Neutrino Theory of Stellar Collapse

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At the very high temperatures and densities which must exist in the interior of contracting stars during the later stages of their evolution, one must expect a special type of nuclear processes accompanied by *the emission of a large number of neutrinos*. These neutrinos penetrating almost without difficulty the body of the star, must carry away very large amounts of energy and prevent the central temperature from rising above a certain limit. This must cause *a rapid contraction of the stellar body* ultimately resulting in a *catastrophic collapse*. It is shown that energy losses through the neutrinos produced in reactions between free electrons and oxygen nuclei can cause a complete collapse of the star within the time period of half an hour. Although the main energy losses in such collapses are due to neutrino emission which escapes direct observation, the heating of the body of a collapsing star must necessarily lead to the *rapid expansion of the outer layers* and the *tremendous increase of luminosity*. It is suggested that stellar collapses of this kind are responsible for the phenomena of *novae* and *supernovae*, the difference between the two being probably due to the difference of their masses.

§1. Introduction

One of the most peculiar phenomena which we encounter in the evolutionary life of stars consists in vast stellar explosions known as "ordinary novae" and "supernovae." It is now well established that, although these two classes of novae possess a great many features in common, they are sharply separated insofar as their maximum luminosities are concerned. The ordinary novae, appearing at the rate of about 50 per year in our stellar system, reach at their maximum a luminosity of the order of magnitude of 10^5 suns. On the other hand, the supernovae, flaring up in any given stellar system only once in several centuries, reach luminosities exceeding that of the sun by a factor of 10^9. The intermediate luminosities have never been observed, and there seems to exist a real gap between these two classes of stellar explosions.

The common features of novae and supernovae may be briefly summarized as follows:

(1) Both ordinary novae and supernovae show a very similar form of luminosity curve (apart from the luminosity scale, of course) with a sharp rise to the maximum within a few days or weeks, and a subsequent slow decline of intensity, decreasing by a factor of two every four or five months.

(2) In both cases the spectrum shows rather high surface temperature (up to 20,000°C for ordinary novae, and probably above 30,000°C for supernovae), and the rapid expansion of the stellar atmosphere which is evidently blown up by the increasing radiative pressure. In the case of Nova Aquilae 1918, for example, the star was surrounded by a luminous gas shell expanding with a velocity of 2000 kilometers per second, whereas the gas masses expelled by the galactic supernova of the year A.D. 1054 (observed by Chinese astronomers) form at present an extensive luminous cloud known as the "Crab-Nebula." It must be noticed here that the large surface area of this blown-up atmosphere is mainly responsible for the observed high luminosities, since the increase of the surface temperature can only account for a factor of several hundreds in the surface brightness.

(3) Whereas the "prenovae," in the rare cases when they have been observed, represent comparatively normal stars of the spectral class A (surface temperature about 10,000°C),[1] the "postnovae," remaining after the flare-up, possess extremely high surface temperatures (spectral class O) and seem to represent highly collapsed configurations, such as the stars of the "Wolf-Rayet" type.

The same evidently holds true for the case of

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[1] The meagerness of observational material makes it impossible to decide whether "prenovae" are located on the main sequence or to the left of it. There seems to be no doubt, however, that, as the result of explosion, the position of the star in the Hertzsprung-Russell diagram is strongly shifted towards higher surface temperatures and smaller radii.

supernovae, since the star found in the center of the "Crab-Nebula," and representing most probably the remainder of the galactic supernova A.D. 1054, shows the typical features of a very dense "white dwarf."

The change of state caused by these stellar catastrophes strongly suggests that the process involved here is not connected with any instantaneous liberation of intranuclear energy due to some explosive reaction, but rather represents a rapid collapse of the entire stellar body as was first suggested by Milne.[2]

It was suggested by Baade and Zwicky,[3] in particular application to supernovae, that such collapses may be due to the formation within the star of a large number of neutrons which would permit considerably closer "packing" in the central regions.

It is easy to see, however, that the possibility of closer packing alone is quite insufficient to explain the rapid collapse of the star, since such a collapse requires the removal of large amounts of gravitational energy produced by contraction in the interior of the star. In fact, quite independent of whether the matter in the center consists of charged nuclei or neutrons, the heat produced by contraction must pass through the entire body of the star, and the rate of energy transport, depending on the opacity of the main body, will be the same in both cases. On the other hand, if we can find some way of removing instantaneously the heat liberated in the central regions, in spite of the opacity of the stellar body, the star will collapse with a velocity comparable to that of "free fall" independent of the kind of particles existing in its interior.

