On the Exchange Properties of the Neutron-Proton Interaction

WILLIAM RARITA* AND JULIAN SCHWINGER
Department of Physics, University of California, Berkeley, California

(Received February 17, 1941)

To determine the exchange properties of the neutron-proton interaction, the existing knowledge of the neutron-proton force in states of even parity, gained from the properties of the deuteron ground state and low energy scattering experiments, must be supplemented by information concerning the interaction in odd parity states, information which can be obtained only by observations on high energy neutron-proton scattering and deuteron photo-disintegration by energetic γ-rays. Calculations have been performed for three types of interactions with the purpose of testing the sensitivity of such experiments to variations of the exchange operator dependence of the interaction. These interactions are analogous, in isotopic spin dependence, to the potentials predicted by three forms of current mesotron theory: (I) "Symmetrical" (II) "Charged" (III) "Neutral." With the interaction in even parity states described by rectangular well potentials with constants adjusted to fit the binding energy and quadripole moment of the deuteron, and the cross section for slow neutron-proton scattering, each of these three potentials makes a definite prediction concerning the interaction in odd parity states. The scattering calculations were performed for a neutron energy of 15.3 Mev. The results for total cross sections and angular distributions in the center of mass system are as follows:

(I) $\sigma=0.621\times10^{-24}$ cm^2,
$\sigma(\vartheta)\sim(1-0.080\cos\vartheta+0.077\cos^2\vartheta)$

(II) $\sigma=0.666\times10^{-24}$ cm^2,
$\sigma(\vartheta)\sim(1+0.126\cos\vartheta+0.042\cos^2\vartheta)$

(III) $\sigma=0.983\times10^{-24}$ cm^2,
$\sigma(\vartheta)\sim(1+0.932\cos\vartheta+0.457\cos^2\vartheta)$.

The energy of the Li+H γ-rays ($\hbar\omega=17.5$ Mev) was adopted for the computations on photo-disintegration. The three theories under discussion predict the following electric dipole total cross sections and angular distributions

(I) $\sigma=0.768\times10^{-27}$ cm^2, $\sigma(\vartheta)\sim(\sin^2\vartheta+0.015)$
(II) $\sigma=0.723\times10^{-27}$ cm^2, $\sigma(\vartheta)\sim(\sin^2\vartheta+0.077)$
(III) $\sigma=0.376\times10^{-27}$ cm^2, $\sigma(\vartheta)\sim(\sin^2\vartheta+0.36)$.

Calculations have also been performed for the small cross sections arising from magnetic dipole and electric quadripole absorption. The spherically symmetrical term in the electric dipole angular distribution is a consequence of the noncentral forces invoked to explain the deuteron quadripole moment. High energy photo-disintegration angular measurements thus constitute the most sensitive test of both the isotopic spin dependence of the neutron-proton interaction and the existence of noncentral forces.

A DETAILED empirical determination of the neutron-proton interaction is fundamental to the development of a mesotron field theory adequate to explain nuclear phenomena. A previous paper[1] has been devoted to the discussion of the neutron-proton interaction in states of even parity, based on properties of the ground state of the deuteron and low energy collision phenomena. Such information should serve to specify the spin coupling between the mesotron field and nuclear particles. To investigate the charge properties of the mesotron field, it is necessary to study the exchange nature of the neutron-proton interaction, or equivalently, the interaction in states of odd parity. The requisite information can be obtained only by consideration of high energy continuum states of the neutron-proton system, realized in experiments on neutron-proton scattering and the photo-disintegration of the deuteron. It is the program of this paper[2] to discuss the experimental implications of several force laws, and guided by their characteristic predictions, to determine the most critical experimental test of an assumed interaction.

EXCHANGE INTERACTIONS

The calculations of this paper will be performed for three types of exchange forces with the purpose of testing the sensitivity of experiments on high energy neutron-proton scattering and deuteron photo-disintegration by energetic γ-rays to the exchange properties of the neutron-proton interaction. The adopted interaction

* On sabbatical leave from Brooklyn College, Brooklyn, New York.

[1] W. Rarita and J. Schwinger, Phys. Rev. **59**, 436 (1941). This paper is referred to as NPI in the text.

[2] Some of the results of this work were published in a short paper written in collaboration with H. A. Nye: W. Rarita, J. Schwinger and H. A. Nye, Phys. Rev. **59**, 209 (1941).

energies are:

$$\text{(I)}\quad V=\tfrac{1}{3}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\{J_0(r)+J_1(r)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+J_2(r)S_{12}\},$$

$$\text{(II)}\quad V=\tfrac{1}{2}(1+\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\{J_0'(r)+J_1'(r)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+J_2'(r)S_{12}\},\quad (1)$$

$$\text{(III)}\quad V=-\{J_0''(r)+J_1''(r)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+J_2''(r)S_{12}\},$$

$$S_{12}=\frac{3\boldsymbol{\sigma}_1\cdot\mathbf{r}\boldsymbol{\sigma}_2\cdot\mathbf{r}}{r^2}-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$$

expressed in terms of isotopic spin operators $\boldsymbol{\tau}_1$, $\boldsymbol{\tau}_2$, the spin operators $\boldsymbol{\sigma}_1$, $\boldsymbol{\sigma}_2$ and the relative position vector $\mathbf{r}$. The particular isotopic spin dependence of the various potentials is motivated by current theories, the potentials (I), (II) and (III) being respectively analogous to the force laws predicted by the symmetrical, charged and neutral mesotron theories. It should be stressed, however, that any evidence offered on behalf of one of these potentials should not necessarily be construed as support for the corresponding mechanism suggested by the present, incomplete mesotron theories. Our previous paper (NPI) constituted a discussion of the interaction in states of even parity based on the simplifying choice of rectangular well potentials for the various coupling terms, of equal range but different depths, i.e.,

