stop until the expelled material brings the mass of the remaining star below the critical value. This process may be compared with the supernovae explosions, in which case the expelled gases form extensive nebulosities (as "Crab-Nebula" or "Filamentary Nebula"), apparently assuming this permanent state after the explosion. This point of view seems also to acquire some confirmation from the fact that the relative number of novae and supernovae occurring in stellar systems is of the same order of magnitude as the relative number of stars having masses smaller and larger than $2 \cdot M(\odot)$.

CONCLUSION

In the present article we have developed the general views regarding the role of neutrino emission in the vast stellar catastrophes known to astronomy. It must be emphasized that, while the neutrinos are still considered as highly hypothetical particles because of the failure of all efforts made to detect them, the phenomena of which we are making use in our considerations are supported by the direct experimental evidence of nuclear physics. In fact, the experiments of Ellis and Wooster¹⁷ and of Meitner and

Orthman¹⁸ leave no doubt that the energy balance does not hold in the processes of radioactive β -transformations, and all later evidence on this subject strongly indicates that this "disappearance of energy" occurs in such a way that it appears to be carried away by particles of almost unlimited penetrability.

Whereas the fundamental ideas of the proposed theory are very simple and the physical part of calculations pertaining to the rate of neutrino emission can be easily carried out on the basis of existing formalism, the problem of the dynamics of the collapse represents very serious mathematical difficulties. It is to be hoped that these difficulties will be overcome by the choice of some suitable simplified model.

It is our pleasant duty to express our thanks to Drs. S. Chandrasekhar, E. Fermi, E. Teller and M. A. Tuve for helpful discussion and interest in the problem. One of us (M. Schoenberg) is grateful to the John Simon Guggenheim Memorial Foundation for the grant which enabled him to carry out this research.

¹⁷ C. D. Ellis and W. A. Wooster, Proc. Roy. Soc. **A117**, 109 (1927). ¹⁸ L. Meitner and W. Orthman, Zeits. f. Physik **60**, 143 (1930).

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Differential Measurement of the Meson Lifetime

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Measurements of the meson lifetime were made by comparing the lead absorption curves of the cosmic radiation at two different elevations, and by compensating the air column at the higher level by a layer of graphite placed on top of the counter telescope. The method does not require any knowledge of the energy distribution or of the height of the producing layer of the mesons. All systematic errors are greatly reduced by the differential method used and can be shown to be insignificant. The value of the lifetime comes out to be appreciably shorter than in previous determinations, viz. $(1.25\pm0.3)\times10^{-6}(\mu c^2/10^8 \text{ ev})$ sec.

1. The Method

A FTER the instability of the mesons was suggested by Yukawa¹ from theoretical $^{-1}$ H. Yukawa, Proc. Phys. Math. Soc. Japan 20, 319 (1938).

reasons, experimental evidence of various kinds for this effect was discussed by several authors, notably by Euler and Heisenberg.² A more

² H. Euler and W. Heisenberg, Ergeb. d. exakt. Naturwiss. 17, 1 (1938).

recent survey of all the earlier observations has been given by Rossi.³ While there seemed to be good support for the decay hypothesis with values of the lifetime of two to four microseconds, it could by no means be said that it was definitely confirmed. Furthermore, all previous methods necessitated assumptions regarding the energy distribution of the mesons or the height of their production or both, in order to obtain a numerical value for the lifetime. This holds true



FIG. 1. Experimental arrangement.

also for all the more recent determinations of the lifetime, which have been published since.⁴⁻⁷ We have tried, therefore, to set up an experiment which would be independent of such additional data and which would be free from objections which might cast doubt on the reality of the effect.

The only feasible method to select a definite energy range in counter experiments is by differential absorption. The mesons seem to follow quite closely a range absorption law⁴ in good agreement with Bloch's formula for energy loss by ionization. Every point on an absorption curve corresponds, therefore, to a definite energy.

