THE

Physical Review

A Journal of Experimental and Theoretical Physics Established by E. L. Nichols in 1893

Vol. 59, No. 7

APRIL 1, 1941

Second Series

The Radioactivity of Mn⁵⁶ and I^{128*}

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The purpose of this work was to examine the β -ray spectra of Mn⁵⁶ and I¹²⁸ as seen in the Wilson cloud chamber in an effort to correlate the measurements with the existing empirically regarded relations of the Fermi or the Konopinski-Uhlenbeck theory. A further study of the γ -rays which accompany these β -rays was made. In all measurements the difficulties of cloud-chamber technique were critically investigated. The results appear to fit the Fermi rather better than the K–U theory. However, the errors of cloud-chamber measurement are seen to be very serious in this type of work.

[•]HE β-ray spectrum of Mn⁵⁶ was first measured in the cloud chamber by Gaerttner, Turin and Crane.¹ Using a magnetic field of 850 gauss, they found the spectrum to consist of a single K-U group having an extrapolated end point at 6.5 mc^2 (2.8 Mev). This spectrum was later measured by Brown and Mitchell.² The fields in their cloud chamber were 375 and 490 gauss. They found that the K-U plot resolves itself into two groups having extrapolated end points at 6.8 mc^2 (3.0 Mev) and 3.4 mc^2 (1.2 Mev). The same spectrum was also measured in a β -ray spectrograph by Alichanow, Alichanian, and Dzelepow.³ They reported the end point to be at 7.3 mc^2 (3.2 Mev), which does not agree with the results given above.

It seemed to us that we might be able to

remove the uncertainties in the previous work by means of additional experiments. As a result of our work, we are able to report that the upper limit of this spectrum is close to the point found by Gaerttner, Turin and Crane;¹ the second group announced by Brown and Mitchell actually does appear to be part of the spectrum; in addition, we find evidence of a third group if the K-U theory be applied.

The γ -rays from Mn⁵⁶ have been investigated by Mitchell and Langer,⁴ using coincidence counters to determine the energies of the Compton electrons ejected from an aluminum plate. They obtained 1.65 Mev as the γ -ray energy, and claimed that their work shows the γ -ray to be monochromatic. Later, Livingood and Seaborg⁵ measured the energy of this γ -ray by finding the thickness of lead necessary to reduce its intensity to one-half. The energy thus obtained was 1.2 Mev. In the spring of 1939,

^{*} An abridgment of the dissertation submitted by R. H. B. in partial fulfillment of the requirements for the Ph.D. degree at New York University.

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¹ Gaerttner, Turin and Crane, Phys. Rev. **49**, 793 (1936). ² M. V. Brown and A. C. G. Mitchell, Phys. Rev. **50**, 593 (1936).

³ Alichanow, Alichanian and Dzelepow, Nature **136**, 257 (1935); Physik. Zeits. Sowjetunion **10**, 78 (1936).

⁴ A. G. G. Mitchell and L. M. Langer, Phys. Rev. **52**, 137 (1937).

⁵ J. J. Livingood and G. T. Seaborg, Phys. Rev. **54**, 391 (1938).



FIG. 1. The effect of the radius of curvature of the tracks upon the observed shape of the spectrum. If a track is to be measured, it must pass inside the circle of radius *B*, or, at least, become tangent to it. As shown in the lower left diagram, one condition for this tangency is given by

$$(\rho + B)^2 = \rho^2 + A^2 - 2\rho A \cos \phi_{\max}$$

 $\therefore \cos \phi_{\max} = (A^2 - B^2 - 2\rho B)/2\rho A.$

From the diagram at the right, it is seen that for tracks whose radius of curvature is less than (A+B)/2, $\phi_{\min}=0$; for tracks whose radius of curvature is greater than (A+B)/2, we have

$$(\rho - B)^2 = \rho^2 + A^2 - 2\rho A \cos \phi_{\min}$$

$$\therefore \quad \cos \phi_{\min} = (A^2 + B^2 + 2\rho B)/2\rho A.$$

When ρ is less than (A-B)/2, the track cannot enter the visible portion of the chamber at all.

Norling⁶ performed a coincidence experiment from which he concluded that there are two γ -ray lines, one of which follows the most energetic β -ray transition.