$$V_{(\text{even})}=-\{1-\tfrac{1}{2}g+\tfrac{1}{2}g\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+\gamma S_{12}\}J(r)\quad (2)$$

with g and γ constants and $J(r)$ a rectangular well function of depth V_0 and range r_0. This specification of the interaction in even states, combined with the special isotopic spin dependence of potentials (I), (II) and (III), provides a complete determination of these several interactions. The elementary isotopic spin dependence of these interactions permits a simple representation of the interaction in odd states in terms of that in even states, *viz.*:

$$\begin{aligned}
&\text{(I)} && {}^3V_{(\text{odd})}=-\tfrac{1}{3}\,{}^3V_{(\text{even})}, \\
& && {}^1V_{(\text{odd})}=-3\,{}^1V_{(\text{even})}, \\
&\text{(II)} && {}^3V_{(\text{odd})}=-{}^3V_{(\text{even})}, \\
& && {}^1V_{(\text{odd})}=-{}^1V_{(\text{even})}, \\
&\text{(III)} && {}^3V_{(\text{odd})}={}^3V_{(\text{even})}, \\
& && {}^1V_{(\text{odd})}={}^1V_{(\text{even})}.
\end{aligned}\quad (3)$$

The various odd interactions thus depend strongly on the total spin in view of their relation to the coupling energies in the singlet and triplet even states:

$$\begin{aligned}
{}^1V_{(\text{even})}&=-\{1-2g\}J(r),\\
{}^3V_{(\text{even})}&=-\{1+\gamma S_{12}\}J(r).
\end{aligned}\quad (4)$$

The constants describing the potential wells have been evaluated in NPI and are rewritten for reference:

$$g=0.0715,\quad \gamma=0.775,\quad V_0=13.89\text{ Mev},\quad r_0=2.80\times10^{-13}\text{ cm}.\quad (5)$$

Scattering of Fast Neutrons by Protons

The discussion of NPI was confined to the treatment of the neutron-proton scattering associated with the ${}^3S_1+{}^3D_1$ continuum state, a valid approximation at low energies. We wish to consider neutrons of sufficient energy that the ${}^3P_0$, ${}^3P_1$, and ${}^3P_2+{}^3F_2$ states are also perturbed. The usual scattering theory must be extended to describe this process, for the interactions, and therefore the radial functions, differ in the various states of total angular momentum J. To develop such a theory it is convenient to have simple operational representations of the ${}^3P_0$, ${}^3P_1$, and ${}^3P_2$ wave functions. The spin and angular dependence of the ${}^3P_0$ and ${}^3P_1$ wave functions may be expressed symbolically by

$${}^3P_0:\quad \tfrac{1}{2}(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot(\mathbf{r}/r)\chi_0^m$$

and

$${}^3P_1:\quad \tfrac{1}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot(\mathbf{r}/r)\chi_1^m.$$

That these forms do indeed correspond to triplet state functions of the correct total angular momentum and magnetic quantum number m is immediately evident from their spin symmetry and rotation properties. The P state character is verified by noticing that they are spherical harmonics of the first order. To perform the scattering analysis, it is necessary that the P component of an incident plane wave $\exp[i\mathbf{k}\cdot\mathbf{r}]\chi_1^m$, *viz:*

$$3i\frac{g_1(kr)}{kr}\frac{\mathbf{k}\cdot\mathbf{r}}{kr}\chi_1^m,\qquad (k=(ME/\hbar^2)^{\frac{1}{2}},\ g_1(\rho)\sim-\cos\rho)\quad (6)$$

be decomposed into its ${}^3P_0$, ${}^3P_1$, ${}^3P_2$ constituents.

The $^3P_0$ and $^3P_1$ components are:

$$^3P_0:\quad i\frac{g_1}{kr}\frac{1}{2}(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\frac{\mathbf{r}}{r}\chi_0{}^m,\qquad {}^3P_1:\quad \frac{3}{2}i\frac{g_1}{kr}\frac{1}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\frac{\mathbf{r}}{r}\frac{1}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\frac{\mathbf{k}}{k}\chi_1{}^m. \tag{7}$$

The latter assumes the simpler form

$$\frac{3}{2}im\frac{g_1}{kr}\frac{1}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\frac{\mathbf{r}}{r}\chi_1{}^m,$$

when the total spin is quantized in the direction of wave propagation. The remainder of the P wave function (6) is then the $^3P_2$ component of the plane wave. The P wave function describing the system under the influence of a coupling is obtained by replacing the radial function of free motion, g_1, by the separate radial functions v_0, v_1 and v_2 in the $^3P_0$, $^3P_1$ and $^3P_2$ constituents of the P wave function. The complete P function then obtained is:

$$i\frac{e^{i\eta_0}v_0(r)}{kr}\frac{1}{2}(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\frac{\mathbf{r}}{r}\chi_0{}^m+\frac{3}{2}im\frac{e^{i\eta_1}v_1(r)}{kr}\frac{1}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\frac{\mathbf{r}}{r}\chi_1{}^m$$
$$+ie^{i\eta_2}\frac{v_2(r)}{kr}\left[3\frac{\mathbf{k}\cdot\mathbf{r}}{kr}\chi_1{}^m-\frac{1}{2}(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\frac{\mathbf{r}}{r}\chi_0{}^m-\frac{3}{2}m\frac{1}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\frac{\mathbf{r}}{r}\chi_1{}^m\right]. \tag{8}$$

The phase factors $e^{i\eta_{0,1,2}}$ are inserted to guarantee that the wave function satisfies the boundary condition of a scattering problem, in virtue of the asymptotic forms of the P radial functions: $v_{0,1,2}\sim-\cos(kr+\eta_{0,1,2})$. It should be remarked that this treatment of the $^3P_2$ state neglects its interaction with $^3F_2$.