The amount of gravitational energy which is liberated when a star of a mass M contracts from the original radius R_0 to the collapsed radius R_c is given approximately by

$$\Delta U \cong GM^2\left(\frac{1}{R_c}-\frac{1}{R_0}\right) \cong \frac{GM^2}{R_c}. \tag{1}$$

Thus, for example, if a star of the mass and radius of our sun contracts to the size of the companion of Sirius ($R_c = R_0/40$),[4] the total liberation of gravitational energy will be of the order of magnitude of 10^{50} erg. On the other hand, the time of the free-fall collapse from the original radius R_0 to any small value of the radius is given approximately by

$$\Delta t \cong R_0^{\frac{3}{2}}/(GM)^{\frac{1}{2}}. \tag{2}$$

In the case of the sun, Δt will be of the order 10^3 sec. (i.e., about half an hour), so that the mean rate of energy removal necessary for such a collapse is about 10^{14} erg/g sec. If we remember that the collapse of novae and supernovae takes place within a few days, we come to the conclusion that the rate of energy removal in these actual cases may be only several hundred times smaller than given above.

As we suggested in a recent publication,[5] this very fast removal of energy from the interior of the star can be understood on the basis of the present ideas on the role of neutrinos in nuclear transformations involving emission or absorption of β-particles. In fact, when the temperature and density in the interior of a contracting star reach certain values depending on the kind of nuclei involved, we should expect processes of the type

$$\begin{cases} {}_ZN^A + e^- \rightarrow {}_{Z-1}N^A + \text{antineutrino} \\ {}_{Z-1}N^A \rightarrow {}_ZN^A + e^- + \text{neutrino}, \end{cases} \tag{3}$$

which we shall call, for brevity, "urca-processes." The neutrinos formed in the above processes[6] absorb a considerable part of the transformation energy (about $\frac{2}{3}$), and escape with practically no difficulty through the body of the star.

As we shall see later, these processes of absorption and reemission of free electrons by certain atomic nuclei which are abundant in stellar matter may lead to such tremendous energy losses through the neutrino emission that

[2] E. A. Milne, Observatory **54**, 145 (1931).

[3] W. Baade and F. Zwicky, Proc. Nat. Acad. Sci. **20**, 259 (1934).

[4] The companion of Sirius possesses a mass which is almost equal to the mass of the sun, and may be considered as representing the type of the white dwarfs obtained by the collapse of the sun.

[5] G. Gamow and M. Schoenberg, Phys. Rev. **58**, 1117 (1940).

[6] We shall use the term "neutrino" both for ordinary neutrinos and antineutrinos involved in the reaction, since there is no noticeable difference in their behavior. It is also clear that one can *neglect the possibility of mutual annihilation of these particles within a stellar body,* since they escape from the star with practically no collisions.

the collapse of the entire stellar body with an almost free-fall velocity becomes quite possible.

Before discussing in more detail the characteristic features of stellar collapse caused by the urca-processes taking place in the hot interior, we must develop the formulas for the dependence of "neutrino losses" on the temperature and density of matter.

§2. Energy Losses Through Neutrino Emission

Since within a contracting star the central temperature gradually increases from comparatively low to very high values, different urca-processes will assume importance during the different stages of the contraction and collapse. In the very beginning, these processes will involve only those nuclei which can capture free electrons with the energy of few kilovolts, whereas at the height of the collapse the nuclei requiring the energy balance of several Mev may become of primary importance. Thus, it becomes necessary to have the formulas both for nonrelativistic and relativistic electrons. In the relativistic calculations we shall limit ourselves to the extreme relativistic case, since the calculations in the intermediate region are necessarily much more complicated. The values for the intermediate temperatures can be obtained with sufficient approximation by a simple interpolation.

We shall, moreover, consider only the case of an ideal electron gas, which, as we shall see later, is of primary importance for the stars having large masses. It is clear that in the case of degeneracy the decreasing number of free energy levels will reduce considerably the rate of urca-processes, and that these processes will stop entirely when the electron gas becomes completely degenerated.

