Positrons from Light Nuclei

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Four nuclei of the type discussed by Wigner, containing one more proton than neutron, have been produced. The nuclei, their half-lives and positron energies are: P^{29}, 4.6 seconds, 3.63 Mev; S^{31}, 3.2 seconds, 3.85 Mev; Cl^{33}, 2.4 seconds, 4.13 Mev; and A^{35}, 2.2 seconds, 4.38 Mev. These energies are in excellent agreement with the calculated values and indicate that (1) the $\pi-\pi$ force is equal to the $\nu-\nu$ force plus the Coulomb repulsion, (2) the nuclear volumes are proportional to the atomic weights, and (3) the nuclear charge is distributed uniformly through the volume. The half-life-energy relation is in good agreement with the Fermi theory. Deviations are in the direction anticipated by the Gamow-Teller modification.

INTRODUCTION

THE difference in binding energy of isobars which differ only in the interchange of the protons with the neutrons has been supposed on theoretical grounds to consist almost entirely of the extra Coulomb energy of the nucleus with the higher charge. Assuming that the $\pi-\pi$ force exceeds the $\nu-\nu$ force only by the Coulomb repulsion, and that the charge on the nucleus is distributed uniformly through a volume proportional to the mass number, one may calculate within a multiplicative constant this binding energy difference, and hence the maximum energy of the positrons emitted in the transition to the lighter nucleus.[1,2] The constant in this proportion, and consequently the nuclear radius, may be evaluated from the experimental data on the energies of beta-rays from nuclei of this particular type, i.e., nuclei which contain one more proton than neutron. We have produced four more of these positron emitting isotopes, which are discussed separately in the following paragraphs.

P^{29}

The production of this isotope, as well as of the others to be discussed, was made possible by increasing our available proton energy from $\sim$6.3 to $>$8.6 Mev. Silicon crystals were pressed into clean lead sheet and bombarded in an atmosphere of hydrogen for times averaging about 3 seconds, with $\sim$0.5 μA of protons. Silicon exists as three stable isotopes, $Si^{28, 29, 30}$. We believe the short period activity produced is P^{29} rather than P^{28} because of the agreement of the half-life and energy with the predictions of theory. However, since P^{30} is also produced when silicon is bombarded with protons, it was necessary to keep the bombardment time to a minimum to eliminate as much as possible of this 2.5-minute activity. The β-ray spectrum was obtained in a cloud chamber filled with helium and alcohol vapor, and with a magnetic field of 714 oersteds. Only one photograph per bombardment was made. The proton

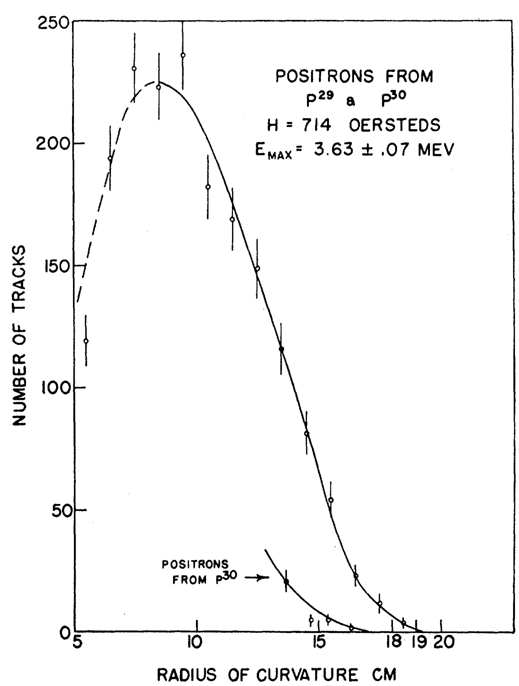

FIG. 1. Positrons from P^{29} and P^{30}. Vertical lines through the points estimate the probable error, as the square root of the numbers of tracks.

[1] E. Wigner, Phys. Rev. **56**, 519 (1939).
[2] White, Delsasso, Fox and Creutz, Phys. Rev. **56**, 512 (1939).