The spherically diverging wave describing the scattered particles becomes asymptotically:

$$i\frac{e^{ikr}}{kr}\left\{3e^{i\eta_2}\sin\eta_2\frac{\mathbf{k}\cdot\mathbf{r}}{kr}\chi_1{}^m+e^{i(\eta_0+\eta_2)}\sin(\eta_0-\eta_2)\frac{1}{2}(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\frac{\mathbf{r}}{r}\chi_0{}^m\right.$$
$$\left.+\frac{3}{2}me^{i(\eta_1+\eta_2)}\sin(\eta_1-\eta_2)\frac{1}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\frac{\mathbf{r}}{r}\chi_1{}^m\right\}. \tag{9}$$

The cross section for scattering through an angle ϑ into unit solid angle is obtained by computing the squared absolute value of the coefficient multiplying e^{ikr}/r and averaging the result over the three magnetic sublevels associated with the quantum number m. This evaluation is simplified by noting that

$$\chi_0{}^m=\tfrac{1}{2}(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot(\mathbf{k}/k)\chi_1{}^m, \tag{10}$$

if $\mathbf{k}$ is the quantization direction. The amplitude of the scattered wave (9) is then of the form of a spin operator A acting on $\chi_1{}^m$. The absolute squares of the amplitudes summed over the triplet states is simply the diagonal sum of A^+A minus the absolute square of the diagonal matrix element of A in the singlet state. The angular scattering cross section, thus evaluated, is:

$$\sigma(\vartheta)=(1/k^2)[(\sin^2\eta_0+3\sin^2\eta_1+5\sin^2\eta_2)\cos^2\vartheta$$
$$-(\tfrac{1}{3}\sin^2(\eta_0-\eta_2)+\tfrac{3}{4}\sin^2(\eta_1-\eta_2))(3\cos^2\vartheta-1)]. \tag{11}$$

The total cross section for the P scattering is composed additively of the cross sections for the several J states, multiplied by appropriate statistical weights.

$$\sigma=3\frac{4\pi}{k^2}\left[\frac{1}{9}\sin^2\eta_0+\frac{3}{9}\sin^2\eta_1+\frac{5}{9}\sin^2\eta_2\right]. \tag{12}$$

To include the interference between ^3P and $^3S_1$ scattering, we must add to (9) the spherically diverging $^3S_1$ wave[3]

$$\frac{e^{ikr}}{r}\frac{e^{2i\delta_0^{(m)}}-1}{2ik}\chi_1{}^m, \tag{13}$$

where

$$\delta_0{}^{(m)}=\kappa_0{}^{(m)}+i\zeta_0{}^{(m)} \tag{14}$$

is the complex $^3S_1$ phase shift associated with the magnetic substate m. By a spin summation calculation similar to that above, we obtain the interference contribution to the angular cross section:

$$\cos\vartheta\frac{1}{k^2}\Big[3\Big(\frac{1}{9}\sin^2\eta_0+\frac{3}{9}\sin^2\eta_1+\frac{5}{9}\sin^2\eta_2\Big)+\frac{1}{3}e^{-2\zeta_0^{(0)}}(\sin\eta_0\sin(2\kappa_0{}^{(0)}-\eta_0)+\sin\eta_2\sin(2\kappa_0{}^{(0)}-\eta_2))$$
$$+e^{-2\zeta_0^{(1)}}(\sin\eta_1\sin(2\kappa_0{}^{(1)}-\eta_1)+\sin\eta_2\sin(2\kappa_0{}^{(1)}-\eta_2))\Big]. \tag{15}$$

The interference between $^3D_1$ and ^3P is also easily evaluated, but the rather complicated expressions will not be written since these interference terms are not numerically significant. A formula equivalent to (11) has been derived by Kittel and Breit.[4] Further our interference formula (15) reduces to theirs when the $^3S_1$ phase shifts are considered real and independent of m (ordinary potential).

To determine the effective potentials in the various ^3P states, the spin-spin operator S_{12} must be evaluated. The states $^3P_0$ and $^3P_1$ are unmixed and must therefore be eigenfunctions of S_{12}. It is immediately verified from the explicit forms of these functions that the eigenvalues are $^3P_0$: $S_{12}=-4$, $^3P_1$: $S_{12}=2$. Indeed these are the only eigenvalues of S_{12} since $(S_{12}+1)^2=9$ in triplet states. Since we shall neglect the coupling of $^3P_2$ with $^3F_2$ the effective potential in the $^3P_2$ state is obtained by inserting the diagonal value of S_{12}. This value is most easily obtained by noting that the diagonal matrix element of S_{12} in a ^3P_J state multiplied by its statistical weight, $2J+1$, and summed over the three possible J values must vanish since this sum equals the diagonal sum of S_{12}. Therefore the required number is $-\frac{1}{5}[1\times-4+3\times2]=-\frac{2}{5}$. Thus, within the range of interaction, the potentials $^3V_{(\text{odd})}$ of the "neutral" theory (III) are $^3P_0$: 29.16 Mev, $^3P_1$: -35.41 Mev, $^3P_2$: -9.58 Mev. The potentials of the "charged" theory (II) have the reversed sign, while those of the "symmetrical" theory (I) are obtained from (II) through division by 3.

Explicit calculations have been performed for a neutron energy of 15.3 Mev. The ^3P phases $\eta_{0,1,2}$ were evaluated for the three potentials with the results indicated below

	η_0	η_1	η_2	
(I)	0.0745	−0.0545	−0.0175	
(II)	0.5310	−0.1146	−0.0462	(16)
(III)	−0.1025	0.9953	0.0731.	