(A) Nonrelativistic case

Let us consider a unit mass of stellar matter containing n_e free electrons and $n_Z{}^0$ nuclei of atomic number Z, which can capture these electrons and go over into unstable nuclei of atomic number $Z-1$. Since the matter in the stellar interior is almost completely ionized, we may write for the number of free electrons

$$n_e = \tfrac{1}{2}\rho/m_H, \tag{4}$$

where ρ is the density of matter and m_H the mass of the hydrogen atom.[7] For the number of nuclei participating in the urca-process we have evidently

$$n_Z{}^0 = c_Z\rho/A_Z\cdot m_H, \tag{5}$$

where c_Z and A_Z are the concentration and atomic weight of the isotope in question.

If the electron gas is not degenerated, the number of electrons with energy between E and $(E+dE)$ is given by the Maxwellian expression

$$n_e(E)dE = 2\pi^{-\frac{1}{2}}n_e\cdot(kT)^{-\frac{3}{2}}\cdot e^{-E/kT}\cdot E^{\frac{1}{2}}dE. \tag{6}$$

The average life of an electron before being captured by the nucleus through the emission of antineutrinos may be calculated on the basis of Fermi's theory of β-decay, and is given by[8]

$$\tau(E) = \pi \lg 2\cdot\hbar^4c^3/g^2n_Z(E-Q)^2, \tag{7}$$

where g is the Fermi constant and Q the maximum electron energy of continuous β-spectrum ($E_{\max}$).

The number of electrons captured by the nuclei in question per unit volume per unit time is

$$N^- = \int_Q^\infty \frac{n_e(E)}{\tau(E)}dE. \tag{8}$$

Using (6) and (7) we get

$$N^- = \frac{2g^2n_Zn_e}{\pi^{\frac{3}{2}}\lg 2c^3\hbar^4}(kT)^{-\frac{3}{2}}\int_Q^\infty e^{-E/kT}(E-Q)^2E^{\frac{1}{2}}dE \tag{9}$$

or

$$N^- = \frac{2g^2n_Zn_e}{\pi^{\frac{3}{2}}\lg 2c^3\hbar^4}(kT)^2I\left(\frac{Q}{kT}\right), \tag{10}$$

where the integral

$$I\left(\frac{Q}{kT}\right) = \int_{x_0}^\infty e^{-x}(x-x_0)x^{\frac{1}{2}}dx \tag{11}$$

is written in respect to a new variable

$$x = (E/kT)(x_0 = Q/kT). \tag{12}$$

Estimating this integral by the saddle-point

[7] This expression presupposes the absence of hydrogen, since the contraction of a star can start only after all the hydrogen is consumed by (C – H) and (H – H) reactions.

[8] This expression differs from the formula given by H. A. Bethe and R. F. Bacher [Rev. Mod. Phys. **8**, 83 (1936)], since the latter was obtained on the basis of the now abandoned Konopinski-Uhlenbeck form of Fermi's theory.

method, we get

$$I(Q/kT)=I(x_0)=2.15(x_0+5/2)^{\frac{1}{2}}\cdot e^{-x_0}. \quad (13)$$

On the other hand, the number of electrons emitted by the unstable nuclei $(Z-1)$, in the energy interval E, $(E+dE)$, is, according to Fermi

$$dN^+=\frac{g^2m^{\frac{3}{2}}n_{Z-1}}{\sqrt{2}\pi^3\hbar^7c^3}(Q-E)^2E^{\frac{1}{2}}dE \quad (14)$$

and the total number of emitted electrons per second per unit volume

$$N^+=0.152\frac{g^2m^{\frac{3}{2}}n_{Z-1}}{\sqrt{2}\pi^3\hbar^7c^3}\cdot Q^{7/2}. \quad (15)$$

In the case of equilibrium, when an equal number of electrons is absorbed and emitted, we must have $N^+=N^-$ which leads to

$$\frac{n_{Z-1}}{n_Z}=\left[\frac{2g^2n_e}{\pi^{\frac{3}{2}}\lg 2c^3\hbar^4}(kT)^2I\left(\frac{Q}{kT}\right)\right]\cdot\left[0.152\frac{g^2m^{\frac{3}{2}}}{\sqrt{2}\pi^3\hbar^7c^3}Q^{7/2}\right]^{-1} \quad (16)$$

or, since $n_{Z-1}+n_Z=n_Z{}^0$

$$n_Z=\frac{6.4\times10^{-3}\cdot\hbar^{-3}m^{\frac{3}{2}}(kT)^{-2}m_H\rho^{-1}\left(\frac{Q}{kT}+\frac{5}{2}\right)^{-\frac{1}{2}}Q^{7/2}e^{Q/kT}}{1+6.4\times10^{-3}\cdot\hbar^{-3}m^{\frac{3}{2}}(kT)^{-2}m_H\rho^{-1}\left(\frac{Q}{kT}+\frac{5}{2}\right)^{-\frac{1}{2}}Q^{7/2}e^{Q/kT}}\times\frac{c_Z\rho}{A_Zm_H} \quad (17)$$