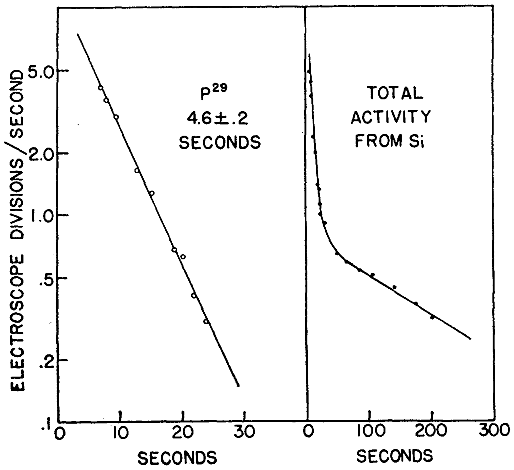

FIG. 2. Decay curve of P^{29}.

beam was shut off during expansion by a cam controlling the ion source, thus eliminating the gamma-ray background of the cyclotron. Targets were placed manually in a well in the chamber within three seconds after bombardment stopped. A large number of targets were used in rotation so that the P^{30} activity would not build up after repeated short exposures to the beam. It was hoped that the energy of P^{29} would be sufficiently higher than that of P^{30} so it could be distinguished. That this was possible may be seen from Fig. 1, where the momentum spectrum of all positrons from the two reactions $Si^{29,30}(p, n)P^{29,30}$ is plotted as well as the upper end of the spectrum of positrons from P^{30} alone, the latter being obtained when the proton energy was below the threshold for production of P^{29}. (The total number of tracks in the two cases was closely the same.) After adding the energy lost in the foil window of the cloud chamber, the upper limit for P^{29} positrons is 3.63 ± 0.07 Mev.

The half-life was obtained by repeatedly photographing a stop watch dial and the image of the fiber of a projection-type Lauritsen electroscope on which the activated target was placed; the cyclotron being turned off after the bombardment was completed. Unfortunately the P^{30} background always had to be subtracted from the decay curves, thus increasing the probable error of the measurement. The half-life, as is seen in Fig. 2, is 4.6 ± 0.2 seconds.

S^{31}

After bombarding iron for various times and finding no appreciable activity, iron-phosphorus alloy was chosen as the target for the production of this isotope. The experimental method was the same as for P^{29}, except that here the problem was much simpler; since P^{31} is the only stable isotope of phosphorus, S^{31} is the only likely radioactive nucleus produced by proton bombardment. From Fig. 3 the upper limit of the positrons is seen to be 3.85 ± 0.07 Mev. Figure 4 shows a typical decay curve, giving a half-life of 3.2 ± 0.2 seconds[3] as the mean of several such curves.

To be certain that the electroscope method is a valid one for the measurement of these short half-lives, in spite of the finite collection time of the ions produced in the instrument, a check was made in the following way. A 15-mC radium source was suspended over the electroscope window by a cord passing over a pulley and thence to a drum whose speed of rotation was proportional to the velocity of a chronograph tape on which the passage of the electroscope fiber over the scale divisions could be recorded. Readings of the activity were first made with the radium stationary at various heights above the

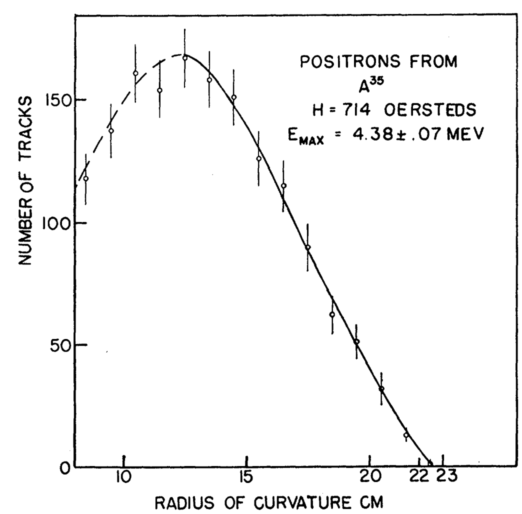

FIG. 3. Positrons from S^{31}.

[3] Since completion of this work the half-life of this isotope, produced by $Si^{28}(\alpha, n)S^{31}$, was reported by King and Elliott at the Chicago meeting of the American Physical Society, November, 1940 (Phys. Rev. **59**, 108A (1941)), as 3.18 seconds. Although their method of measurement was different from ours, their value agrees well with ours.

electroscope. A plot of activity against height was then made. This plot was compared with that obtained when the radium source was in constant motion away from the window. The second case corresponds roughly (insofar as the rate of change of ionization in the electroscope is concerned) to a decaying source. Comparison of the two curves shows whether or not the rate of decrease of activity measured by the electroscope actually corresponds to the rate at which the incident radiation is decaying. The agreement was excellent up to speeds corresponding to half-lives of three seconds and was within 5 percent for a half-life of 1.6 seconds. This fact has been considered in estimating our probable errors.