It may be noted that the signs of these phases are in accordance with the attractive or repulsive nature of the corresponding potentials. The P angular cross sections computed from Eq. (11), with the phases appropriate to the three potentials are:

$$\begin{aligned}&(\text{I}) && \sigma(\vartheta)=(1/k^2)[0.0038+0.0045\cos^2\vartheta],\\&(\text{II}) && \sigma(\vartheta)=(1/k^2)[0.1028-0.0020\cos^2\vartheta],\\&(\text{III}) && \sigma(\vartheta)=(1/k^2)[0.4871+0.6871\cos^2\vartheta]\end{aligned} \tag{17}$$

with

$$4\pi/k^2=0.6823\times10^{-24}\text{ cm}^2. \tag{18}$$

[3] The notation conforms to that of NPI.
[4] C. Kittel and G. Breit, Phys. Rev. **56**, 744 (1939).

The four complex phases describing the $^3S_1+^3D_1$ scattering have been evaluated by the procedure outlined in NPI, with the following numerical results:

$$\kappa_0^{(0)}=-1.6618,\quad \kappa_0^{(1)}=-1.6735,\quad \kappa_2^{(0)}=-0.0035,\quad \kappa_2^{(1)}=-0.0152,$$
$$\zeta_0^{(0)}=\zeta_2^{(1)}=-0.0811,\quad \zeta_0^{(1)}=\zeta_2^{(0)}=0.0532. \tag{19}$$

The $^3S_1+^3D_1$ angular cross section computed from Eqs. 31, 35 and 37 of NPI is then:

$$\sigma(\vartheta)=(1/k^2)[0.9764+0.0464\cos^2\vartheta]. \tag{20}$$

The complete triplet differential angular cross sections, including ^3P, $^3S_1+^3D_1$, and $^3S_1$, ^3P interference scattering, become:

$$\text{(I)}\qquad \sigma(\vartheta)d\Omega=0.6804[0.9830+0.0017\cos\vartheta+0.0510\cos^2\vartheta](d\Omega/4\pi)\times 10^{-24}\text{ cm}^2,$$
$$\text{(II)}\qquad \sigma(\vartheta)d\Omega=0.7464[0.9865+0.1931\cos\vartheta+0.0406\cos^2\vartheta](d\Omega/4\pi)\times 10^{-24}\text{ cm}^2, \tag{21}$$
$$\text{(III)}\qquad \sigma(\vartheta)d\Omega=1.1654[0.8569+0.8484\cos\vartheta+0.4294\cos^2\vartheta](d\Omega/4\pi)\times 10^{-24}\text{ cm}^2.$$

The angular factors in brackets are so normalized that their average over all directions is unity; the initial multiplicative numbers thus represent the total cross section in 10^{-24} cm^2 units.

These triplet cross sections must be appropriately averaged with singlet cross sections to give numbers suitable for comparison with experiment. Within the range of interaction, the ^1P interaction energy of the "neutral" theory (III) equals that in the ^1S state: -11.90 Mev. The interaction energy of "charged" theory (II) has the opposite sign, while that of the "symmetrical" theory (I) is three times larger than (II). The $^1S+^1P$ scattering cross sections were computed by the usual scattering theory, with the following results:

$$\text{(I)}\qquad \sigma(\vartheta)d\Omega=0.4445[0.9392-0.4380\cos\vartheta+0.1824\cos^2\vartheta](d\Omega/4\pi)\times 10^{-24}\text{ cm}^2,$$
$$\text{(II)}\qquad \sigma(\vartheta)d\Omega=0.4236[0.9855-0.2402\cos\vartheta+0.0437\cos^2\vartheta](d\Omega/4\pi)\times 10^{-24}\text{ cm}^2, \tag{22}$$
$$\text{(III)}\qquad \sigma(\vartheta)d\Omega=0.4369[0.9553+0.4981\cos\vartheta+0.1338\cos^2\vartheta](d\Omega/4\pi)\times 10^{-24}\text{ cm}^2.$$

The total cross sections obtained by combining the singlet and triplet cross sections in the ratio of statistical weights are for the three theories:

$$\text{(I)}\qquad \sigma=0.621\times 10^{-24}\text{ cm}^2,$$
$$\text{(II)}\qquad \sigma=0.666\times 10^{-24}\text{ cm}^2, \tag{23}$$
$$\text{(III)}\qquad \sigma=0.983\times 10^{-24}\text{ cm}^2,$$

while the angular distributions are of the form:

$$\text{(I)}\qquad \sigma(\vartheta)\sim(1-0.080\cos\vartheta+0.077\cos^2\vartheta),$$
$$\text{(II)}\qquad \sigma(\vartheta)\sim(1+0.126\cos\vartheta+0.042\cos^2\vartheta), \tag{24}$$
$$\text{(III)}\qquad \sigma(\vartheta)\sim(1+0.932\cos\vartheta+0.457\cos^2\vartheta).$$

The only measurements which have been performed at this energy are those of Salant, Roberts and Wang, and Salant and Ramsey,[5] who employed the energetic neutrons emitted in the Li+D reaction. The scattering cross section obtained by these authors is $\sigma=(0.66\pm 0.07)\times 10^{-24}$ cm^2, referring to a neutron energy of 15 Mev. This result is consistent with both theory I and theory II, but would appear to definitely exclude the potential III.