and

$$n_{Z-1}=\left[1+6.4\times10^{-3}\cdot\hbar^{-3}m^{\frac{3}{2}}(kT)^{-2}m_H\rho^{-1}\left(\frac{Q}{kT}+\frac{5}{2}\right)^{-\frac{1}{2}}Q^{7/2}e^{Q/kT}\right]^{-1}\times\frac{c_Z\rho}{A_Zm_H}. \quad (18)$$

The energy taken up by antineutrinos ejected in the process of electron capture per unit time per unit volume can be easily calculated and found to be

$$W^{(1)}=\frac{8.5g^2n_e\cdot n_Z}{\pi\lg 2\hbar^4c^3}\cdot\left(\frac{Q}{kT}+\frac{7}{2}\right)^{\frac{1}{2}}(kT)^3e^{-Q/kT}, \quad (19)$$

while the energy of neutrinos accompanying electron emission is

$$W^{(2)}=\tfrac{2}{3}Q\lambda n_{Z-1}, \quad (20)$$

where λ is the decay constant of the unstable nucleus $(Z-1)$. Comparing (19) and (20), and bearing in mind the expression for λ, we get

$$W^{(1)}\cong5.5(kT/Q)W^{(2)}, \quad (21)$$

so that for the total energy of neutrino radiation per unit time per unit volume we have

$$W=W^{(1)}+W^{(2)}$$
$$\cong(1+5.5\cdot kT/Q)\cdot\tfrac{2}{3}Q\cdot\lambda\cdot n_{Z-1}. \quad (22)$$

(B) Extreme relativistic case

In this case the energy distribution between the thermal electrons is given by the expression

$$n_e(E)dE=n_ef(kT)\cdot e^{-E/kT}p_eEdE, \quad (23)$$

which is generally correct also for the intermediate velocities. Here p_e is the momentum, and E includes the rest energy mc^2 of the electron. The function $f(kT)$ has a simple form only in the extreme relativistic case, and is then given by

$$f(kT)=c\cdot(kT)^{-3}$$
$$\times[1+(1+m_0c^2/kT)^2]^{-1}\cdot e^{m_0c^2/kT}. \quad (24)$$

Formula (7) for the capture of free electrons contains only the energy $(E-Q)$ of the ejected neutrinos, which were treated relativistically because of their negligible mass, and may also be used directly in the present calculations. Thus, combining (7) with (23) and (24) we get

$$N^-_{\text{relat}}=\frac{g^2n_Zn_e}{\pi\lg 2\hbar^4c^4}f(kT)\cdot(kT)^5I, \quad (25)$$

where $Q'=Q+mc^2$ (i.e., the total emission energy of the electron), and

$$I=\int_0^\infty e^{-x}\left[x^4+2x^3\frac{Q'}{kT}+x^2\left(\frac{Q'}{kT}\right)\right]^2dx. \quad (26)$$

This integral can be easily calculated and is given by

$$I=[24+12Q'/kT+2(Q'/kT)^2]e^{-Q/kT}. \quad (27)$$

For the number of electrons emitted in the energy interval E, $(E+dE)$ we have the relativistic expression

$$dN^+=\frac{mc^2}{2\pi^3\hbar}\left(\frac{gm^2c}{\hbar^3}\right)^2\epsilon(\epsilon^2-1)^{\frac{1}{2}}(\epsilon_0-\epsilon)^2d\epsilon \quad (28)$$

with $\epsilon=E/mc^2$, $\epsilon_0=Q'/mc^2$. Integrating we have

$$N^+=\frac{g^2n_{Z-1}}{60\pi^3\hbar^7c^6}\cdot Q^5. \quad (29)$$

The number of stable and unstable nuclei in equilibrium will now be given by

$$n_Z=\frac{1.2\times10^{-3}\hbar^{-3}c^{-3}m_H\rho^{-1}[12(kt)^2+6Q'kT+Q'^2][1+(1+m_0c^2/kT)^2]Q^5e^{-Q/kT}}{1+1.2\times10^{-3}\hbar^{-3}c^{-3}m_H\rho^{-1}[12(kT)^2+6Q'kT+Q'^2][1+(1+m_0c^2/kT)^2]Q^5e^{-Q/kT}}\times\frac{c_Z\rho}{A_Zm_H} \quad (30)$$

and

$$n_{Z-1}=[1+1.2\times10^{-3}\hbar^{-3}c^{-3}m_H\rho^{-1}[12(kT)^2+6Q'kT+Q'^2][1+(1+m_0c^2/kT)^2]Q^5e^{-Q/kT}]^{-1}\times\frac{c_Z\rho}{A_Zm_H}. \quad (31)$$