The procedure adopted was the following one. The absorption curves in lead were taken at two different elevations. In place of the absent air column at the higher level a layer of graphite of equal mass was put on top of the counter telescope. As the absorbing power of carbon and air is practically the same, according to Bloch's theory, the only difference in the two series of observations consists in the compression of the air mass into the smaller volume of the graphite where the mesons have less chance for spontaneous decay. If N(z) is the number of counts per unit time at a lead thickness z, then $\partial N/\partial z$ will be proportional to the number of particles corresponding to the energy of particles of range z in lead. The attenuation factor A, measuring the fraction of particles surviving at the lower level *I* is therefore given by

$$A(E) = \left(\frac{\partial N_I}{\partial z} \right) \frac{\partial N_{II}}{\partial z}_{z=z(E)},$$

where the indices I and II refer to lower and upper level, respectively.

Recently Fermi⁸ has pointed out that the density of packing of the absorbing material has a direct influence on its stopping power. The effect is due to partial screening of the contributions of atoms at some distance from the passing particle. Formulas for this effect have been developed by Fermi and by Halpern and Hall.⁹ A correction for this effect can be easily applied in our case, and it turns out to be quite small.

A preliminary run with the described arrangement¹⁰ gave, simultaneously with the somewhat similar measurements by Rossi and collaborators,⁵ the first unambiguous proof for the decay effect. In the present series we have tried

TABLE I. Counting rate for different lead absorber thicknesses.

HR.	ст./нг. 41.9±0.3	ст.	нк. 66 5	CT./HR.
198	41.9 ± 0.3	4179	66.5	62 8 1 0 6
225 25			00.0	U2.0±0.0
233.23	32.3 ± 0.3	2989	70.25	42.5 ± 0.5
271.5	29.4 ± 0.2	4226	113	37.4 ± 0.4
81.33	29.6 ± 0.4			
196.25	28.1 ± 0.2	4627	135.75	34.1 ± 0.3
140.75	26.9 ± 0.3	3606	115.25	31.3 ± 0.3
150.75	25.95 ± 0.3	4129	141.5	29.2 ± 0.3
99.75	25.7 ± 0.3			
	271.5 81.33 196.25 140.75 150.75 99.75	$\begin{array}{ccccc} 271.5 & 29.4 \pm 0.2 \\ 81.33 & 29.6 \pm 0.4 \\ 196.25 & 28.1 \pm 0.2 \\ 140.75 & 26.9 \pm 0.3 \\ 150.75 & 25.95 \pm 0.3 \\ 99.75 & 25.7 \pm 0.3 \end{array}$	$\begin{array}{cccccc} 271.5 & 29.4 \pm 0.2 \\ 81.33 & 29.6 \pm 0.4 \\ 196.25 & 28.1 \pm 0.2 \\ 140.75 & 26.9 \pm 0.3 \\ 150.75 & 25.95 \pm 0.3 \\ 99.75 & 25.7 \pm 0.3 \\ \end{array}$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$

⁸ E. Fermi, Phys. Rev. **56**, 1242 (1939); **57**, 485 (1940). ⁹ O. Halpern and H. Hall, Phys. Rev. **57**, 459 (1940). ¹⁰ W. M. Nielsen, C. M. Ryerson, L. W. Nordheim and

K. Z. Morgan, Phys. Rev. 57, 158 (1940).

⁸ B. Rossi, Rev. Mod. Phys. 11, 296 (1939)

 ⁴ M. A. Pomerantz, Phys. Rev. 57, 3 (1940).
 ⁵ B. Rossi, Norman Hilberry and J. Barton Hoag, Phys.

<sup>Rev. 57, 461 (1940).
M. Ageno, G. Bernardini, N. B. Cacciapuoti, B. Ferretti and G. C. Wick, Ricerca Scient. 10, 1073 (1939); Phys. Rev. 57, 945 (1940).</sup>

⁷ H. V. Neher and H. G. Stever, Phys. Rev. 58, 766 (1940).

to hold all corrections for side effects to a minimum and we hope to have obtained a more reliable determination of the lifetime than heretofore available.

2. Experimental Procedure and Data

The absorption curves of the cosmic radiation of vertical incidence were measured for lead from 0 to 15 inches, in Durham, North Carolina (elevation 125 meters, mean barometric pressure 75 cm Hg) and on Mount Mitchell, North Carolina (elevation 2040 meters, mean barometric pressure 60 cm Hg). In the latter observations, the vertical air column of 15 cm Hg=203 g/cm² between the elevations of Durham and Mount Mitchell was compensated for by a layer of graphite of a thickness of 200 g/cm². As shown in Fig. 1, the graphite filled completely the sensitive cone of the counter telescope. Observations were made at the higher elevation in the Park Warden's cabin on Mount Mitchell.