Recently, Powell, Heitler and Champion[6] have measured the angular distribution of the recoil protons produced by collisions with the 8.7-Mev neutrons emitted in the B+D reaction. Their data, although consistent with spherically symmetrical scattering in the center of gravity system, do not preclude small deviations from isotropy. In particular, the angular distribution predicted by the "neutral" theory, the most asymmetric of the three:

$$\sigma(\vartheta)\sim(1+0.229\cos\vartheta+0.159\cos^2\vartheta) \tag{25}$$

cannot be considered in definite disagreement with the observations.

[5] E. O. Salant, R. B. Roberts and P. Wang, Phys. Rev. **55**, 984 (1939). E. O. Salant and N. Ramsey, Phys. Rev. **57**, 1075 (1940).

[6] C. F. Powell, H. Heitler and F. C. Champion, Nature **146**, 716 (1940).

The Photo-Disintegration of the Deuteron

This section will be devoted to a discussion of deuteron disintegration produced by electric dipole, magnetic dipole and electric quadripole γ-ray absorption. The cross section for the absorption of a γ-ray of energy $\hbar\omega$ incident in the direction of the unit vector $\boldsymbol{\kappa}$ with unit polarization vector $\mathbf{e}$, inducing a transition of the deuteron system from an initial state Ψ_i to any final state Ψ_f describing emission of the disintegration products within the solid angle $d\Omega$ about the propagation vector $\mathbf{k}$, is expressed by

$$\frac{e^2}{\hbar c}\frac{\hbar\omega}{Mc^2}\left(\frac{Mc}{\hbar}\right)^2 k\frac{1}{3}\sum_{m,\,m'}\left|\left(\Psi_f,\left(\frac{1}{2}\mathbf{e}\cdot\mathbf{r}+\frac{\hbar}{2Mc}\boldsymbol{\kappa}\times\mathbf{e}\cdot\mathbf{M}+i\frac{\omega}{8c}\mathbf{e}\cdot\mathbf{r}\boldsymbol{\kappa}\cdot\mathbf{r}\right)\Psi_i\right)\right|^2\frac{d\Omega}{4\pi}. \tag{26}$$

The required average over the initial magnetic substates and summation over the final substates is indicated in the formula. The three absorption processes under discussion are thus described by the matrix elements of $e\mathbf{r}/2$, the electric dipole vector, $(e\hbar/2Mc)\mathbf{M}$, the magnetic dipole vector, and $e\mathbf{rr}/4$, the quadripole moment dyadic of the system. For completeness we shall list all the various final states associated with the three processes:

Electric dipole: $^3P_0$, $^3P_1$, $^3P_2+{}^3F_2$,

Electric quadripole: $^3S_1+{}^3D_1$, $^3D_2$, $^3D_3+{}^3G_3$,
Magnetic dipole: $^1S_0$, $^1D_2$, $^3S_1+{}^3D_1$, $^3D_2$.

It should be noted that both triplet and singlet states may serve as final states for the magnetic dipole absorption process. The total photo-cross section is composed additively of the separate cross sections for these processes. In treating the angular distribution of the disintegration products, however, it is necessary to include interference between states of the same total spin; triplet and singlet states do not interfere, of course. At the high energies contemplated in this paper, the photoelectric disintegration is the dominant process and we shall include only the interference of the ^3P states. We shall also calculate the cross sections for the electric quadripole and magnetic dipole disintegration, chiefly for their theoretical interest.

The initial state function Ψ_i is the ground state wave function of the deuteron (NPI Eq. (2)):

$$\Psi_i=(4\pi)^{-\frac{1}{2}}\{u/r+2^{-\frac{3}{2}}S_{12}w/r\}\chi_1{}^m. \tag{27}$$

The final state of the electric dipole transition is described by the Eq. (18), but with the modification that the phase factors $e^{i\eta_{0,1,2}}$ are replaced by $e^{-i\eta_{0,1,2}}$ to satisfy the asymptotic boundary condition of a plane wave $\exp[i\mathbf{k}\cdot\mathbf{r}]$ and a *converging* spherical wave,[7] appropriate to this problem. In evaluating the matrix element of the electric dipole moment, the operator S_{12} must be replaced by the numbers -4, 2, $-\frac{2}{5}$ when acting on the $^3P_0$, $^3P_1$ and $^3P_2$ wave functions, respectively. The further technique of calculation is identical with that employed in the treatment of scattering. The resultant differential cross section, restricted to unpolarized γ-rays, is then

$$\begin{aligned}\sigma(\vartheta)d\Omega=\frac{\pi}{3}\frac{e^2}{\hbar c}\frac{M\omega}{\hbar k}\Big\{&\frac{1}{9}(I_0{}^2+3I_1{}^2+5I_2{}^2)\\&-\frac{1}{72}(3\cos^2\vartheta-1)(3I_1{}^2+7I_2{}^2\\&+8I_0I_2\cos(\eta_0-\eta_2)\\&+18I_1I_2\cos(\eta_1-\eta_2))\Big\}\frac{d\Omega}{4\pi},\end{aligned} \tag{28}$$

expressed in terms of ϑ, the angle of emission with respect to the direction of the γ-ray, and the three radial integrals

$$\begin{aligned}I_0&=\int_0^\infty rv_0(u-2^{\frac{1}{2}}w)dr,\\I_1&=\int_0^\infty rv_1(u+2^{-\frac{1}{2}}w)dr,\\I_2&=\int_0^\infty rv_2\left(u-\frac{1}{5}2^{-\frac{1}{2}}w\right)dr.\end{aligned} \tag{29}$$

[7] Cf. N. F. Mott and H. S. W. Massey, *The Theory of Atomic Collisions* (Oxford University Press, New York, 1933), p. 258. The proof of this reference applies to ordinary potentials. The required generalization is given in Appendix II.