The total energy taken up by antineutrinos ejected in the process of electron capture is

$$W^{(1)}=\frac{g^2n_Zn_e}{\pi\lg 2\hbar^4c^3}(kT)^3e^{-Q/kT}\left[120+48\frac{Q'}{kT}+6\left(\frac{Q'}{kT}\right)^2\right]\cdot\left[1+\left(1+\frac{m_0c^2}{kT}\right)^2\right]^{-1}, \quad (32)$$

whereas the neutrino-energy in emission

$$W^{(2)}=\tfrac{1}{2}Q\lambda n_{Z-1}, \quad (33)$$

where λ is again the decay constant. Expressing $W^{(1)}$ through $W^{(2)}$,[9] we get

$$W^{(1)}=2\frac{kT}{Q}\frac{60+24(Q'/kT)+3(Q'/kT)^2}{12+6(Q'/kT)+(Q'/kT)^2}\cdot W^{(2)} \quad (34)$$

and, after some simplifications, we obtain for the total energy of neutrino radiation per unit volume per unit time

$$W=W^{(1)}+W^{(2)}\cong(1+8kT/Q)\frac{Q}{2}\cdot\lambda\cdot n_{Z-1}. \quad (35)$$

(C) Nonrelativistic gas and relativistic β-electrons

Of some special interest is the case when the temperature of electron gas is not yet high enough to treat the main bulk of particles relativistically, whereas, on the other hand, the electrons responsible for urca-process are those from the relativistic Maxwell distribution tail.

It can be easily calculated that in such case the number of nuclei in Z and $Z-1$ states will be given by

$$n_Z=\frac{0.98\times10^{-3}(\hbar c)^{-3}m_H\rho^{-1}Q^{9/2}(kT)^{-\frac{3}{2}}e^{Q/kT}}{1+0.98\times10^{-3}(\hbar c)^{-3}m_H\rho^{-1}Q^{9/2}(kT)^{-\frac{3}{2}}e^{Q/kT}}\cdot\frac{c_Z\rho}{m_H} \quad (36)$$

and

$$n_{Z-1}=\frac{1}{1+0.98\times10^{-3}(\hbar c)^{-3}m_H\rho^{-1}Q^{9/2}(kT)^{-\frac{3}{2}}e^{Q/kT}}\cdot\frac{c_Z\rho}{m_H}. \quad (37)$$

[9] By means of (26), (32), and the equilibrium relation $N^+=N^-$.

For $W^{(2)}$ we have as before

$$W^{(2)}=n_{Z-1}\cdot\lambda\cdot\tfrac{1}{2}Q, \tag{38}$$

whereas $W^{(1)}$ is given by

$$W^{(1)}=7.1\frac{kT}{Q}\frac{(Q/kT+7/2)^{\frac{1}{2}}}{(Q/kT+5/2)^{\frac{1}{2}}}W^{(2)}. \tag{39}$$

Thus for the total energy of neutrino emission per gram per second we have in this case approximately

$$W\cong(1+7.1\cdot kT/Q)\cdot\tfrac{1}{2}n_{Z-1}\lambda Q. \tag{40}$$

§3. Urca-Processes with Various Elements

Since practically any stable nucleus leads to an unstable isobar after capturing one electron, there must be, altogether, many hundreds of possible urca-processes presenting a large display of all possible energies and decay constants. The dependence between the decay energy and the decay constant permits arranging all such processes in one row, beginning with very slow processes, at comparatively low temperatures, and ending with the very fast ones requiring extremely high thermal velocities.[10]

Since our present list of the known Fermi elements is very limited and contains only a small fraction of all possible β-active nuclei, the choice of a particular reaction, or group of reactions, which would be of special importance for the theory of stellar collapse, presents serious difficulties. This choice can be, however, somewhat simplified if we remember that the importance of any given reaction depends not only on its absolute efficiency but also, to a large extent, on the abundance of the element in question in stellar material.