An additional layer of wood was placed on the roof of the small shack on the Duke University campus where the observations at the lower elevation were performed. In this way the amount of wood above the apparatus in both cases was of the same thickness of $1\frac{1}{2}$ inches.

Fourfold coincidences were recorded between counters arranged as shown in Fig. 1 with various thicknesses of absorbing lead between them. The counters, the shielding lead at the sides, and the absorbing lead layers between, were all supported on an angle iron frame not indicated in the figure. The same frame was, of course, used in both series of observations, and in both cases the axis of the counters made approximately the same angle with respect to the east-west direction. The three bottom counters were shielded by four inches of lead on each side. The upper counter was shielded by two inches of lead on each side as indicated in the figure. The pieces of shielding lead and the absorbing layers between counters were approximately three times the active length of the counters.

The counters were approximately 5 cm in diameter and had an effective length of 20 centimeters. They were prepared and filled with hydrogen in a manner somewhat similar to that suggested by Shonka.¹¹ Tests showed that the counters had an efficiency of 95 percent or more. The counter and recording circuits will be published elsewhere. In a series of measurements of this kind it is quite essential that the over-all efficiency of the counting apparatus be constant over both series of measurements. In addition to the frequent checks on counting rates of the individual counters we have assured ourselves of the reliability of the apparatus by repeating the



FIG. 2. Counting rate as a function of lead absorber thickness for Durham (curve I) and Mount Mitchell (curve II).

low altitude observation for two absorbing thicknesses of 6" and 15" after the observations on Mount Mitchell were completed. These repeat measurements are the lower entries of the Durham data for the corresponding thicknesses as shown in Table I. The two series of observations made in Durham which bracket the Mount Mitchell observations agree well within the statistical errors of the observation. The measurements here reported are also in very satisfactory agreement with the results reported earlier.¹⁰

The complete 1940 data for both series of observations are shown in Table I and the absorption curves are plotted in Fig. 2.

3. Evaluation of the Lifetime

The energy lost by ionization in an air column of 1900 m is of the same order of magnitude as

¹¹ See J. B. Hoag, *Electron and Nuclear Physics* (D. Van Nostrand, New York, 1938), p. 432.

in about 10 inches of lead. In the evaluation of the lifetime in terms of the observed attenuation factor, the variation of energy along the path has to be taken into account. The probability of decay w in an interval dx is

$$wdx = (dx/c\tau)(\mu c^2/\rho c), \qquad (2)$$

where τ is the lifetime at rest, μ the rest mass and p the momentum of the meson. We assume for the loss of momentum a linear law, i.e., the momentum given at a point x backwards from the point of observation will be

$$pc = p_0 c + \beta x, \tag{3}$$

where the specific loss in air β can be considered as constant for kinetic energies of the order of the rest energy and over. The attenuation factor becomes then

$$A(p_0c) = \exp\left(-\int_0^{x_0} w dx\right)$$
$$= \exp\left\{-\frac{1}{\beta}\int_{p_0c}^{p_0c+\beta x_0} \frac{1}{c\tau} \frac{\mu c^2}{\rho c} d(\rho c)\right\}$$
$$= \left(\frac{p_0c}{p_0c+\beta x_0}\right)^{\mu c^2/c\tau\beta}$$
(4)

In this formula p_0 is the final momentum of the particle and βx_0 the amount of energy lost by ionization in the air column. In (4) the variation with altitude of the density of the air has been neglected. We have not introduced this refinement, as it only would give a correction of the order of one percent in A.