We have measured this γ -ray spectrum by examining the Compton electrons ejected from a thin sheet of Bakelite placed along a diameter of the cloud chamber. The distribution of the observed electrons indicates γ -ray lines at about 0.6 or 0.7 Mev and at about 1.7 Mev. There is also some indication of other lines not clearly resolved.

As this work was being completed, Dunworth⁷ reported another measurement based on coincidence counting. From his work he concluded that there are two lines in the γ -ray spectrum: one at about 0.6 Mev and the other at about 1.7 Mev. The relative intensities given are in agreement with our results. It should be noted that these γ -ray line intensities are not in accord with the relative population of the two β -ray groups as determined either by Brown and Mitchell² or by ourselves. Two months later, Langer, Mitchell and McDaniel⁸ reported a $\beta - \gamma$ coincidence counter measurement from which it may be inferred that there are at least two lines in this spectrum, but nothing is stated about their energies. A later measurement is that of Curran, Dee and Strothers.⁹ They determined the distribution of the Compton electrons ejected from an aluminum window in a modified β -ray spectrograph. Their results indicate γ -ray lines at 0.91 and 2.03 Mev, the less energetic γ -ray being twice as intense as the harder.

The end point of the β -ray spectrum of I¹²⁸ was first determined by Alichanow, Alichanian and Dzelepow.3 Our measurements agree with theirs as to the position of this end point. From the shape of the spectrum we find two groups having upper limits at 1.05 and 2.10 Mev. However, we were unable to find any γ -ray of sufficient intensity to account for two groups in this spectrum. Roberts and Irvine¹⁰ also came to the conclusion that if there be any γ -ray accompanying this radioactivity, there must be less than one photon for every ten electrons observed. Later, a γ -ray of about 0.4 MeV was reported by Livingood and Seaborg,¹¹ but neither the energy nor the intensity agrees with the β -ray spectrum. Further work was done by Tape.¹² His observations on the γ -ray agree with ours; those on β -particles do not.

Apparatus

The cloud chamber consists of an aluminum piston, 7 inches in diameter and 6 inches high, moving in a brass cylinder. The expansion ratio is controlled by screwing the cylinder into or out of the brass cup which forms the base of the chamber. The joint between cylinder and cup is sealed with Apiezon Q. To the top of the cylinder is fitted a removable head, sealed in place with Apiezon Q.

Toward the end of an expansion, the chamber was illuminated by six 1000-watt clear glass

- ¹¹ J. J. Livingood and G. T. Seaborg, Phys. Rev. 54, 777 (1938).
- ¹² G. F. Tape, Phys. Rev. 56, 965 (1938).

⁶ F. Norling, Naturwiss. 27, 432 (1939).

⁷ J. V. Dunworth, Nature 143, 1065 (1939).

⁸ Langer, Mitchell and McDaniel, Phys. Rev. 56, 422 (1939). ⁹ Curran, Dee and Strothers, Proc. Roy. Soc. A174, 546

^{(1940).} ¹⁰ A. Roberts and J. W. Irvine, Jr., Phys. Rev. **53**, 609 (1938).

photoflood lamps, made specially for us by The Westinghouse Electric and Manufacturing Company. The life of these lamps was in excess of 30,000 expansions. Cylindrical lenses and suitable slits were used to collimate a flat beam of light about 1 cm high across the chamber. The pictures were taken along the lines of force of the magnetic field. The cycle of operations was controlled by a commutator whose shaft also operated the camera through a pair of universal joints.

The coils for producing the magnetic field have already been described.¹³ They are more efficient than a Helmholz-Gaugain arrangement—only 10 amperes at 240 volts were required to produce a uniform field of 880 gauss throughout the volume of the chamber.

EXPERIMENTAL DETAILS

The magnetic field was measured in the usual way by means of a ballistic galvanometer, standard mutual inductance, and suitable exploring coils of accurately known constants. The error of these measurements was less than 2 percent throughout the volume of the chamber.

The radius of curvature of each track was determined by reprojecting the photographic image to full size on a sheet of paper and matching the track to the edge of one of a series of brass disks. These disk gauges were prepared with radii from $\frac{1}{2}$ to 25 cm in half-centimeter steps. As each track was measured, an arc was drawn in red pencil over its image. This method was found helpful in preventing the omission or the double tally of a track.