As an example of urca-process which would take place at a comparatively low temperature, we may mention the case of the light helium isotope. The process will take place according to the scheme

$$\begin{cases} He^3+e^-\rightarrow H^3+\text{antineutrino} \\ H^3\rightarrow He^3+e^-+\text{neutrino.} \end{cases} \tag{41}$$

The decay of H^3 was recently investigated by O'Neal and Goldhaber[11] who found 15 ± 3 kv for the maximum energy of β-particles and 7×10^{-10} sec.$^{-1}$ for the decay constant (decay period being about 30 years). Since the energy of 15 kv corresponds to the temperature of 1.2×10^{8}°C, i.e., only six times higher than that in the center of the sun, we see that the reaction must take place with some speed even under solar conditions.

However, because of the small value of λ, the neutrino losses due to this reaction are comparatively unimportant. In fact, even when most of the nuclei He^3 are transformed into H^3, the energy of neutrinos emitted per gram per second will be given by

$$W^{(2)}_{\text{sat}}=\frac{2}{3}Q\cdot\lambda\cdot\frac{c_Z}{3\cdot m_H}\cong2\times10^{6}\frac{\text{erg}}{\text{g sec.}}\times c_Z, \tag{42}$$

a comparable amount being carried away by the antineutrinos emitted in the capture process. Thus, even for a star constructed entirely of He^3 ($c_Z=1$), the energy losses are several million times lower than are necessary for a rapid collapse. There is, however, every reason to believe that He^3 is not very abundant in the stellar interior because of its transformation into ordinary helium through the capture of a thermal proton. The only other known β-decaying nucleus with low energy balance is that of the still less abundant radioactive element RaD ($E_{\max}\cong35$ kv, $\lambda\cong1.4\times10^{-9}$ sec.$^{-1}$), but there must surely be more Fermi elements of this class which are as yet undiscovered because of their comparatively weak activity.

As an example of an intermediate case we can give the urca-process with the main iron isotope which reacts according to the scheme

$$\begin{cases} Fe^{56}+e^-\rightarrow Mn^{56}+\text{antineutrino} \\ Mn^{56}\rightarrow Fe^{56}+e^-+\text{neutrino.} \end{cases} \tag{43}$$

This iron isotope forms probably about 12 percent of stellar matter and the nucleus Mn^{56} is known to possess a maximum electron energy of

[10] The effect of permitted and nonpermitted β-transitions impairs somewhat the above regularity, and must be investigated separately in each particular case.

[11] R. D. O'Neal and M. Goldhaber, Phys. Rev. **58**, 574 (1940).

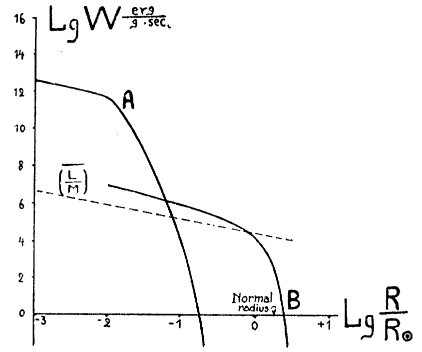

FIG. 1. Energy-losses through the neutrino emission in a contracting star with $M=5M(\odot)$ and $\mu=1.7$ (H completely substituted by He). Curve A for the urca-process in Fe^{56} ($E=1.7$ Mev; $\lambda=7.7\times10^{-5}$ sec.$^{-1}$; concentration 10 percent). Curve B for the urca-process in He^3 or similar element ($E=15$ kv; $\lambda=7\times10^{-10}$ sec.$^{-1}$; assumed concentration one percent). Broken line represents the average radiative energy losses per unit mass calculated from the ordinary luminosity formula for the contracting model. "Normal radius" represents the radius which the star had while still in the main sequence.

1.7×10^6 ev and the decay constant of 7.7×10^{-5} sec.$^{-1}$ (decay period 2.5 hours).[12]

This reaction, which becomes saturated at the temperature of 10^{10}°C, will remove in the form of neutrino radiation about 10^{11} erg per second per gram of stellar matter.

As the final example we may take ordinary oxygen representing about 30 percent of stellar material. In this case the urca-process takes place according to the scheme

$$\begin{cases} O^{16}+e^-\rightarrow N^{16}+\text{antineutrino} \\ N^{16}\rightarrow O^{16}+e^-+\text{neutrino}. \end{cases} \tag{44}$$

The energy balance and the decay constant of N^{16} are 6×10^6 ev and 7.7×10^{-2} sec.$^{-1}$ (decay period 9 sec.), respectively, which leads to the energy losses of about 10^{16} erg per second per gram at the saturation point.