Solving (4) for τ we obtain

$$\tau = \frac{\mu c^2}{c\beta} \lg \frac{p_0 c + \beta x_0}{p_0 c} / \lg \frac{1}{A}.$$
 (5)

The specific energy loss in air for particles with a kinetic energy slightly larger than their rest energy is $\beta = 1.9 \times 10^6 \text{ ev/g}$, i.e., $2.3 \times 10^8 \text{ ev/km}$ air. With $x_0 = 1.92$ km and referring τ to a standard mass of 10^8 ev and measuring it in microseconds, we obtain

$$\tau = \gamma \left(\frac{\mu c^2}{10^8 \text{ ev}}\right) \times 10^{-6} \text{ sec.}, \qquad (6)$$

$$\gamma = 1.45 \, \lg \frac{pc + 4.4}{pc} \Big/ \lg \frac{1}{A}, \tag{7}$$

where pc is expressed in units 10^8 ev.

The energy loss in lead of a meson is $= 1.2 \times 10^6$ ev/g, i.e., 0.35×10^8 ev/inch. For kinetic energies

 TABLE II. Attenuation factor, average energy and lifetime for various intervals on lead absorption curve.

INTERVAL INCHES PB	1/A	¢c _{Av}	γ
0 to 3	2.12		
3 to 6	1.76	2.45	2.62
6 to 9	2.64	3.50	1.22
9 to 12	2.35	4.55	1.14
12 to 15	2.13	5.6	1.12

greater than 10^8 ev the *pc* scale as a function of lead thickness is

$$bc = (0.9 + 0.35z) \times 10^8 \text{ ev.}$$
 (8)

The end correction has been obtained from the range-energy relation as given by Bloch's formula for particles with mass ~ 200 m.

Table II gives the values of 1/A, i.e., the ratio of slopes of curves I and II for various intervals of lead thicknesses together with the average value of pc and the resulting γ (lifetime in microseconds).

The values of γ agree very well for the intervals between 6 and 15 inches. The interval between 3 and 6, however, seems to give a much larger value. This behavior finds its natural explanation in the role of the soft component for not too large thicknesses of the lead absorber. An inspection of the absorption curves of Fig. 2 shows a sharp upturn in the interval in question. This suggests that the soft component makes a noticeable contribution, of the order of 3 to 4 percent, to the counting rate at 3 inches, as indicated by the dotted extrapolation lines in Fig. 2. An effect of this magnitude is indeed to be expected from the theory of the soft component. It can be seen from an estimate by Heitler¹² that a fraction of order of $\frac{1}{10}$ of the

¹² W. Heitler, Nature 140, 235 (1937).

soft radiation should be able to produce coincidences with 80 g/cm^2 lead interposed, and this would give approximately 1/30 of the total counting rate. At 6 inches, however, the contribution of the soft component should be quite negligible. The correction for the soft component might even be larger for curve I (Durham) since electrons coming from the meson decay in the air column would give an additional contribution. If we subtract from the counting rate at 3 inches on both curves the same amount of 1.4 counts per hour (1/30 of 42) we would bring the value of γ down to 1.6, and the sketched extrapolation of the curves would bring it to 1.35. The effect of the soft component is thus certainly of the order of magnitude required to remove the above discrepancy. The exact value of the correction can, of course, not be determined, but we can safely discard the point at 3 inches for the evaluation of the lifetime.

For the intervals between 6 and 15 inches the value of γ is practically constant, while the attenuation factor itself varies appreciably (compare Table II). This behavior means a confirmation of the Lorentz time factor, i.e., the energy dependence of the mean free path against decay.

In order to obtain an estimate of the probable error introduced by the errors in the counting rates we can proceed as follows. In view of the smoothness of the curves we can consider the whole interval between 6 and 15 inches. The average value of pc in this interval is 4.4×10^8 ev.¹³ With the probable errors listed in Table I we obtain for this interval $1/A = 2.38 \pm 0.32$. The corresponding values of γ are

$$\frac{1/A}{\gamma} = \frac{2.06}{1.38} = \frac{2.38}{1.16} = \frac{2.70}{1.01}.$$

We feel safe, therefore, to give as a final result, apart from systematic errors to be discussed in the next section

$$\gamma = 1.2 \pm 0.3$$
.