In the course of this work, it appeared that the shape of the β -ray spectrum was different for different fields. An approximate geometric solution for this effect is shown in the legend of Fig. 1: ϕ =the angle which the horizontal component of the electron's initial momentum makes with the horizontal tangent to the emitting surface. A=inside radius of the cloud chamber = radius of the emitting surface = 9 cm. B= radius of the visible portion of the cloud chamber. ρ =radius of the horizontal projection of the electron track.

If an electron is to pass into the visible portion of the chamber, it must cross the circle of radius B, or at least become tangent to it. The angles of emission between which this is possible are indicated in Fig. 1. B is a function of the position of the point of emission on the surface of radius A: this is clear from the upper portion of Fig. 1.

The probability of an electron path of given radius passing inside the circle of radius *B* is proportional to $(\phi_{\max} - \phi_{\min})$, where the ϕ 's are, in turn, functions of the position of the point of emission. The curve in Fig. 2 shows this theoretical probability plotted against the radius of curvature of the tracks. In the same figure is shown the value of the ratio:

number of tracks actually observed

	in a given momentum interval	
number given	for that interval by the	•

best K-U curve to fit the entire data

Finally, for the special runs described in the next paragraph, the ratio:

number of tracks observed at

		850 ga	uss in	a giv	en interv	al
number to	be expe	cted in	that			

interval from the 319 gauss curve

is denoted by the points marked \oplus .

The apparent shape of the spectrum was found to depend also upon the position and size



FIG. 2. Comparison of the quantity $(\phi_{max} - \phi_{min})$ (smooth curve) with the ratio

observed number of tracks
number demanded by the best K-U curve
The points denoted by \oplus are the plot of the ratio
observed number of tracks at 850 gauss

number to be expected from the 319-gauss curve'

In the case of I¹²⁸, the conditions applying to the calculation of $(\phi_{\max} - \phi_{\min})$ were not constant throughout the measurement of this spectrum, so no computed curve is shown.

¹³ R. H. Bacon, Rev. Sci. Inst. 7, 423 (1936).



FIG. 3. K–U plot of the energies of 7200 electrons emitted by Mn⁵⁶.

of the source. In our early work the Mn⁵⁶ was deposited on filter papers 3.5×11 cm which were affixed to the glass ring of the chamber top. In our later work, filter papers 2×11 cm were used. These narrower sources always showed a higher proportion of low energy tracks. For comparison purposes a series of runs was made with a wide and a narrow filter paper on opposite sides of the chamber head. Each run consisted of ten pictures at 850 gauss followed by ten at 319 gauss. A total of 2000 tracks obtained in this manner were measured. It was found that the yield of low energy tracks was the same for each source whereas the yield of high energy tracks was proportional to the width of the source.

THEORY

Experiments may be compared with the theory of Fermi14 or with that of Konopinski and Uhlenbeck¹⁵ by a method due to Kurie, Richardson and Paxton.¹⁶ The theory may be expressed in the form

$$(N/f)^{1/k} = K [(1+\eta_0^2)^{\frac{1}{2}} - (1+\eta^2)^{\frac{1}{2}}], \qquad (1)$$

where N = number of electrons having momentum $\eta = H\rho/1700$, $\eta_0 =$ momentum of the fastest electron emitted,

$$f = f(Z, \eta) = \frac{2\pi\alpha Z \eta (1+\eta^2)^{\frac{1}{2}}}{1-e^{-2\pi\alpha Z (1+\eta^2)^{\frac{1}{2}}/\eta}},$$



FIG. 4. Fermi plot for Mn⁵⁶ of the same data as used in Fig. 3. Although the plotted points between 3.4 and 5.1 mc² fall quite accurately on a straight line, it is not possible to resolve this spectrum further by this method, as the Fermi plot of the differential is a smooth curve through which it is impossible to pass a significant straight line.

K = constant of proportionality, depending uponthe total number of electrons counted in the experiments, k=2 in Fermi's theory, =4 in that of Konopinski and Uhlenbeck.

A plot of $(N/f)^{1/k}$ against $(1+\eta^2)^{\frac{1}{2}}$ should result in a straight line, depending on which theory fits the experimental facts. A plot of $(N/f)^{\frac{1}{2}}$ against $(1+\eta^2)^{\frac{1}{2}}$ does result in a straight line for many problems, at least in the middle energy range. In some cases, however, the plotted points do not arrange themselves in a straight line, but along a curve that suggests the β -rays might be divided into two or more groups.