Cl^{33}

This isotope was reported by Hoag,[4] produced by $S^{32}(D, n)Cl^{33}$. In our laboratory sulfur was melted onto lead strips and bombarded with protons to obtain the reaction $S^{33}(p, n)Cl^{33}$. This activity, like the preceding, was a "clean" one, i.e., no other periods were produced in any quantity. Sufficient proton energy is not available* to produce $S^{32}(p, n)Cl^{32}$, while Cl^{34} and Cl^{36} gave no difficulty because of their long lifetimes. An additional possibility is that the isotope produced is P^{29} by $S^{32}(p, \alpha)P^{29}$, and that the supposed P^{29} obtained when we bombard Si is actually P^{28}. These two coincidences with the

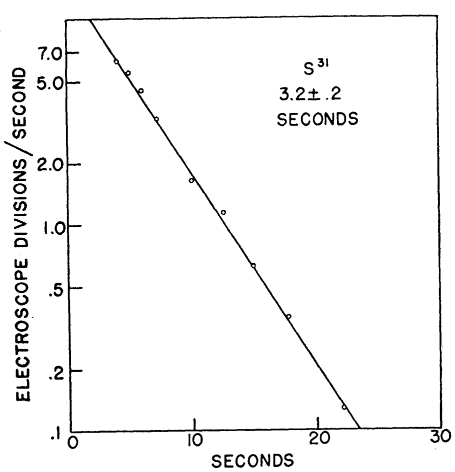

FIG. 4. Decay curve of S^{31}.

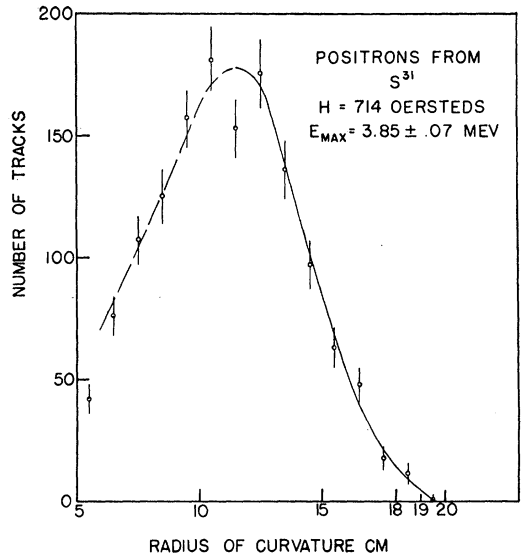

FIG. 5. Positrons from Cl^{33}.

energies expected from Cl^{33} and P^{29} (see discussion) would, however, seem very improbable. The upper limit of the positrons is seen from Fig. 5 to be 4.13 ± 0.07 Mev. Figure 6 shows a half-life curve. The value is 2.4 ± 0.2 seconds.

As in all experimental work, the accurate determination of the upper limits of beta-ray spectra by the cloud-chamber method requires that extreme precautions be taken to guard against subjective errors. It was expected, before these experiments were made, that the energies would increase with the atomic number of the element concerned. Since the same cloud-chamber magnetic field was used for all four elements, it would be very easy unconsciously to "look for" tracks of slightly higher radius of curvature with each successive element. A very large percentage of the tracks photographed are not measured because they are diffuse, scattered, too short, or do not clearly originate in the target. Therefore whether or not a track is counted is necessarily very much a matter of personal choice in deciding whether or not it is a "good" one. Probably the best way to insure against any unintentional cheating would be to mix all the pictures of all the elements before measuring them, so that the observer would not know whether he "ought" to find high energy tracks on a certain film or not. This was not done, but various other precautions were taken. The reprojected tracks were com-

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[4] J. Barton Hoag, Phys. Rev. **57**, 937 (1940).

* From Barkas' masses, the threshold for this reaction is ~13 Mev.