Comparing this value with the energy losses necessary for the free-fall collapse, we find that such a collapse can take place even if only one percent of the stellar body participates in the urca-process.

In order to get some idea of the energy losses in different stages of stellar contraction, we calculate numerically the rate of the urca-process with iron in a contracting star of mass equal to ten sun masses. The temperature and density distribution within a gravitationally contracting star is very closely represented by the Emden's polytrope with the index 3, and the values in the center are given by the formulas[13]

$$T_c=1.2\times10^{-15}M/R, \tag{45}$$

$$\rho_c=13M/R^3, \tag{46}$$

where M and R are the mass and the radius of the star. Taking $M=5\cdot M(\odot)\cong1\times10^{34}$ g, and expressing R in terms of solar radius $R(\odot)\cong7\times10^{10}$ cm we get

$$T_c=1.7\times10^8\cdot[R/R(\odot)]^{-1}\cdot\,^{\circ}\text{C}, \tag{47}$$

$$\rho_c=385\cdot[R/R(\odot)]^{-3}\cdot\text{g/cm}^3. \tag{48}$$

Assuming the data given above for the iron reaction, and using formulas (18), (22), (31) and (35), we get for the energy losses through neutrino emission (per second per gram) the values represented in Fig. 1. Since the luminosity of a contracting star, as the function of its mass and radius, is given by the expression[14]

$$L=6\times10^{34}[M/M(\odot)]^{5.5}\times[R/R(\odot)]^{-0.5}\text{ erg/sec.} \tag{49}$$

we see that, even if the urca-process is limited to one percent of the stellar mass, neutrino losses become comparable to radiative losses at $R=0.1R(\odot)$. For $R=0.01R(\odot)$ the losses through neutrino emission will reach the tremendous value of 10^{12} erg/g sec.

§4. DYNAMICS OF THE COLLAPSE

Since the hydrodynamical equations describing the collapse of a stellar body, cooled from the center by neutrino emission, are necessarily very complicated, we shall limit ourselves in the present article to the entirely qualitative discussion.

It must be clear first of all that in different stages of stellar contraction the most important

[12] See, for example, G. Gamow, *Structure of Atomic Nuclei* (Oxford University Press, 1937), Table *F*.

[13] See, for example, A. Eddington, *Internal Constitution of Stars* (Cambridge University Press, 1926). We take here the mean molecular weight of stellar matter to be equal to 1.7, which corresponds to 35 percent helium, and 65 percent of heavier elements.

[14] A. Eddington, reference 16. In this expression, as well as formulas (45) to (48), we assume $\mu=1.7$, and the opacity equal to that of the Russell mixture, forming 65 percent of stellar matter (the rest being helium).

role is played by urca-processes with different elements. In the very beginning, when the central temperature is comparatively low, neutrino cooling can be produced only by the elements with the low value of energy balance. As we have seen above, energy losses due to such elements are rather small, even at the saturation point, so that one may expect here only a slight acceleration of normal gravitational contraction. However, as the central temperature rises, new and more powerful urca-processes are introduced, the rate of contraction increases, and there results a catastrophic collapse.

It is important to keep in mind that, since urca-processes take place in the regions of highest temperature, the energy flow towards these points, either through radiative or through convective mechanisms, is entirely excluded.

Thus, in order to maintain a central pressure necessary for the support of the outer layers, *the star must develop in its interior a rapidly growing central condensation.* The heat which will be generated in compressing the extra material into this small central region will be rapidly removed by neutrino emission, and the whole process will be somewhat analogous to the condensation of water vapor contained in a vertical vessel with the bottom cooled by liquid air.

However, whereas in the above example the vertical vapor column, unsupported from beneath, will simply fall down as a whole, gas masses forming the body of a collapsing star will be strongly heated by compression.