The total photoelectric disintegration cross section is simply:

$$\sigma=\frac{\pi}{3}\frac{e^2}{\hbar c}\frac{M\omega}{\hbar k}\frac{1}{9}(I_0{}^2+3I_1{}^2+5I_2{}^2). \tag{30}$$

It should be remarked that there is no interference between $^3P_0$ and $^3P_1$ states, for in virtue of their property of being eigenfunctions of S_{12} associated with different eigenvalues, the spin scalar product vanishes. The formulae (28) and (30) reduce correctly to the Eqs. (75) and (76) of NPI when the particles in P states are regarded as unperturbed $v_{0,1,2}=g_1$.

The photomagnetic transition to the singlet state including the interference of $^1S_0$ and $^1D_2$ has already been discussed in NPI (Eqs. (79) and (80)), but subject to the approximation of disregarding the effect of forces on the $^1D_2$ wave function. This approximation is easily removed; the modifications required are the replacement of g_2 by the exact wave function $w_0(\sim -\sin(kr+\delta_2))$ in the radial integrals, and the substitution of $\cos(\delta_0-\delta_2)$ for the $\cos\delta_0$ occurring in the interference term of NPI Eq. (79).

The treatment of both electric quadripole and magnetic dipole transitions to the triplet state need not be more than approximate, for these effects are quite small. In the electric quadripole transition we shall disregard the effect of spin-spin forces on the D states, describing them by ordinary potentials. We shall further neglect the transitions between the continuum D states and $^3D_1$ portion of the ground state. The total cross section for this process is then:

$$\frac{\pi}{240}\frac{e^2}{\hbar c}\frac{|E_0|}{Mc^2}\left(\frac{\hbar\omega}{|E_0|}\right)^3\frac{\alpha^4}{k}\left\{\left(\int_0^\infty r^2uw_1dr\right)^2+\frac{1}{5}\left(\int_0^\infty r^2wu_1dr\right)^2\right\}, \tag{31}$$

wherein the u_1 and w_1 are, respectively, the ^3S and ^3D continuum radial wave functions. The magnetic dipole transition to the triplet state is of some interest, for its very existence is a consequence of the spin-spin forces, insofar as these forces affect the ground state of the deuteron. The cross section, computed with an approximation regarding the continuum D states similar to that above, is simply:

$$2\pi\frac{e^2}{\hbar c}(\mu_n+\mu_p-\tfrac{1}{2})^2\frac{\hbar\omega}{Mc^2}\frac{1}{k}\left(\int_0^\infty ww_1dr\right)^2. \tag{32}$$

The numerical evaluations of these cross sections and angular distributions have been performed at the energy of the γ-rays emitted in the Li+H reaction: $\hbar\omega=17.5$ Mev. The total cross sections for photoelectric dipole absorption are computed to be:

$$\text{El. dip.}\quad\begin{array}{ll}(\text{I}) & \sigma=0.768\times10^{-27}\text{ cm}^2\\ (\text{II}) & \sigma=0.723\times10^{-27}\text{ cm}^2\\ (\text{III}) & \sigma=0.376\times10^{-27}\text{ cm}^2.\end{array} \tag{33}$$

To explain the small cross section obtained from the "neutral" theory (III) it is necessary to observe that a strong attractive force in the final state diminishes the electric dipole matrix element, when the wave-length of the dissociated particles is comparable with the size of the deuteron. This effect, which is particularly enhanced in the $^3P_1$ state of the "neutral" theory, accounts for the diminution of the cross section. For comparison, we include the total cross section computed by treating the dissociated particles as unperturbed: 0.774×10^{-27} cm^2, and that obtained by describing the interaction in the P state by an ordinary attractive potential: 0.529×10^{-27} cm^2.

A specific characteristic of the spin-spin forces is the modification of the usual electric dipole $\sin^2\vartheta$ angular distribution by the introduction of a spherically symmetric term. Proceeding from Eq. (28), the angular distributions have been evaluated for the three theories:

$$\text{El. dip.}\quad\begin{array}{ll}(\text{I}) & \sigma(\vartheta)\sim\sin^2\vartheta+0.015\\ (\text{II}) & \sigma(\vartheta)\sim\sin^2\vartheta+0.077\\ (\text{III}) & \sigma(\vartheta)\sim\sin^2\vartheta+0.36.\end{array} \tag{34}$$

An appreciable isotropic term will occur when there is a large disparity in the values of the radial integrals I_0, I_1, I_2; a strong attractive force in one of the states most readily produces such an effect. Although this situation is realized in the $^3P_0$ state of the "charged" theory (II), the small statistical weight of this state makes the effect less important than that produced by the strong attraction in the $^3P_1$ state of the "neutral" theory.

The interference between the ^1S and ^1D states resulting from photomagnetic absorption to the singlet state produces an appreciable departure from spherical symmetry. The angular distribution predicted by the modified Eq. (79) of NPI:

$$\text{Mag. dip. } (1\rightarrow 0): \sigma(\vartheta)\sim(1-0.57\cos^2\vartheta)$$

represents a significant reduction[8] in the forward intensity. However the total cross section for the process:

$$\text{Mag. dip. } (1\rightarrow 0): \sigma=0.95\times 10^{-29}\text{ cm}^2,$$

is at most three percent of the electric dipole cross section.

Although the cross sections for magnetic dipole and electric quadripole transitions to the triplet state are negligibly small, *viz.*:

$$\begin{aligned}&\text{Mag. dip. } (1\rightarrow 0): &&\sigma=0.048\times 10^{-29}\text{ cm}^2\\ &\text{El. quad.} &&\sigma=0.074\times 10^{-29}\text{ cm}^2,\end{aligned}$$

it is of interest that the magnetic process, a consequence of the small $^3D_1$ component of the ground state, is comparable in magnitude with the electric quadripole process, which originates primarily in the principal component, $^3S_1$, of the ground state.

