulations as compared with the period of the system ($\sim 10^{-10}$ sec. for B states depressed by 10^{-4} ev) would make excitation to continuum states extremely improbable. There would thus be no resistance to deformation, no mechanism for dissipating flow energy, and hence no viscosity. While this model has some physical plausibility to recommend it, it must be regarded as tentative because of the great difficulty of calculating the several parameters that appear in it in terms of atomic properties.

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