It should be emphasized here that the process of decomposing the β -ray spectrum into groups is independent of the validity of the K-U theory. This process was first suggested by Ellis and Mott.¹⁷ They found that the RaC spectrum could be built of five components, each having the same shape as the RaE spectrum, the latter being regarded by them as single. The end points and relative populations of these five components were correlated with the known energies and intensities of the γ -rays accompanying the RaC \rightarrow RaC' disintegration.

Now it has been shown by several workers¹⁸ that, except at the end points, the β -ray spectrum

¹⁴ E. Fermi, Zeits. f. Physik 88, 161 (1934).

¹⁵ E. J. Konopinski and G. E. Uhlenbeck, Phys. Rev. 48, (1935)

¹⁶ Kurie, Richardson and Paxton, Phys. Rev. 49, 370 (1936).

¹⁷ C. D. Ellis and N. F. Mott, Proc. Roy. Soc. A141, 502

^{(1934).} ¹⁸ E. M. Lyman, Phys. Rev. **51**, 1 (1937); L. M. Langer and M. D. Whitaker, Phys. Rev. **51**, 713 (1937); J. S. O'Conor, Phys. Rev. **52**, 303 (1937).

of RaE closely follows the K-U theory. Therefore, if the process suggested by Ellis and Mott be valid, and if the RaE spectrum be simple, then the K-U theory affords a convenient implement for effecting such a resolution into groups, whether the theory itself be valid or not. But if the assumption that the RaE spectrum is simple be incorrect, then it is not correct to interpret departures from a straight line in the K-U plot as evidence of group structure. In the following procedure nothing is assumed as to the theoretical validity of the K-U theory: Equation (1) is used merely as a convenient method of effecting the process proposed by Ellis and Mott.

Results

The work on the β -ray spectrum of Mn⁵⁶ may be divided into two parts: During the first part of this work we had but 50 mg of radium mixed with beryllium as a neutron source for activating the manganese, which was deposited on filter papers 3.5×11 cm. The chamber was filled with helium (sometimes with hydrogen) for some of the runs, and left filled with air for others. Runs were taken at four magnetic fields: 850, 637, 425 and 319 gauss. The results of the runs with the three higher fields, as already announced,¹⁹ may be summarized as follows:

The 850-gauss run duplicated almost exactly the curve of Gaerttner, Turin and Crane:¹ The observed electrons seemed to fit a single K-U group having an upper limit at $6.58\pm0.10 mc^2$ $(2.84\pm0.05$ Mev). The runs at 637 and 425 gauss seemed to support the work of Brown and Mitchell:² The observed electrons seemed to fit into two groups, having upper limits at 6.55 and 3.25 mc^2 (2.80 and 1.15 Mev), respectively. The end points given by Brown and Mitchell are 6.8 and 3.4 mc^2 (2.9 and 1.2 Mev), respectively.

A short run at 319 gauss showed more slow electrons than those demanded by these two K-U groups. This led us to pursue the matter further.

Meanwhile, we acquired the new neutron source containing 200 mg radium mixed with beryllium. We now cut our filter papers into strips 2×11 cm. Runs were taken at 850, 637 and 319 gauss. As mentioned above, the results of this part of the work differed from those of the previous summer only in that the relative populations of the low energy groups now seemed to be greater than they did at first, and the great excess of slow electrons observed in the weakest field seemed to fit a third K-U group having an end point at about 2.2 mc^2 (0.6 Mev). See Figs. 3, 4 and 10.

The Gamma-Rays Accompanying the Decay of Mn⁵⁶

The solution of NaMnO₄ was irradiated in the regular way overnight and filtered in the morning. The still moist filter paper was then rolled into as tight a wad as possible (about 5 mm in diameter), and placed in a rest between the field coils, in the midplane of the cloud chamber. Owing to the weakness of this source, as compared with those used by Richardson and Kurie,²⁰ for instance, it was not feasible to place it as far



FIG. 5. Observed distribution of 400 electrons ejected from Bakelite strip by the γ -rays accompanying the decay of Mn⁵⁶. Upper half: Open circles, data obtained by Richardson and Kurie, Phys. Rev. 50, 999 (1936). Electrons ejected from radiator 40 mg/cm² by annihilation radiation from N¹³, 250 gauss. Closed circles, electrons ejected from radiator 100 mg/cm² by annihilation radiation from Cu isotopes excited by bombarding Ni with protons, 425 gauss. Lower half: Electrons emitted from radiator 100 mg/cm² by gamma-rays accompanying the decay of Mn⁵⁶, 425 gauss.