Conclusion

The two types of measurements which may be made on neutron-proton scattering and on the photo-disintegration of the deuteron concern the total cross sections, and the angular distributions of the scattered or disintegrated particles. It is clear from the preceding discussion (cf. (23) and (24)) that a total cross section measurement of not too great accuracy can distinguish between the "neutral" theory and the two theories I and II. However, to decide between the latter theories by total cross section measurements would require difficult, precise, observations. We wish to stress that valuable information can more easily be obtained by angular distribution studies, in particular, by the investigation of the angular distribution of the photo-disintegration particles. The angular distribution of the protons recoiling under neutron impact will also differentiate easily between theories III and I, II (cf. (24)), but again the predictions of the latter theories are too similar to be resolved by anything short of a painstaking measurement. Although the situation would appear to be identical for the angular distributions describing photo-disintegration (Eq. (34)), it is relatively simpler to perform and analyze such experiments, for the energy of the disintegration products is essentially independent of angle, in contrast with the great variation occurring in neutron-proton scattering. Furthermore, photo-disintegration angular measurements test not only the isotopic spin dependence of the interaction, but in addition the presence of the spin-spin coupling energy, for the existence of photoelectric emission in the forward direction is a consequence of only such noncentral forces.

We thank Professor J. R. Oppenheimer for his interest in this work.

Appendix I

The discussions of this paper and our previous paper NPI have been based, mathematically, upon special spin techniques which are peculiarly adapted to the treatment of states with zero or unit angular momentum, but which cannot be extended to states of higher angular momenta. We shall sketch a general method which employs the operational methods developed for the study of the states of a mesotron with unit spin.[9] The basic idea is that the three functions required to specify the states of a system with unit spin may, in a suitable representation of the spin matrices, be regarded as the components of a space vector. The three types of normalized spin angular functions ($L=J$, $L=J\pm 1$) now appear as vector operators acting on the spherical harmonic P_J^m:

$$\boldsymbol{\Phi}_{L=J}=[J(J+1)]^{-\frac{1}{2}}\mathbf{L}P_J^m$$

$$\boldsymbol{\Phi}_{L=J+1}=[(J+1)(2J+1)]^{-\frac{1}{2}}\left\{(J+1)\frac{\mathbf{r}}{r}+i\frac{\mathbf{r}}{r}\times\mathbf{L}\right\}P_J^m$$

$$\boldsymbol{\Phi}_{L=J-1}=[J(2J+1)]^{-\frac{1}{2}}\left\{-J\frac{\mathbf{r}}{r}+i\frac{\mathbf{r}}{r}\times\mathbf{L}\right\}P_J^m.$$

The operator S_{12}, when written in terms of the total spin:

$$S_{12}=6\left(\mathbf{S}\cdot\frac{\mathbf{r}}{r}\right)^2-4,\qquad \mathbf{S}=\tfrac{1}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)$$

is easily translated into the vector notation:

$$S_{12}\boldsymbol{\Phi}=2\boldsymbol{\Phi}-6\frac{\mathbf{r}}{r}\left\{\frac{\mathbf{r}}{r}\cdot\boldsymbol{\Phi}\right\}.$$

The matrix elements of S_{12} are obtained immediately by

[8] It may be noted that the direction of the deviation from spherical symmetry involves the sign of the deuteron quadripole moment. If the moment were negative, the intensity would be increased in the forward direction.

[9] H. C. Corben and J. Schwinger, Phys. Rev. **58**, 953 (1940).

elementary vector algebra. Thus

$$S_{12}\mathbf{\Phi}_{L=J}=2\mathbf{\Phi}_{L=J} \qquad (\mathbf{r}\cdot\mathbf{L}=0)$$

$$S_{12}\mathbf{\Phi}_{L=J-1}=\frac{-2(J-1)}{2J+1}\mathbf{\Phi}_{L=J-1}+6\frac{[J(J+1)]^{\frac{1}{2}}}{2J+1}\mathbf{\Phi}_{L=J+1}$$

$$S_{12}\mathbf{\Phi}_{L=J+1}=\frac{-2(J+2)}{2J+1}\mathbf{\Phi}_{L=J+1}+6\frac{[J(J+1)]^{\frac{1}{2}}}{2J+1}\mathbf{\Phi}_{L=J-1}.$$

We may note that the diagonal matrix elements S_{JL} satisfy two simple sum rules

(1) $\sum_{L=J,\,J\pm1} S_{JL}=0$ (with the exception of $J=0$)

(2) $\sum_{J=L,\,L\pm1} (2J+1)S_{JL}=0.$

Further, the nondiagonal matrix element, N_J, which connects the states $L=J\pm1$, is represented in terms of either diagonal element by

(3) $$N_J^2=9-(S_{J,\,J\pm1}+1)^2.$$

Sum rules (1) and (3) are an immediate consequence of the simple algebraic property of S_{12}:

$$S_{12}^2=8-2S_{12}.$$

Sum rule (2) has been mentioned briefly in the text. Its proof is elementary:

$$\sum_J (2J+1)S_{JL}=\sum_{Jm}(LJm|S_{12}|LJm)=\sum_{m_Lm_S}(Lm_Lm_S|S_{12}|Lm_Lm_S)=0$$

for both orbital and spin quantum number sums vanish.

Appendix II

The photo-dissociation calculations of the text are based upon a perturbation formula in which the final wave function, describing the dissociated particles, satisfies the boundary condition that it represent a plane wave plus a converging spherical wave. Although this theorem is well known for ordinary potentials, it requires justification when the potential involves spin operators in such manner that the interaction differs for the various states of total angular momentum J.