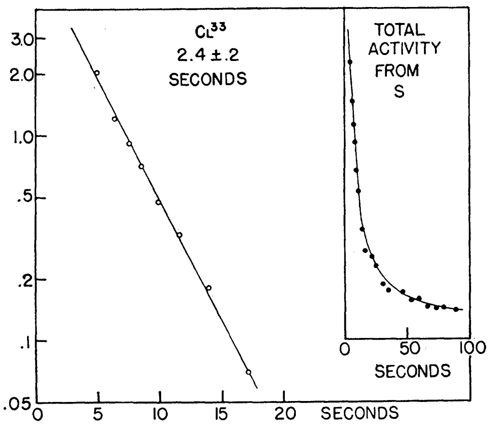

FIG. 6. Decay curve of Cl^{33}.

pared with arcs of circles printed in black ink on white cards, each card having un-numbered arcs of radii from 5 cm to 25 cm in $\frac{1}{2}$-cm steps. When a track was found to fit a certain arc better than the next one on either side, a pencil mark was made on that arc. As soon as enough marks had been made on one card so that the observer began to be aware of any "shape" to their distribution, a new card was taken. Thus for any spectrum, from 15 to 30 cards were used. Furthermore, it is quite difficult, if not impossible, to tell from looking at a track whether its radius is 18, 20, or 22 cm for instance, until it is compared with a measured arc. All distributions were plotted before the theoretical energies were calculated. The curves were finally drawn in through the points on sheets with the abscissae un-numbered and the spectra un-named. Under these conditions different people well within the probable error obtained the same upper limit from the data.

A^{35}

Since bombardment of lead has not shown appreciable activity, lead chloride was used as the target material to produce the reaction $Cl^{35}(p, n)A^{35}$. The only contaminating activity was of long period, possibly A^{37}. The momentum spectrum of the positrons accompanying the decay back to Cl^{35} is shown in Fig. 7, giving an upper limit of 4.38 ± 0.07 Mev. The half-life, as averaged from curves such as that given in Fig. 8, is 2.2 ± 0.2 seconds.[5] In this case, as well as in the preceding, consideration must be made of the failure of the electroscope to follow such rapidly decaying activity. This correction has been made, amounting to about 0.1 second.

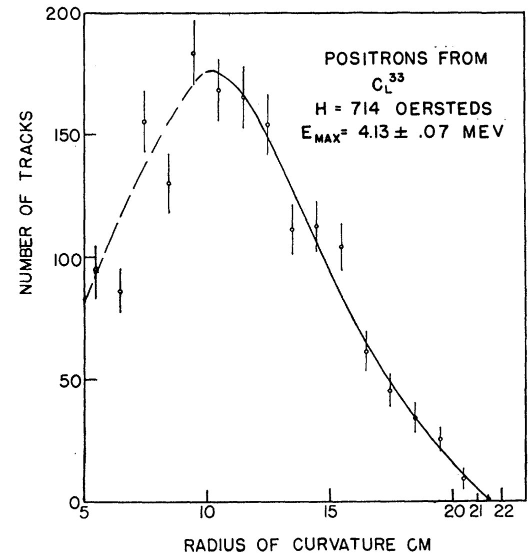

FIG. 7. Positrons from A^{35}

DISCUSSION

All nuclei of this type which can be made by (p, n) reactions, with the exceptions of Na^{21} and Ca^{39} have now been studied.[6] The half-life but not the energy of Na^{21} has been measured.[7] In Fig. 9 the measured positron energy of these

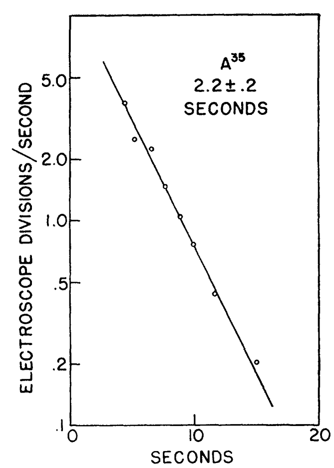

FIG. 8. Decay curve of A^{35}.

[5] L. D. P. King and D. R. Elliott, reference 3, quote 1.91 seconds for the half-life of this nucleus, as produced by $S^{32}(\alpha, n)A^{35}$.

[6] L. D. P. King and D. R. Elliott, reference 3, report 0.87 second for the half-life of Sc^{41}, made by $Ca^{40}(D, n)$.