¹⁹ Bacon, Grisewood and van der Merwe, Phys. Rev. 52, 668 (1937).

²⁰ J. R. Richardson and F. N. D. Kurie, Phys. Rev. 50, 999 (1936).



FIG. 6. Observed distribution of 1300 electrons emitted by I^{128} .

from the chamber as they did. There is thus introduced into the measurement of the angle between the observed direction of the initial motion of the Compton electron and the assumed direction of motion of the incident photon a small error of not more than 5° in the worst instance.

As a check on this work, we measured the energy of the annihilation radiation from the Cu positron emitters formed by the bombardment of a nickel foil by protons in the Columbia cyclotron.²¹ As metallic nickel would disturb the magnetic field, the irradiated foil was dissolved in HNO₃, and the small mass of Ni(NO₃)₂ was laid in the same place as had been the Mn⁵⁶ before. The results of these measurements are shown in Fig. 5. For comparison, there is also drawn to appropriate scale the result obtained by Richardson and Kurie²⁰ who used a lamina of about two-fifths the surface density of ours to measure the annihilation of radiation from N¹³.

The results²² of our measurements, as shown in Fig. 5 seem to indicate a γ -ray line at about 600 or 700 kev, and perhaps one at about 1.7 Mev. However, the complete interpretation is not clear; it is probable that there are other lines also. For the relative intensities, we obtain

Sum of the intensities of all lines

below 0.7 Mev Sum of the intensities of all lines above 0.7 Mev



FIG. 7. K-U plot of the data shown in Fig. 6.



FIG. 8. Fermi plot of the data shown in Fig. 6.

These results agree with those of Dunworth;⁷ the rest of our analyses of these data do not.

These estimates of the γ -ray energies, however, do not agree with those of Curran, Dee and Strothers.⁹ A new measurement was therefore made in this laboratory by Titone,²³ using a radiator of only 25 mg/cm², and taking advantage of a total of 600 mg radium mixed with beryllium, which was at that time available. His results confirm ours.

Discussion of Results of Measurements of the Radioactivity of Mn⁵⁶

The γ -rays from Mn⁵⁶ are quite intense compared with the total activity. For the arrange-

²¹ We take this opportunity to express our appreciation to Professor J. R. Dunning and Dr. E. T. Booth for their kindness and interest.

²² Bacon, Grisewood and van der Merwe, Phys. Rev. 56, 1168 (1939); Phys. Rev. 57, 240 (1940).

²³ L. V. Titone, essay for the degree of Master of Science, New York University, 1940.

ment of filter papers used, about one twentyfifth of the number of electrons measured could have been secondaries ejected from the source by γ -rays. As most of these secondary electrons are of low energy, it is probable that a portion of the "third group" really consists of secondary electrons. Furthermore, the filter papers were placed so close to the glass ring that a number of electrons must have been reflected back from the ring into the visible portion of the chamber. Again there is the effect of scattering in the source itself. All three effects tend to enhance the low energy end of the spectrum.

It might be argued that the "third group" represents the partial internal conversion of a γ -ray line at about 600 or 700 kev. This possibility cannot be definitely excluded, but it seems unlikely, as the observed unconverted intensity of this line is quite strong already, and unless there are several lines of about this energy in the spectrum, the possibility of internal conversion seems small.

In the coincidence counting experiments, Norling,⁶ Dunworth⁷ and Langer, Mitchell and McDaniel⁸ find that there is at least one γ -ray line accompanying the highest energy electron group; and Dunworth finds that this γ -ray line is the one having energy about 600 kev, and then supposes that the second β -ray transition is followed by two γ -rays, the 1.7 and the 0.6 Mev in turn. The findings of Langer, Mitchell and McDaniel also suggest this.

Now, the second group of β -rays is twice as intense as the most energetic group. The high energy γ -ray, therefore, should, according to Dunworth's picture be two-thirds as intense as the low energy γ -ray. Dunworth, however, and we also, find that the high energy γ -ray is only about two-fifths as intense as the low energy γ -ray.