The required proof is easily given with the aid of projection operators $O_L{}^J$ which decompose a wave function associated with orbital angular momentum L into its J components. More precisely, the projection operators are defined by their action on normalized spin angular functions of total angular momentum J, $F_L{}^{Jm}$:

$$O_L{}^JF_L{}^{J'm}=\delta_{JJ'}F_L{}^{Jm},\quad \sum_J O_L{}^J=1.$$

Such operators are easily constructed from the operator J^2. The wave function describing scattering is then constructed from the plane wave:

$$\exp[i\mathbf{k}\cdot\mathbf{r}]\chi_1{}^m=\sum_L(2L+1)i^L\frac{g_L(kr)}{kr}P_L(\mathbf{k},\mathbf{r})\chi_1{}^m,$$

by isolating the J component of the L wave: $O_L{}^JP_L(\mathbf{k},\mathbf{r})\chi_1{}^m$ and replacing the radial function of free motion, $g_L(kr)$, by the appropriate radial function

$$\exp[i\delta_L{}^J]u_L{}^J(r),\quad (u_L{}^J(r)\sim\sin(kr-\tfrac{1}{2}\pi L+\delta_L{}^J)).$$

Hence

$$\Psi_{\text{scatt}}(r)=\sum_{J,L}(2L+1)i^L\exp[i\delta_L{}^J]\frac{u_L{}^J(r)}{kr}O_L{}^JP_L(\mathbf{k},\mathbf{r})\chi_1{}^m.$$

If a time dependent perturbation

$$V(r)e^{-i\omega t}+V^*(r)e^{i\omega t}$$

act on the deuteron system in the initial state Ψ_i, the function describing the outgoing wave, obtained by the usual procedure, is:

$$\frac{M}{\hbar^2}\frac{e^{ikr-iEt/\hbar}}{r}\sum_{JmL}F_L{}^{Jm}(\mathbf{r},\sigma)\exp[i\delta_L{}^J-\tfrac{1}{2}\pi L]\times\left(F_L{}^{Jm}(\mathbf{r}')\frac{u_L{}^J(r')}{kr'},\ V(r')\Psi_i(\mathbf{r}')\right),$$

indicating explicitly the spin coordinate σ of the outgoing particles. (For electric dipole absorption, $V(r)=-ie\times(2\pi\hbar\omega)^{\frac{1}{2}}\mathbf{e}\cdot\frac{1}{2}\mathbf{r}$.) Upon inserting projection operators,

$$\exp[-i\delta_L{}^J]\frac{u_L{}^J(r')}{kr'}F_L{}^{Jm}(\mathbf{r}')=\left[\sum_{J'}\exp[-i\delta_L{}^{J'}]\frac{u_L{}^{J'}(r')}{kr'}O_L{}^{J'}\right]F_L{}^{Jm}(\mathbf{r}'),$$

the outgoing wave becomes

$$\frac{M}{\hbar^2}\frac{e^{ikr-iEt/\hbar}}{r}\sum_{J'L}\left(\left[i^L\exp[-i\delta_L{}^{J'}]\frac{u_L{}^{J'}(r')}{kr'}O_L{}^{J'}\right]\times\sum_{Jm}F_L{}^{Jm}(\mathbf{r}',\sigma')F^{Jm}(\mathbf{r},\sigma)^*,\ V(r')\Psi_i(\mathbf{r}',\sigma')\right).$$

The functions $F_L{}^{Jm}(\mathbf{r},\sigma)$ are obtained from the set $P_L{}^{m_L}(\mathbf{r})\chi_1{}^{m_S}(\sigma)$ by a unitary transformation, and therefore

$$\sum_{Jm}F_L{}^{Jm}(\mathbf{r}',\sigma')F_L{}^{Jm}(\mathbf{r},\sigma)^*=\sum_{m_Lm_S}P_L{}^{m_L}(\mathbf{r}')P_L{}^{m_L}(\mathbf{r})^*\chi_1{}^{m_S}(\sigma')\chi_1{}^{m_S}(\sigma)^*=\frac{2L+1}{4\pi}P_L(\mathbf{r},\mathbf{r}')\chi_1{}^{\sigma}(\sigma')$$

by the spherical harmonic addition theorem and the fact that

$$\chi_1{}^{m_S}(\sigma)=\delta_{\sigma,m_S}.$$

The amplitude of the outgoing wave, *viz.:*

$$\frac{M}{4\pi\hbar^2}\frac{e^{ikr-iEt/\hbar}}{r}\left(\sum_{J'L}(2L+1)i^L\times\exp[-i\delta_L{}^{J'}]\frac{u_L{}^{J'}(r')}{kr'}O_L{}^{J'}P_L(\mathbf{r},\mathbf{r}')\chi_1{}^{\sigma},\ V(r')\Psi_i(\mathbf{r}')\right),$$

is thus in the form of a matrix element with the final wave function:

$$\Psi_f(\mathbf{r}')=\sum_{JL}(2L+1)i^L\exp[-i\delta_L{}^J]\frac{u_L{}^J(r')}{kr'}O_L{}^JP_L(\mathbf{r},\mathbf{r}')\chi_1{}^{\sigma},$$

which has the form of the scattering wave function describing propagation in the direction of $\mathbf{r}$, the direction of observation, with magnetic quantum number σ, the spin coordinate of the outgoing wave; but with the phase factor $\exp[-i\delta_L{}^J]$. It will, therefore, describe a plane wave and a *converging* spherical wave. It is of some interest that Ψ_f is obtained from Ψ_{scatt} by the combined operations of time and space reflection.