 13 This corresponds to 10 inches Pb. The most consistent way to determine the average momentum for a finite interval is to take the geometrical mean

$$\frac{1}{p_{AV}} = \frac{1}{p_2 - p_1} \int_{p_1}^{p_2} \frac{dp}{p} = \frac{1}{p_2 - p_1} \lg \frac{p_2}{p_1}.$$

5. Systematic Errors. Comparison with Other Observers

Our results are not very sensitive against changes in the assumed values of stopping power of air and lead, and corrections for such changes could easily be applied. It might, however, seem somewhat more doubtful to identify the energy scale on the lead absorber with the values from the ionization losses alone. The correct procedure would be, of course, to take the geometrical average of the energy of the distribution taken out by all occurring processes in a unit layer of lead. Any additional effect besides ionization, i.e., scattering, nuclear absorption and such, would increase the meson energy and therefore tend to lower the value of the lifetime. In view of the behavior of the geometrical mean this would, however, introduce a serious deviation only if the number of particles of high energy absorbed would be at least comparable with those lost by ionization.

In our treatment we assume that no mesons are generated between Mount Mitchell and Durham. We also assume that there is no appreciable admixture of protons.

There are three differences in the operation of the counter telescope at the two locations at different altitude, which might give rise to systematic errors and which have to be discussed.

(I) The mesons, before entering the telescope, emerge from different materials.-It will be much more likely that a meson is accompanied by soft secondaries when coming from a dense material. It might happen therefore, that the meson misses the first counter, which, however, responds to a secondary. As the probability for such an event is independent of the lead absorber between the counters this effect would give a constant percentage correction to the curve II (at the higher elevation). It would reduce, therefore, the value of 1/A by the same fraction. A one percent change in 1/A would produce only 1.5 percent change in the lifetime. This effect is therefore wholly negligible, even if we should allow one percent for it as Rossi does.

(II) The soft component is of different intensity outside the cone of the telescope; Auger showers.— Owing to the heavy shielding of the counter telescope only showers of very high energy could possibly give a contribution to the counting rate. The rate of increase of large showers and bursts from sea level to an altitude of 2000 m is about¹⁴ 4 : 1. Therefore, in case the lead filters between the counters have a noticeable effect on the chance of recording such showers a correction for these counts must be made.

In our preliminary measurements,¹⁰ observations were made without any side shielding and with side shielding of the lowest counter. The reduction of the counting rate (amounting to one count per hour) by the shielding was practically independent of the lead thickness between the counters, and we concluded, therefore, that the large air showers did not affect appreciably our lifetime determination. To make sure of this for our present experiment, we made measurements in Durham with one of the counters placed out of line as shown in Fig. 1. The counting rate then was very low, 0.13 ± 0.03 per hour with 6 inches Pb between the counters, and 0.09 ± 0.02 per hour with 15 inches Pb. It seems doubtful whether the slight dependence on lead thickness is at all significant. However, if we take these figures as a true measure of the contributions of large sidewise showers in Durham and admit an increase by a factor 4 for the rate on Mount Mitchell, the correction to the lifetime would only be one percent, and if we take the limits 0.16 count per hour at 6 inches Pb and 0.07 at 15 inches Pb, the correction would still be only 3 percent. We feel sure, therefore, that the effect of large showers can be entirely neglected compared to the statistical errors.

(III) The stopping power of the air and carbon columns of equal weight is not quite the same.— The differences due to the different nuclei in ionization losses and probably also nuclear absorption if any, should be negligible. As already mentioned, however, the packing density itself has some influence on the stopping power, as discussed by Fermi.⁸ It is particularly important to consider this effect carefully as it cannot be distinguished readily from a true decay.

Formulas for the change in stopping power due to the density of the traversed material have been given by Fermi⁸ and by Halpern and Hall.⁹ In sufficient approximation, the specific energy loss is diminished compared to the values given by the Bloch formula by

$$\Delta \frac{dE}{dx} = \frac{2\pi N e^4}{mv^2} \left[\lg \frac{\epsilon - 1}{1 - v^2/c^2} - 1 \right]; \quad \frac{\epsilon - 1}{1 - v^2/c^2} \gg 1. \quad (9)$$

Here N is the number of electrons per unit volume and ϵ an effective dielectric constant. According to Halpern and Hall ϵ is given by

$$\epsilon - 1 = \frac{Ne^2}{\pi m \nu_{Av}^2};$$

$$\nu_{Av} \cong 13.5Z \text{ ev} = 3.5Z \times 10^{-15} \text{ sec.}^{-1}$$
(10)

where ν_{Av} is the geometrical average over the atomic frequencies. This formula gives a good approximation if

$$Ne^2/\pi m v^2 > 1 - v^2/c^2$$

for the largest frequencies occurring. Otherwise the effect is reduced considerably.