Thus, in order to correlate the γ -ray measurements with those of the β -rays, it is necessary to postulate more than two β -ray transitions, or more than two γ -ray transitions, or both. Hence, the energy level diagram given by Norling or by Dunworth is incomplete.

Finally we have tried the Fermi plot in Fig. 4. Assuming that the distortion above $3 mc^2$ (1 Mev) is small, we found by the method of least squares the best straight line among the points plotted between 3.4 and 5.1 mc^2 (1.2 and 2.1 Mev). Having done this, one can postulate two groups of energy 5.5 and 3.4 mc^2 (2.3 and 1.2 Mev). But the fit is so bad that it is impossible to say anything about either relative populations or the existence of any lower groups.

The Radioactivity of I^{128}

The results²⁴ shown in Figs. 6, 7 and 8 may be summarized as follows: The observed electrons appear to have energies up to about 1.85 Mev. The K-U plot apparently shows two groups having upper limits at 3.06 and $5.10\pm0.10 \ mc^2$ (1.05 and 2.10 ± 0.05 Mev). The supposition of two groups in the β -ray spectrum of I¹²⁸ suggests that either the groups have different periods, or else that γ -rays exist whose energy accounts for the energy difference between the two groups.

We measured the period on a Geiger counter several times, following one run through twelve half-lives (see Fig. 9). The period was found to be single to within one minute. The value was 26 ± 1 minute. We also looked for a γ -ray. The conclusion was that there was less than one photon for every 10 electrons.



²⁴ Bacon, Grisewood and van der Merwe, Phys. Rev. 54, 315 (1938).



FIG. 10. The high energy end of the β -ray spectrum of Mn⁵⁶, drawn to much enlarged scale. It will be seen from this figure, that the predictions of the two theories are quite indistinguishable over an appreciable range.

Conclusions

Our results appear to lend somewhat greater support to the K-U theory than to the original Fermi theory of beta-decay. This is in accord with the findings of most observers. This apparent agreement with the K-U theory has been ascribed by Konopinski²⁵ to the distortion of the true spectrum by the scattering of the electrons in the β -ray source. In support of this explanation is the recent work of Tyler^{26} and of $\mathrm{Lawson},^{27}$ who find that as the source is made thinner, the observed spectrum departs from the K-U and begins to approach the Fermi distribution; against this explanation is the work of Richardson and Leigh-Smith,²⁸ who upon measuring the β -ray spectrum of ThC as a gas (bismuth trimethyl) in the cloud chamber, find close agreement with the K-U theory, even to the large

number of very slow electrons postulated by the K-U theory for elements of high atomic number.

Perhaps it might be mentioned in passing that, at the other end of the periodic table, Fowler, Delsasso and Lauritsen²⁹ find that the K-U end point of the N13 spectrum agrees with the disintegration data, whereas a lower end point does not.

If we attempt to choose between the Fermi and the K-U theory by considering only the upper region of the spectrum, where the scattering contemplated by Konopinski²⁵ is probably very small, we find that in the case of Mn⁵⁶ the two theories can be brought into practical agreement over the range from 1.2 to 2.1 Mev. This is clearly brought out in Fig. 10 which shows on enlarged scale the data used in constructing Fig. 3, together with the best Fermi distribution to fit the data between the limits mentioned.

In view of this, it might perhaps best be said that the cloud chamber is incapable of distinguishing between the two theories, at least in this case.

Confidence in cloud-chamber measurements is further shaken by consideration of the disagreement between the chamber and the β -ray spectrograph reported by DuBridge and Marshall.³⁰ Their work shows that the cloud chamber can make some electrons appear to have more energy than they really possess.

The agreement between the 1.7-Mev γ -ray line and the difference in energy of two groups in the Mn⁵⁶ spectrum is quite striking, and would seem to support the K-U analysis of this spectrum.

²⁵ E. J. Konopinski, Phys. Rev. 57, 68 (1940).
²⁶ A. W. Tyler, Phys. Rev. 56, 125 (1939).

I. L. Lawson, Phys. Rev. 56, 131 (1939).
 H. O. W. Richardson and Alice Leigh-Smith, Proc. Roy. Soc. 162, 391 (1937).

²⁹ Fowler, Delsasso and Lauritsen, Phys. Rev. 49, 561 (1936).

³⁰ L. A. DuBridge and J. Marshall, Phys. Rev. 56, 629 (1939).