[7] Creutz, Fox and Sutton, Phys. Rev. **57**, 567 (1940).

nuclei is plotted against that calculated on the assumptions stated in the introduction, namely that (1) the $\pi-\pi$ force is equal to the $\nu-\nu$ force plus the Coulomb repulsion, (2) the charge is uniformly distributed throughout the volume, and (3) the nuclear volume is proportional to the mass number. The theoretical values are calculated under the assumption that the nucleus has Coulomb energy[1]

$$E=\tfrac{1}{2}\times 2.35\times z(z-1)A^{-\frac{1}{3}}mc^2.$$

The difference in this quantity for two nuclei of charge z and $z-1$ is then

$$\Delta E=2.35(z-1)A^{-\frac{1}{3}}mc^2.$$

The constant was chosen to give the best visual fit to the data. To obtain the expected upper limit of these beta-ray spectra, one must subtract from this available energy two electron masses and the neutron-proton mass difference. Converting to Mev this gives for the upper limit, since $A-1=2(z-1)$,

$$E_{\text{max}}=6.0(A-1)A^{-\frac{1}{3}}-1.78 \text{ Mev},$$

provided the decay is to the ground state of the final nucleus. This is assumed since in previous work on similar nuclear types no gamma-radiation was definitely found.[2,8] Deviation from a straight line would indicate that these assumptions are invalid. The agreement in most cases is good enough to make another investigation of the worst cases (15, 25, 27) worth while.

The transition probability for β-decay, according to Fermi's theory,[9] is given by

$$\lambda=GI(\omega),$$

where G depends on the matrix element for the transition and ω is the maximum total positron energy. I is roughly $\omega^5/30$. For this reason the half-lives should be about proportional to ω^{-5}. In Fig. 10 are plotted the observed half-lives to the minus one-fifth power, against the observed ω. $I(\omega)$ has actually been more accurately calculated from formulas 1(a) and (2) of reference (1), and in Table I are given values of this quantity as well as the observed half-lives multiplied by $I(\omega)$. Figure 11 shows a plot of $I(\omega)\cdot t$ against z. It may be of some interest to point out that the proton and neutron form an isobaric pair of this series. If we assume the Fermi theory to hold for the decay of the neutron by electron emission, using an average value of It from Fig. 11 with the neutron-proton mass difference of $1.5mc^2$, we estimate the half-life of the neutron to be roughly one-half hour.

The following discussion was written by Professor Wigner.

The fact that the quantity in the last column in Table I is essentially constant can be most simply interpreted by assuming that the matrix element of the transition to the normal state of

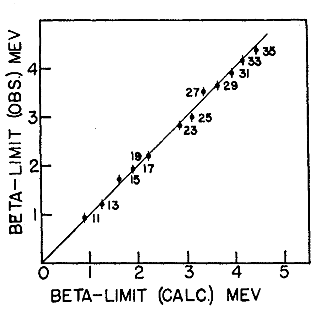

FIG. 9. Observed and calculated energies of positrons from nuclei with one more proton than neutron. Vertical lines estimate the probable error.

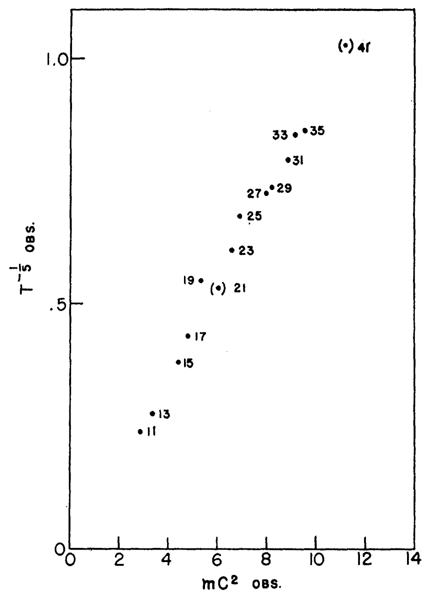

FIG. 10. (Observed half-life)$^{-1/5}$ plotted against the observed total positron energy. Calculated energies were used at points 21 and 41.

[8] Richardson presented cloud-chamber evidence for gamma-radiation from N^{13} (Phys. Rev. **53**, 610 (1938); **55**, 609 (1939)). Valley, using a magnetic spectrograph, failed to find such radiation (Phys. Rev. **56**, 839 (1939)).

[9] E. Fermi, Zeits. f. Physik **60**, 320 (1934).

the product nucleus is essentially constant throughout the series and that this transition determines at least the order of magnitude of the lifetime. Both these assumptions follow from Fermi's theory of β-decay and also from Gamow and Teller's modified form if one assumes, in the latter case, that the effect of the spin-dependent forces on the wave functions is small.