Since this heat cannot escape towards the center (except from the layers in the immediate vicinity of the urca-region), part of it must remain in the collapsing body and part be radiated from the surface, thus increasing the stellar luminosity. It is not difficult to see that, under such conditions, inner parts of a stellar body will continue to move towards the center, whereas *the outer regions will begin to expand.*

Such an expansion of the outer parts of the star, resulting in the decrease of their opacity and the increase of radiating surface, must necessarily lead to a rapid rise of luminosity which we observe as the "flare-up" in the nova phenomena. It must be noticed that this "flare-up" probably takes place as soon as the collapse of the interior has progressed to a high degree. At a certain stage the rapidly increasing radiative pressure overbalances the weight of stellar atmosphere, and large masses of gas begin to flow outside forming the luminous shell characterizing all nova and supernova explosions. We must stress here the point that, according to the proposed theory, the increase of luminosity during stellar collapse is an entirely secondary phenomenon, so that in the first approximation these changes of luminosity may be easily neglected.

Let us turn now to the question of the ultimate fate of a collapsing star, and the possible reasons for the difference between the ordinary novae and supernovae.

It was shown by Chandrasekhar[15] that, whereas the stars with mass smaller than $5.75M(\odot)/\mu^2$ $(=2\cdot M(\odot)$ for the molecular weight $\mu=1.7)$ possess stable configurations with a certain minimum radius (white dwarfs), the stars having the largest masses are subject to unlimited contraction.[16] In view of this fact, it is natural to expect that, for the star masses smaller than the above-given critical value, the collapse will occur on a much smaller scale than for heavier bodies. The increasing condensation in the interior of such light collapsing stars will sooner or later lead to electron degeneracy which will slow down and finally stop all urca-processes. For these small scale collapses we should expect that the final state will be represented by a white dwarf of a mass comparable to that of the original star, and that the amount of material thrown out in the form of a gas shell will be comparatively small. This expectation seems to be in good agreement with observations of ordinary nova phenomena, where the expelled gas shell takes up only about one-hundredth of a percent of stellar mass, and is completely dissolved in space several years after the explosion.

On the other hand, the stars possessing a mass larger than the critical one will undergo a much more extensive collapse, and their ever-increasing radiation will drive away more and more material from their surface. The process will probably not

[15] See, for example, S. Chandrasekhar, *Introduction into the Study of Stellar Structure* (Chicago University Press, 1939), Chapter IV.

[16] This results from the fact that in very heavy stars the velocities of electrons in the generated Fermi gas approach the velocity of light, after which such a gas is unable to support the weight of the stellar body.

stop until the expelled material brings the mass of the remaining star below the critical value. This process may be compared with the supernovae explosions, in which case the expelled gases form extensive nebulosities (as "Crab-Nebula" or "Filamentary Nebula"), apparently assuming this permanent state after the explosion. This point of view seems also to acquire some confirmation from the fact that the relative number of novae and supernovae occurring in stellar systems is of the same order of magnitude as the relative number of stars having masses smaller and larger than $2\cdot M(\odot)$.

Conclusion

In the present article we have developed the general views regarding the role of neutrino emission in the vast stellar catastrophes known to astronomy. It must be emphasized that, while the neutrinos are still considered as highly hypothetical particles because of the failure of all efforts made to detect them, the phenomena of which we are making use in our considerations are supported by the direct experimental evidence of nuclear physics. In fact, the experiments of Ellis and Wooster[17] and of Meitner and Orthman[18] *leave no doubt that the energy balance does not hold in the processes of radioactive β-transformations*, and all later evidence on this subject strongly indicates that this "disappearance of energy" occurs in such a way that it appears to be carried away by particles of almost unlimited penetrability.

Whereas the fundamental ideas of the proposed theory are very simple and the physical part of calculations pertaining to the rate of neutrino emission can be easily carried out on the basis of existing formalism, the problem of the dynamics of the collapse represents very serious mathematical difficulties. It is to be hoped that these difficulties will be overcome by the choice of some suitable simplified model.

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Differential Measurement of the Meson Lifetime

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Measurements of the meson lifetime were made by comparing the lead absorption curves of the cosmic radiation at two different elevations, and by compensating the air column at the higher level by a layer of graphite placed on top of the counter telescope. The method does not require any knowledge of the energy distribution or of the height of the producing layer of the mesons. All systematic errors are greatly reduced by the differential method used and can be shown to be insignificant. The value of the lifetime comes out to be appreciably shorter than in previous determinations, *viz.* $(1.25\pm0.3)\times10^{-6}(\mu c^2/10^8\ \mathrm{ev})$ sec.

1. The Method

After the instability of the mesons was suggested by Yukawa[1] from theoretical reasons, experimental evidence of various kinds for this effect was discussed by several authors, notably by Euler and Heisenberg.[2] A more

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