A correction of this type disturbs, of course, the balancing of the air column for curve I by the carbon for curve II. We can restore this balance by shifting the curves against each other, i.e., by comparing the slopes not at the same lead thickness, but at points where the total energy loss of the mesons just stopped is the same. As our curves show only a very small curvature in the range used (6 to 15 inches Pb) the error committed in neglecting the Fermi effect can only be rather small.

Expression (10) gives for carbon $\epsilon = 1.13$. The total deficiency in stopping power of 200 g of carbon for particles with $1/(1-v^2/c^2)^{\frac{1}{2}} = E/\mu c^2$ =6.6 is then calculated from (9) to be 0.108 $\times 10^8$ ev, which corresponds to 0.31 inch of Pb. This is, therefore, the amount the two curves have to be shifted against each other. The above value for v corresponds to the average energy in the carbon absorber of those particles which are stopped by 10 inches of lead. The differences in counting rate for the three intervals used are 3.3, 2.8, 2.18. The second differences are thus 0.5 and 0.67, or an average of about 0.6 per three inches on a three-inch interval, i.e., 0.6 per inch shift for a nine-inch interval. The counting rate difference of 8.2 between 6 and 15 inches has,

¹⁴ Compare for instance, Norman Hilberry, Chicago meeting, Phys. Rev. **59**, 112A (1941).

therefore, to be corrected by $0.31 \times 0.6 = 0.19$ to 8.04, giving an 1/A of 2.32 in place of 2.38 as formerly. This corresponds to an increase in the lifetime γ by 3 percent only. The comparatively small value of the dielectric constant ϵ seems to agree fairly well with the measurements of Crane and collaborators¹⁵ which showed only a rather small deviation from the Bloch formula in carbon for electrons with $E/mc^2 = 20$. Even if we would take $\epsilon = 2$, which value is very improbable, the correction to be applied to our value of γ would be only 11 percent.

All the systematic effects discussed here give, thus, corrections much smaller than the probable limits of error due to statistics. We believe, therefore,

$$\tau = (1.25 \pm 0.3) (\mu c^2 / 10^8 \text{ ev}) \times 10^{-6} \text{ sec.}$$

to be a fair representation of our results.

Our value for the lifetime is thus considerably shorter, about one-half, than heretofore assumed. A direct comparison with previous observations can only be made with those of Rossi⁵ et al. We obtain a mean free path

$$L = \frac{\delta x}{\lg 1/A} = 1.92 \text{ km} \frac{37.4}{2.94} = (8.0 \pm 0.8) \text{ km}$$

for mesons which can traverse 15.2 cm Pb, as compared to Rossi's value of (9.4 ± 1.6) km for particles which can traverse 12.7 cm Pb. Our value is somewhat smaller, the more so in view of the thicker lead absorber, but well within the limits of error. This means that the average energy of the mesons must be definitely larger than the value calculated by Rossi from Blackett's energy distribution.

Similar remarks apply to the failure of other observers to find a lifetime as short as one microsecond by inclination measurements and other methods. All these measurements imply a knowledge of either the height where the mesons are produced, or their average energy or both. It is generally assumed that most of the mesons are produced near the maximum of the soft component, but apart from a hypothetical genetic connection, there are no facts in support of this assumption. The observations of Schein and collaborators¹⁶ seem to indicate, on the contrary, that the production of mesons extends to considerably lower altitudes. The new value of the lifetime will, however, be of importance for the discussion of the temperature effect and of the fraction of the soft component due to meson decay and similar questions.

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¹⁵ H. R. Crane, N. L. Oleson and K. T. Chao, Phys. Rev. 57, 664 (1940).

¹⁶ M. Schein, E. O. Wollan and Gerhart Groetzinger, Phys. Rev. **58**, 1027 (1940). ¹⁷ W. M. Nielsen and K. Z. Morgan, Phys. Rev. **54**, 245

^{(1938).}