There are several phenomena which seem to support and others which appear to contradict the above theories. *We shall discuss them here only insofar as the experiments have a bearing on them.*

Fermi's original theory would require that the numbers in the last column be all exactly equal if the $\pi-\pi$, $\nu-\nu$ and $\pi-\nu$ interactions are all equal. The variations within the last column must be interpreted then on the Fermi theory as being caused by modifications of the wave function resulting from differences between the above forces. The variation within the last column is perhaps somewhat bigger than one would expect on this basis. In this theory, from the normal state of any of the radioactive nuclei of this series, only the transition into the normal state of the product nucleus is "allowed."

In the Gamow-Teller modification of Fermi's theory one has to consider the doublet structure of the normal states. From a $^2P_{3/2}$ normal state of, say, C^{11}, not only the transition into the $^2P_{3/2}$ normal state of B^{11} is "allowed" but also the transition into the $^2P_{1/2}$ state of this nucleus. The situation is similar in the other cases. The matrix elements of both transitions can be calculated.

TABLE I. *Values of $I(\omega)$ and of $I(\omega)\times t$.*

Z	$(\omega)_{OBS}$ (mc^2)	$I(\omega)$	t_{OBS} (SEC.)	$I(\omega)\times t_{OBS}$
11	2.86	3.46	1230	4330
13	3.37	10.2	630	6430
15	4.37	42.1	125	5260
17	4.78	66.3	64	4230
19	5.31	116	20.3	2360
21	6.00*	222	23	6920
23	6.52	351	11.6	4070
25	6.85	452	7	3160
27	7.93	967	4.9	4730
29	8.16	1100	4.6	5060
31	8.75	1590	3.2	5080
33	9.14	2010	2.4	4680
35	9.57	2550	2.2	5610

* Calculated.

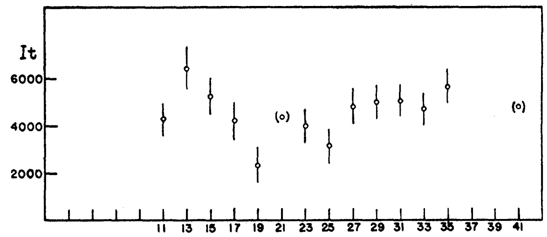

FIG. 11. Fermi's $I(\omega)$ calculated from observed values of ω (=total positron energy), multiplied by the observed half-life. The calculated ω was used for the points at 21 and 41.[10]

Since the energy of transition into the normal state of the product nucleus is measured, the transition probability into this state can also be calculated. Since, however, the energy of transition into the upper state is, in general, not known, the transition probability into this state cannot be evaluated. As the lifetime of the radioactive nucleus is the reciprocal of the sum of both transiton probabilities, it cannot be evaluated exactly. However, in most cases, the transition into the normal state will play a predominant role and one will expect, therefore, that the total transition probability will be relatively largest where the matrix element of the transition into the normal state is largest.

This appears to be in agreement with observation. The matrix element is largest ($\frac{3}{4}$) in the case of a $^2S_{1/2}-{}^2S_{1/2}$ transition. One has reason to suppose that this takes place in the case of the $Ne^{19}-F^{19}$ transition, and, in fact, the last column (which is inversely proportional to the transition probability) is smallest in this case. The matrix element is smallest ($\frac{1}{12}$) in the case of the $^2P_{1/2}-{}^2P_{1/2}$ transition, which is assumed to take place for $n=13$ and $n=15$. The last column shows large values in these cases. In case of the other transitions the matrix element has intermediate values.

[10] References for Fig. 11: $A=11$, Delsasso, White, Barkas and Creutz, Phys. Rev. **58**, 586 (1940); $A=13$, Kikuchi, Watasi, Itoh, Takeda and Yamaguchi, Proc. Phys. Soc. Japan **21**, 52 (1939) and Lyman, Phys. Rev. **51**, 1 (1937); $A=15$, Fowler, Delsasso and Lauritsen, Phys. Rev. **49**, 561 (1936); $A=17$, Curie, Richardson and Paxton, Phys. Rev. **49**, 368 (1936); $A=19$, 23, 25, White, Delsasso, Fox and Creutz, Phys. Rev. **56**, 512 (1939); $A=21$, Creutz, Fox and Sutton, Phys. Rev. **57**, 567A (1940); $A=27$, McCreary, Kuerti and Van Voorhis, Phys. Rev. **57**, 351 (1940), Barkas, Creutz, Delsasso, Sutton and White, Phys. Rev. **58**, 383 (1940).